Theory of Elasticity

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Lecture - 57
Photo - Elasticity (Contd.)

Welcome, this is the fourth lecture of our Theory of Elasticity's, module number 11. In the last three lecture, we have introduced Photo-Elasticity and some basic concepts of optics. Essentially these optics theory boils down to our knowledge of simple harmonic motion. So, basically we learn two things mainly; that when you have a simple harmonic wave and if it is coming from two such direction in a perpendicular direction then if you superpose those waves and then what will be the resultant wave and then if these two waves are in a plane then what if the superposition will lead to.

So, these two theories or these two wave superposition we have learned. Another, the reason we learned this because the photo-elastic experiment uses the instrument called polariscope and polariscope is based on these principle, the superposition of wave. So, what happens is when you basically put a photo-elastic material under means testing in a polariscope; then it shows a characteristics of birefringence.

The birefringence is that the material when not stressed or not loaded essentially it shows the isotropic optical property but when stressed it exhibits the anisotropic optical property. What does this anisotropic optical property means? The anisotropic optical property means that the refractive index in different direction of the material is different. But when you remove the load and when it become unstressed condition then refractive index remains same in all three directions, all three mutually perpendicular directions.

So, this property is essentially known as the temporary double refractive refraction or temporally temporary doubly the refraction. And this kind of material is suitable for photo elastic experiment and this phenomenon is known as the birefringence. So, basically with this property the whole photo elastic experiments rely or it uses that this property.

And this type of experimentation or this type of photo elastic experiment is also known as the transmission photo elasticity. So, we will discuss what is non transmission photo elasticity in the last lecture in a brief outline will give. And then once we are equipped with these basic optics in this lecture essentially, we will find out the what is the theory of photo elasticity and what are the basics laws involving it.

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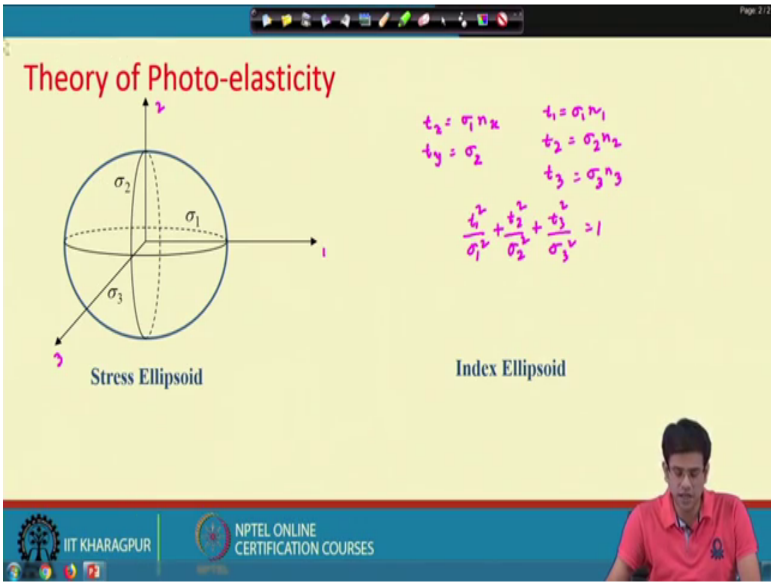

So, the basic laws in photo elasticity is the stress optics law. So, before we get into stress optics law; so stress at a point essentially has 6 independent component. And then if these components coincides with the orthogonal or the principal direction essentially; then shear stress vanishes. And the stress at a point and be represented in terms of sigma 1, sigma 2 and sigma 3. So, sigma 1, sigma 2, sigma 3 are essentially the principle directions and the stress at a point can be expressed in terms of three principal stresses.

Now, these three principal stresses the resultant stress also we can express and these resultant stress is essentially your through the Cauchy's formula, we can express. For instance, the resultant stress that is attraction along x direction is sigma 1 n x. And sigma t y is sigma 1 sigma 2 n y or we can just simply write it in terms of this thing also; t 1 is essentially sigma 1 in 1 and sigma t 2 is sigma 2 in 2 and so on.

So this sigma 1, sigma 2 and sigma 3, and t 1, t 2, t 3, represents a stress ellipsoid. So, if you manipulate these are if you try to remove this n 1, n 2, n 3 that is your cos x or cos the direction cosines essentially, the angle between x and n axis or one direction and the plane of normal, the normal direction. So, this n 1, n 2, n 3 if you try to remove and then express this t 1, t 2, t 3 in c1 say sigma 1, sigma 2, sigma 3, then it represents an equation of ellipse.

And this equation of ellipse is essentially t 1 squared, sigma 1 squared and plus t 2 square sigma 2 square and t 3 square into c sigma 3 squared equals to 1. So, this equation of ellipse is essentially this stress ellipsoid. So, for this each axis of these ellipsoid represents the value of principal stresses. So, this is another definition in terms of principal stresses at stress at a point.

Now, this concept can be in case of a birefringent material or the material which exhibits the anisotropic behaviour of refractive index temporarily at least. So, these can be extended to index ellipsoid or this formula is essentially valid for your index ellipsoid or this formula can also be equivalent to the refractive index.

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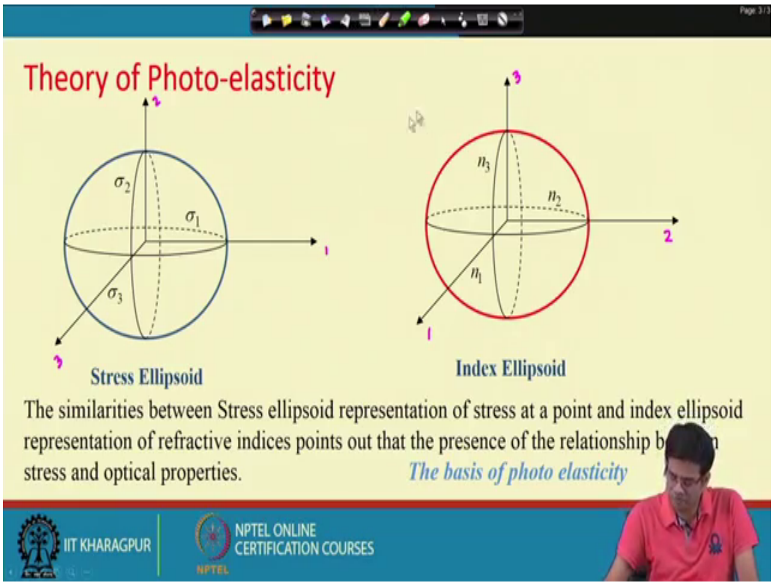

So, if you have the refractive index of n 1, n 2 and n 3 are three directions. Then this can be represented in n 1 this index ellipsoid is very similar to the stress ellipsoid. So that naturally these two things are very much coordinated. So, what I mean the correlation is that if you know the index ellipsoid you can have the stress ellipsoid or vice versa. So, that means, the stress ellipsoid and index ellipsoid are related and this is the basic principle of photo elasticity or essentially the transmission photo elasticity.

So, the similarities between stress ellipsoid representation of a stress at a point and index ellipsoid representation of refractive indices at a point that presents that points out the presence of a relationship between stress and optical properties. So, it is naturally because in the last class also we have discussed this; that the material which exhibits or which shows the temporary double refractive properties or temporary double refraction properties. Then this kind of material is when it is loaded shows that anisotropic behaviour of repair refractive and indices. So, naturally when you load that body is stressed. So, naturally this stress is connected with the refractive indexes of the body.

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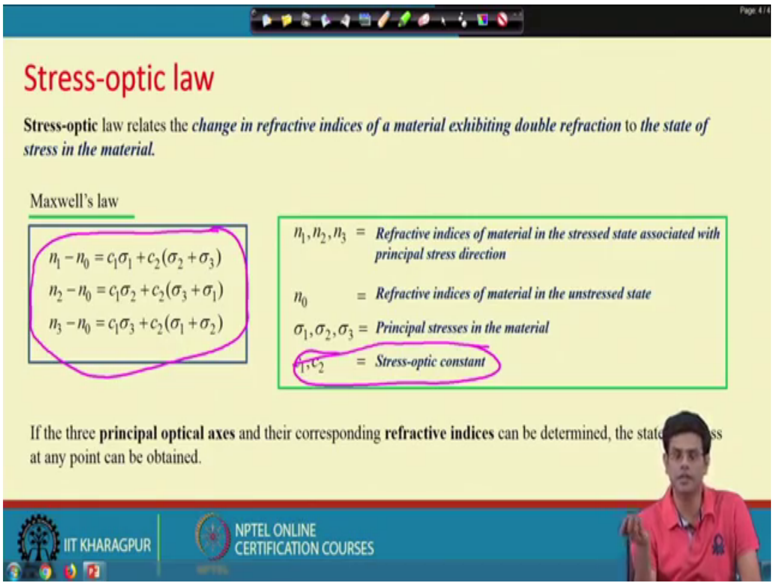

So, this finally, leads to the Stress-optics law in photo elasticity and this stress of this law is also known as a Maxwell's law. So, this stress optics law relates the change in refractive indices of a material exhibiting doubly double refraction or double refraction to a state of stress or in material. So, essentially what it means that that refractive indices are related with the stresses. And how it is related the relation is that if you have n 1 n 2 and n 3 are the three refractive indices and the difference with the absolute n 0 that is the refractive index of material in unstressed rest.

So, so relative change in the refractive indices when the body is stressed it is equals to c 1 sigma 1 and c 2 sigma 2 plus sigma 3 so along the one direction. Similarly for the other direction is c 1 sigma 2 plus c 2 sigma 3 plus sigma 1. Similarly n 3 minus n 0 is c 1 sigma 3 plus c 2 sigma 1 plus sigma 2 so this change or this relation is essentially known as the Maxwell's relation; and where the n 1 n 2 n 3 are principle refractive indices in this stress state associated with principal stress direction. So, an n 0 is the isotropic refractive indices of the material when the body is not at all stressed.

Now, sigma 1 sigma 2, and sigma 3 are principal stresses in the material and c 1 and c 2 are stress optics constant. So, this is a material constant essentially this c 1 and c 2. So, this constant is a material property. So, stress optics constant. So, this three thing if we know or now the refractive indices and the stress optic constants, then we can find out the principal stresses with these three equations so this is the basics basic essence of the Maxwell's stress optics law. Now if three principal axis and their corresponding refractive indices can be determined in the stress state of stress at any point can be obtained. So, this is the basic formula or the basic laws of relation between stress and refractive indices.

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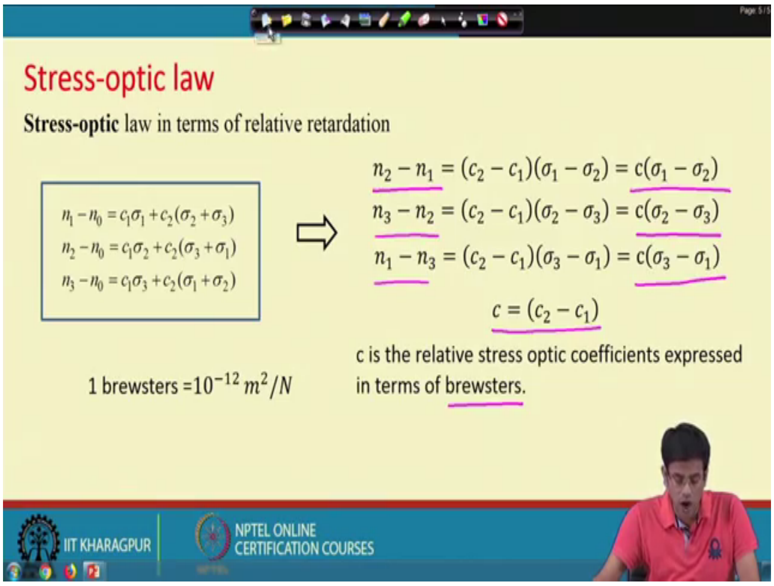

Now, with this law we can essentially modify the stress optic law in terms of retardation. So, what does this retardation means is that when if I just now represent these stress optics law in terms of n 1 n 2 and n 3. That is we try to remove n 0 that is the unstressed refractive indices then the stress optics law represents.

This and this c is known as the straight relative stress optics coefficient; which is c 2 minus c 1 and this is expressed in terms of Brewster. The Brewster is the units of stress optic coefficient so essentially it is 1 by stress if you can see from this so it is meter square per Newton So, this blue star is essentially the unit of stress optics coefficient.

So, finally, my stress optics law in terms of relative refractive indices is this. So, c into sigma 1 minus sigma 2, c into sigma 2 minus sigma 3 and c into sigma 3 minus sigma 1. So, this the stress optics law is used essentially for finding out the stress difference or principal stress difference in case of a isochromatic fringes or isoclinic fringes so that we will see later.

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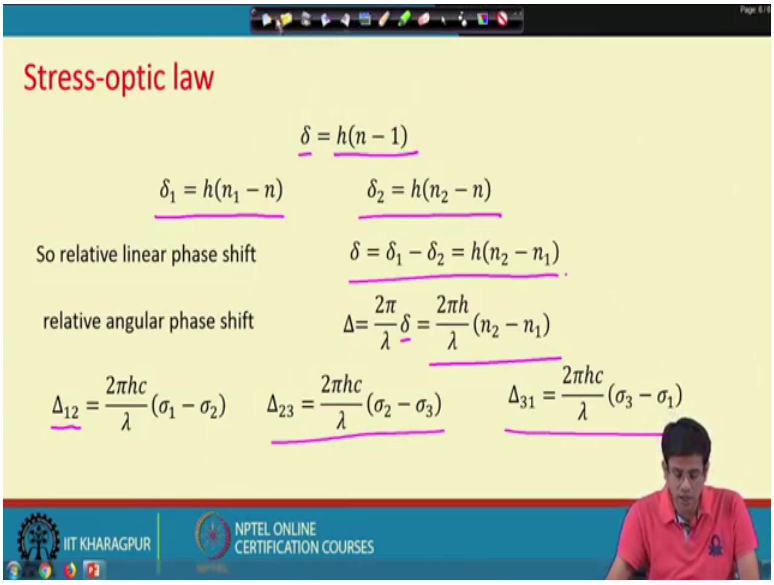

Now, once we know this we can also find out in terms of retardation. How the retardation means. So, as we know from the phase shift in a absolute air to the material if the refractive indices is, is n then the linear phase shift delta I can expressed in terms of this.

This is very simple formula because if the light is propagating through the material then delta is essentially h by if h is the thickness of the material h, h by v is the delta. And then h by v can be replayed the delta is essentially h by v minus 1 that is h because the relay it is the relative distance. Now, similarly, if there are two material the two axis in two axis delta 1 and delta 2 or phase shift. Then the relative phase shift can also be expressed in terms of these two expression.

So, essentially delta is the relative phase shift and there are delta 1 minus delta 2 is h n 2 minus n. So, this n 1 and n 2 are the medium refractive indices or the direction of refractive indices. If it is 1 and the n 1 is representing the one direction one refractive indices, n2 represents the direction 2 refractive indices. So, linear phase shift similar to the angular velocity we can also define a term called angular phase shift. This angular phase shift is essentially 2 pi by lambda into the linear phase shift.

So, these 2 pi by lambda and delta I can express in terms of 2 pi h lambda n 2 minus n 1. Now if you use this and with the previous stress optics law that we have discussed in the previous slide. So, then delta 1 2 for n 2 minus n 1 we can write it in this form. So, this is essentially the stress optic flow in terms of retardation. So, what does this delta 1 2 means? Let us see was does delta want mean.

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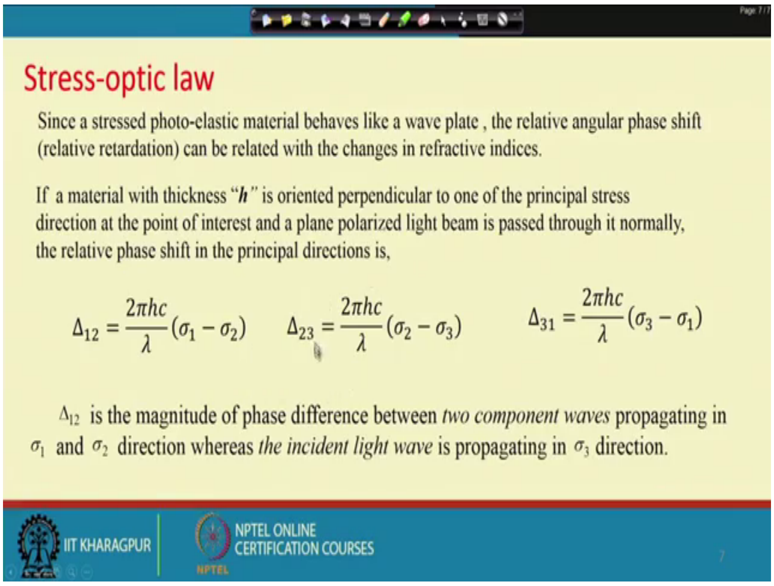

So, since a stress photo elastic material behaves like a wave plate. So, what does this mean this means that it shows a different refractive indices or essentially the propagation of light in different direction is different that means, since the refractive indices becomes different in different direction so the velocity of light essentially becomes different and then the distance travelled by the light within the medium in different direction becomes different.

So, the relative angular phase shift or relative retardation which is dependent on the refractive indices on that on the different axis can be related with the changes of the refractive indices. So, if a material with thickness h is oriented perpendicular to one of the principal stress direction at a point of interest. And plane polarized light beam is passed to it normally then the relative phase shift of the principal direction is this.

So, what does this delta 1 2 means delta 1 2 is a magnitude of phase difference between two components of the wave propagating in sigma 1 and sigma 2 direction whereas, the incident light wave is propagating in the sigma 3 direction. So, essentially delta 1 2 means that it is the phase shift between the 2 orthogonal direction when the light is propagating along sigma 3 direction or essentially the third direction light is propagating.

And the two orthogonal direction what is the relative phase shift, so this is the sense of the stress optics law. So, stress optics law here we have expressed in terms of retardation; that is, the change or the how slow the light is along to orthogonal directions. But this is remember that if the refractive indices are not different along the one and two direction the phase shift will not happen. Because the stress optics lost rate states that the refractive indices or the relative refractive indices is proportional to the relative change in the principal stresses.

So, essentially delta 1 2 will be 0, if the material law is unstressed because the refractive indices will be isotropic in all direction. So that means, so the birefringence property or the property of double E refraction along different directions is essential for the photo elastic experiment.

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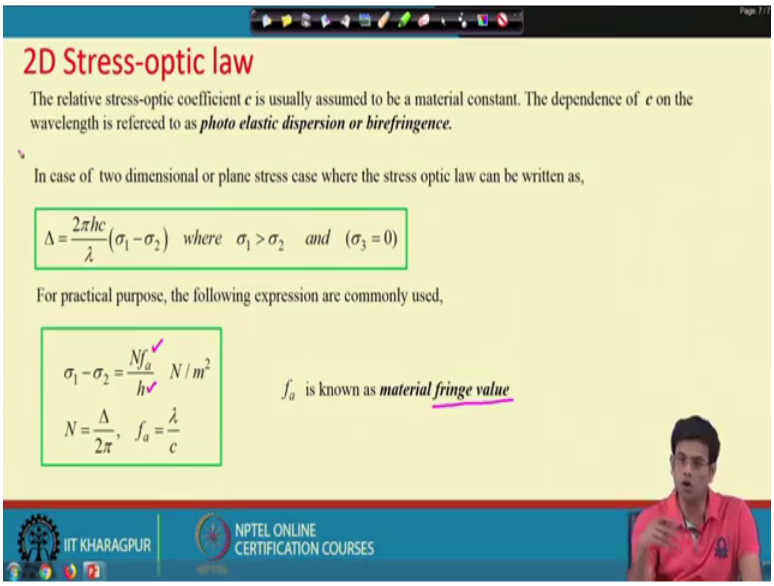

So, now in case of a 2 d plane stress problem we can just simply write it in this form. Because in a sigma 1 and sigma 2 if we take sigma 3 then this phase shift can be written as 2 pi h c by lambda and sigma 1 minus sigma 2. Now this 2 pi h c by lambda is where c the your Brewster. So, Brewster is not commonly used so what we write is that lambda by c, we define first which is known as the material fringe value.

So, this lambda by c is essentially material fringe value, which is a material constant essentially; so material property of the photo elastic material. So, what it states that how the light will propagate through a photo elastic material? So, now if we write this so this sigma 1 minus sigma 2 we write this N f a by h. Now N is delta by 2 pi and f a is lambda by c which is material fringe value. So, if you know material fringe value if you know the angular phase shift relative angular phase shift on. That means, delta is angular field shift delta by 2 pi if you know and then if you know the thickness of the material.

So, essentially you can calculate the relative change in the principal stresses. And with that you can really find out the principal stresses in different direction. So, essentially this is the stress optics law that we require. Now, this can also be extended for the if you know if you want to have the purely elastic material for instance which is not true always because we see the viscoelastic behaviour of photo elastic material. So, even if we assume under low stress state or the relatively elastic regime purely elastic regime. So, plane stress state we can find out the strains with this laws because let us see how we can find out the strains.

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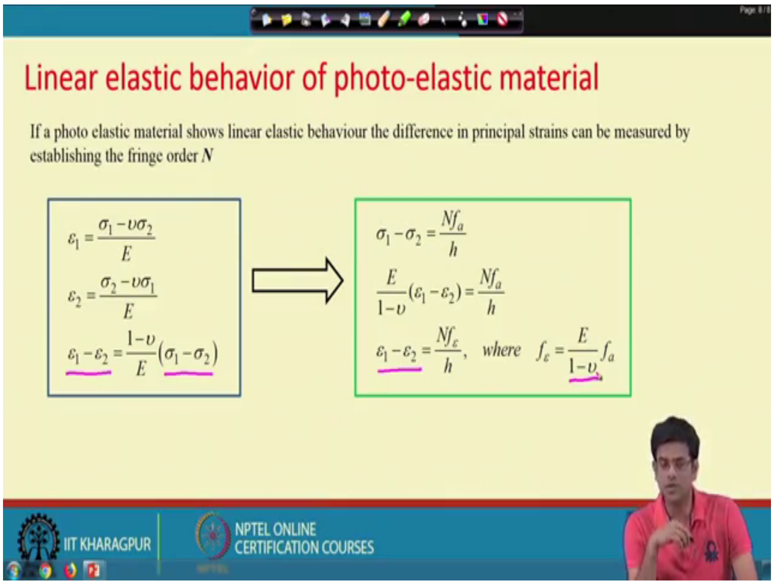

So, this is it if a photo elastic material shows linear elastic material behaviour then difference of principle strain can also be measured. So, we know from the basic elasticity relation what are the stress strain relation. So, this stress strain relations can be expressed in terms of this. So, these epsilon 1 epsilon 2 is the principal stress a principal strain and which is related with the principal stress sigma 1 and sigma 2.

So, this the difference in principal stress sigma a epsilon 1 and epsilon 2 can be expressed in terms of principal stresses difference in principal stresses. So, this if we plug into these stress optic law finally, then we can find out that difference in principle strain is in if epsilon by h. Now N epsilon is essentially the material fringe constant with a material parameter involving it. So, that E by 1 minus nu 1 minus mu is the plane stress and for plane stress condition.

This v by 1 minus nu into f a is essentially your material fringe constant for stream principle strain difference. So, finally, we can find out the difference in principle strain from the stress optics law. So, what are the basic things that we learned here now? We learned here the two important concept; one is that the stress at a point can also be expressed in terms of principal stress principle stress; which is the very common argument.

Because if the directions the orientation of the axis in such that it is coincides with the principal strain situation. Then the resultant stress or the stress vector essentially or distraction vector we can also express in terms of these principal stresses through the Cauchy's principle or the Cauchy's law. So, with this Cauchy's law we can express the attraction and the stress in terms of normal. So, now, this expression can also be if we try to remove N1 N2 N3 or the direction cosines in different directions.

So, we can get a relationship between the traction and the principal attraction values and the principal stresses. So, this relation represents an equation of ellipse and this ellipse is known as the stress ellipsoid which represents stress at a point. So, now similar to this phenomenon it has been observed that refractive indices also follows some ellipsoidal description. So, which is in case of a principal stress it is N1 N2 and N3 so it is known as the index ellipsoid.

Now, this stress ellipsoid and index ellipsoids are called coordinated and this is the basic principle of photo elasticity. Because in a photo-elastic material when you stress it, it represents anisotropic behaviour of the refractive indices, what it means? The means that refractive indices at different directions are different. Now this property goes off when you unload the material or you remove the stress so it becomes a isotropic refractive indices.

That means, at all direction the refractive indices are constant. Now this property is known all also known as the birefringence. So, based on this birefringence property we derived the stress optics law or it is known as the Maxwell's law. Which essentially states that the relative changes of the refractive indices is proportional with the relative changes in the principal stresses. Now this stress optics law can also be expressed in terms of your retardation.

That means, if light propagates in one direction and then what is the what are the retardation in two other orthogonal direction. So; that means, we can compute the phase shift between the two orthogonal direction. So, we have derived the linear phase shift we have derived the angular phase shift. And with this angular phase shift a linear phase shift we finally, write the stress optics law in a material with the help of material fringe value.

And the material fringe value is essentially c by lambda by c. And then this or c by lambda whatever is that. So, this material fringe value can essentially represents the how light will propagate through photo elastic material. So; that means, the material which has to be the birefringent capability has to be there for photo elastic experiments.

So, now, this in is essentially the angular shift or relative angular shape which is delta by 2 pi and then h is the your thickness of the material. Now with these values we can essentially calculate the difference between principal stresses. And if the material is purely elastic material we can also find out the principal difference in principal strength. So, in that case the principal the material fringe constant will be modified in terms of material constant which is f epsilon equals to E by 1 minus nu into f a.

So, I stop here today so in the next class actually you will try to learn what is Isochromatic fringe and isoclinic fringe. And what is the effect on this stress optics law and how essentially we can calculate the fringe isochromatic fringes so. And what are the things that will it will depend on. So, I stop here for today.

Thank you.