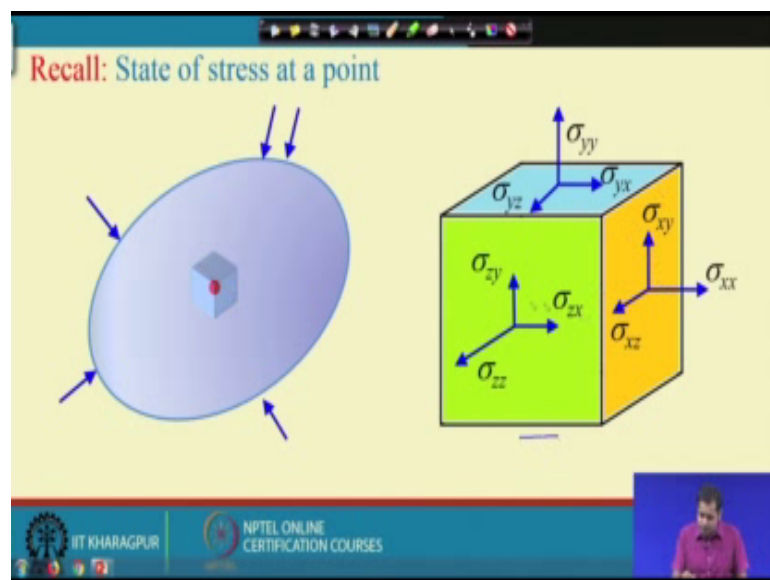


Theory of Elasticity
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Lecture – 09
Concept of Stress and Strain (Contd.)

Hello everyone. This is the 3rd lecture of this week. Today's topic is Traction, but before going into that to start with let us spend a couple of minutes time to revise whatever we have learnt so far in the last two classes.

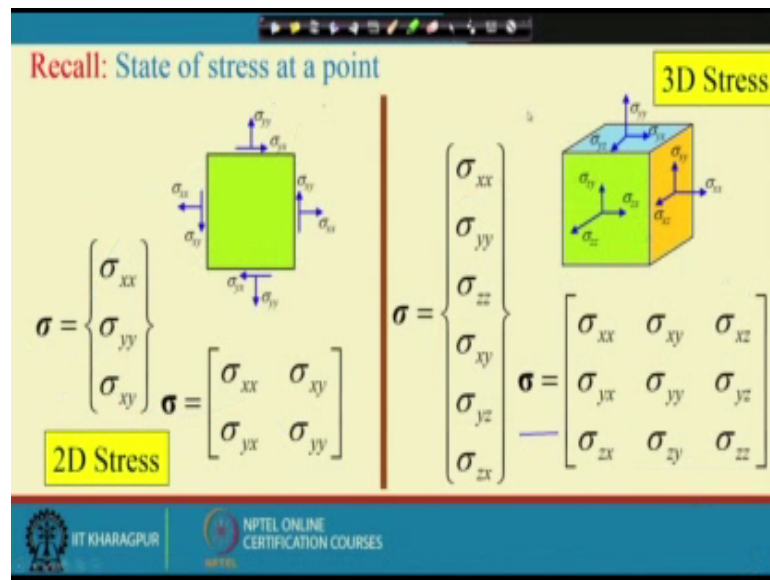
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Ok. What we have seen is that we define, what is stress at a point. If we take a three-dimensional object which is which is subjected to some kind of external loading and we discuss the stress is a way is a response of the material to the external agitation ok. Now, if we take a point say up any point here and if we want to represent this stress at this point, then we consider infinitesimal volume element around this point and then, these stresses are represented as this ok.

So, this is essentially these volume elements. So, this element is essentially this volume element ok. Now, stresses are represented on 3 planes of this volume element; in the other 3 planes also you have the same plane same stresses whatever stresses we have this on this plane is it is mirror plane. We also have the similar stresses which is not shown here for clarity ok.

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Then, we also discussed that the stresses at a point can be represented in various ways. This is the graphical representation, a schematic representation of the stress on one element and then, these stresses can be represented in a vector form like this and then a tensor form like this.

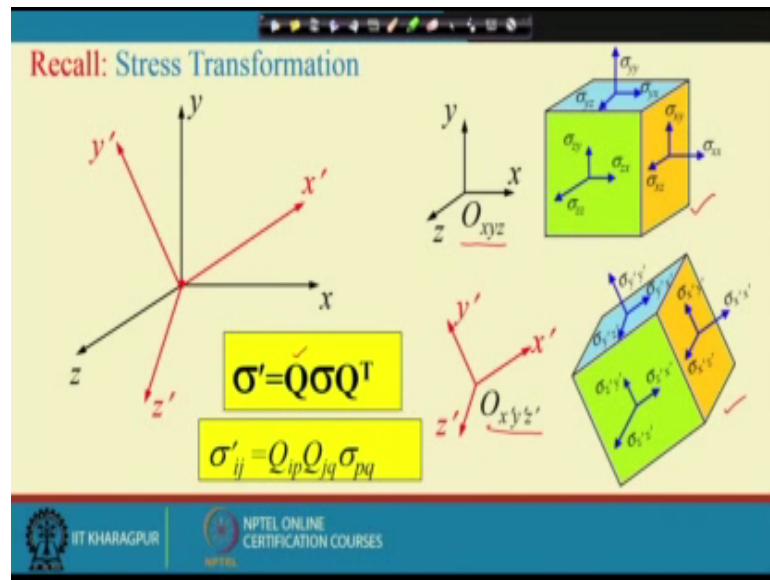
Now, if you see these there are total 9 components of stresses; but if you remember, we discussed because of the momentum balance, the off-diagonal terms in this stress tensor are symmetric, the shear terms are symmetric. So, essentially we have 6 stress components. All the diagonal stresses are the normal stresses on 3 planes and the off-diagonals are the shear stresses on different planes.

Similarly, if I have if I represent the stress at in a two-dimensional object, then the other components are 0; but here you make a star and you see there is nothing like two-dimensional object; all objects are essentially in 3 dimensions. But in some cases, we can idealize this object, idealize the material, idealize the structure as a 2-dimensional structure.

So, when you have how we idealize; how we idealize that three-dimensional object into 2D, there are depending on that we have we have some principle called Plane Stress Plane Strain Theory will discuss that in a subsequent week.

But, to start with do not consider these 2 dimension representation of the state. This is a general representation of the stress and how to project this 2D's, this how to project these 3 D of 3 D stress the presentation to 2 D depending on the idealization, will discuss that in the subsequent week ok.

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Now, we also discussed that if we whenever we represent a stress; for instance this is the stress it is always associated with certain coordinate axis ok.

So, we choose a coordinate axis and with respect to that coordinate axis stresses are measured and stresses are represented. Now, if we have a point here. Suppose, this is the this is the point; then at this point, we can have at this point we can have say infinite number of coordinate axis orthogonal coordinate axis.

For instance, if you take any arbitrary coordinate, suppose this prime coordinate. Now, if we now and the and the stresses are suppose stresses are represented with respect to prime coordinate as the sigma prime O_{xyz} is the xyz coordinate system and $O_{x'y'z'}$ is the prime coordinate system.

So, this is the representation of stress with respect to this coordinate system and this is the representation of stress with respect to this coordinate system. Now, these 2 representation can be related to each other through a transformation matrix and these transformation matrix is Q . Now, Q is the transformation matrix ok. Q is Q has a

property is an orthogonal matrix it is an orthogonal transform by orthogonal transformation.

Now that also we discussed and then, if you see if I have to write this in indicial form, then this form becomes this is the written in indicial form. So, if we know the stress at a point with respect to any coordinate axis, then if we can have infinite number of coordinate axis at the point and then for any coordinate axis, we can obtain the stress components by this transformation.

Now, you see there are now there is next what we learned is, next we learn the concept of stress invariant.

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Recall: Stress Invariants

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

$$I_{\boldsymbol{\sigma}} = \sigma_{ii} = \text{tr } \boldsymbol{\sigma}$$

$$II_{\boldsymbol{\sigma}} = \frac{1}{2}(\sigma_{ii}\sigma_{jj} - \sigma_{ij}\sigma_{ji}) = \frac{1}{2}[(\text{tr } \boldsymbol{\sigma})^2 - \text{tr}(\boldsymbol{\sigma}^2)]$$

$$III_{\boldsymbol{\sigma}} = \epsilon_{ijk}\sigma_{i1}\sigma_{j2}\sigma_{k3} = \det \boldsymbol{\sigma}$$

The slide also features a diagram of a stress element with normal and shear stresses, and logos for IIT Kharagpur and NPTEL ONLINE CERTIFICATION COURSES.

Now, you see for instance for instance if I take an object if I take suppose this is water body and if I take some object here ok; what would be the pressure on this object? See hydrostatic pressure on this object is these are the pressure on this object right. This is the pressure on this object.

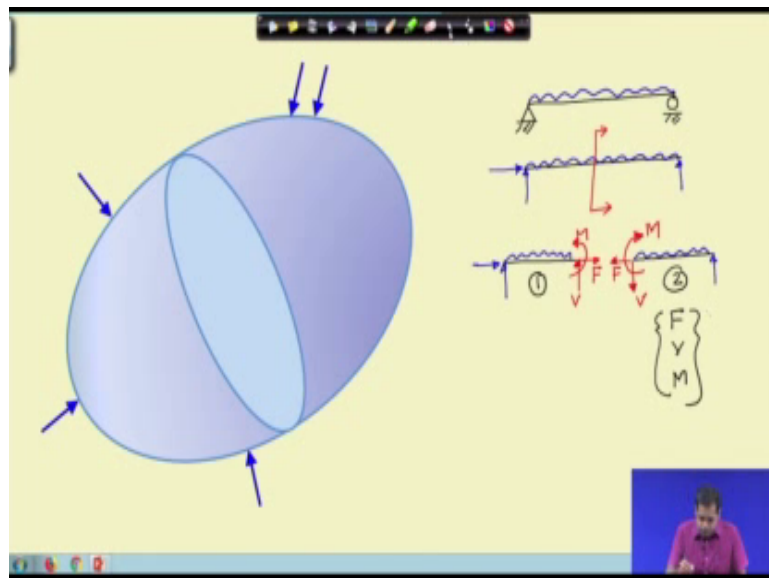
And this pressure do not have any, this pressure does not have any specific direction, any direction. We take this pressure is same. Therefore, if I take this coordinate if I take this coordinate axis or this coordinate axis or any other coordinate axis this pressure is going to be same. So, pressure this pressure remains unchanged do not vary way even if we change the coordinate axis. So, this is called the this is called Invariant.

So, for instance the energy, now energy of a system; if in any coordinate system, we take energy this energy at a point energy and a small volume do not change right. Now, the thing is energy depends on the stress. So, energy can be explained in terms of stresses. So, some combination of stress gives you energy and therefore, some combination of stress must remain unchanged even if you change the coordinate axis.

Some combination of stress gives me pressure and therefore, since pressure is the by definition, it should be unchanged even if you change the coordinate system. So, some coordinate some combination of this stress component should remain unchanged and those combinations are called Stress Invariant. There are 3 stress invariants for a symmetric second order tensor here.

So, we have the first invariant which is the stress of this matrix or the tensile second invariant is the sum of the cofactor and the third invariant is the determinant of a sigma; that is also discussed we have not discussed the physical interpretation of these invariants, will discuss shortly. It was the end of this, end of this end of this week. Now, so this is we discussed in the last 2 classes.

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Now, let us understand what is Traction? What is the topic, today we are going to discuss. Now, first try to understand traction from what is the physical interpretation; physical meaning of traction. Then, once we understand that then, will see how the traction can be determined by knowing the stress at a point ok.

Now, consider three dimensional object here and take any point suppose take any cross section, before that you see if you recall your first year mechanics or structural analysis, if we have a say if we have a any beams; a simply supported beam consider a simply supported beam and which is subjected to some external loading, some external loading right.

Now, if we draw the free body diagram of this beam; the free body diagram will be something like this. We have external loading on this and then, we have reactions at this end is a hinge support that is where 2 reactions and the roller support we have one reactions ok. And then, now if we divide the beam into if you take a section at this point and this point if you take a section and divide the beam into 2 parts ok.

Now, this is the one part and this is another part ok. Now, what is the free body diagram of first part, free body diagram of first part will be this and we have external loading on this and then, in addition to that we have an axial force on this and then, we have shear force and then, we have moment right.

And what is the free body diagram of the second part external load external loading and in addition to that we have internal forces. Internal forces this and then, shear stress like this and then moment like this ok. If it is V , it is F ; it is minus V , it is V ; it is F and it is M and this is M .

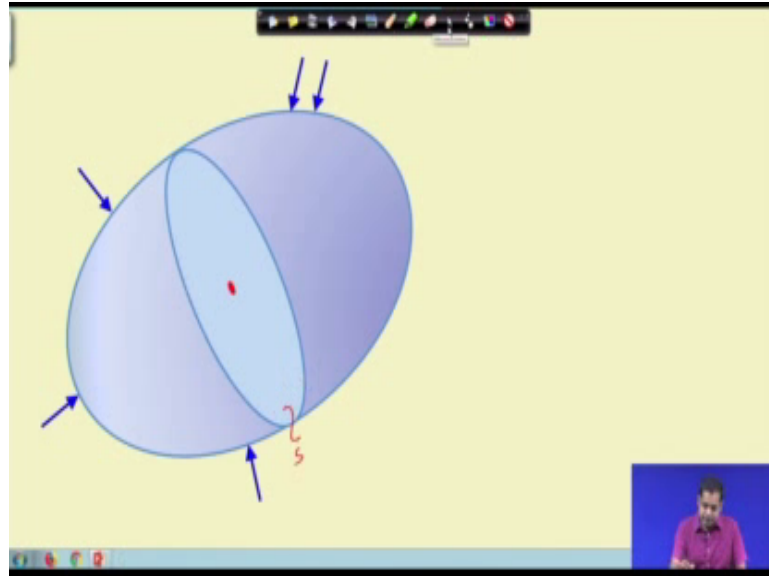
So, what this M V and F ? If we have a if we define a field like this, if we define a field like this F V and M ; what essentially they mean F V and M ? It means that if we have 2; this is part one and this is part two, then these two part if we divide the entire structure into two part; part one and part two, these 2 structure these 2 structure interact with each other through these internal forces or through this field, they interact with each other.

Now, when they interact with each other it when I say the body is in equilibrium every point in the body is in equilibrium; so, is this point as well. So, in order to maintain the equilibrium, this force and this force should be opposite direction; this force the shear force and shear force at this plate should be an opposite direction, equal and opposite and same is the same is for bending moment. So, therefore, the equilibrium is maintained.

So, these two object interact with each other through this through this field right. Now, similarly if we take and any three-dimensional object and if we cut it, then these 2 object

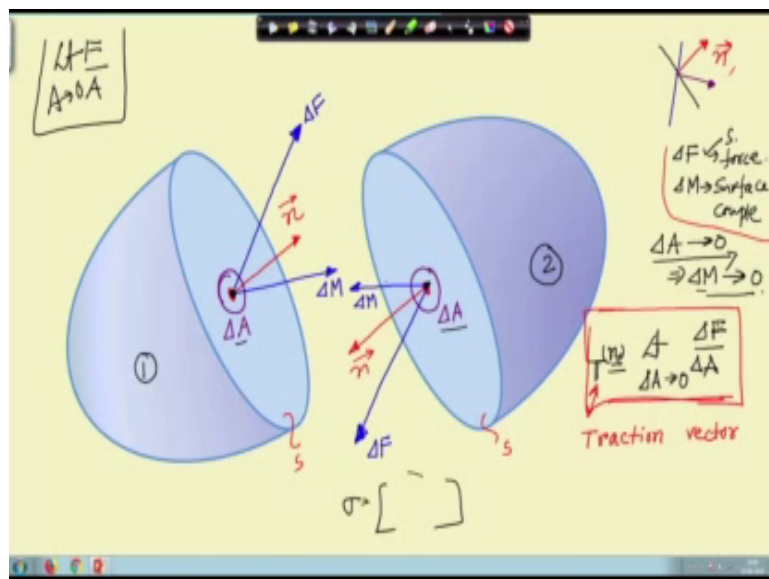
should interact with each other through some forces and couple right on this plane ok. Now let us see that.

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So, let us take a small point here ok, any small point and then divide this entire object into two parts being this as the common plane ok. Now, suppose this plane is S this plane is S ok. Now, let us divide it into 2 parts; this is 2 parts.

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So, this plane is S; this is S ok. Now this is important when we say a plane is defined by and by a vector which is normal to this plane which is called normal vector.

For instance, if this is a plane, this plane is defined by a vector called n . This is the normal vector. This is defined the same as this in n normal vector. For instance, if we have say this is one plane, this is one plane, this is defined by a normal like this ok. So, if we know this, if you do know this normal n ; then, this n define this plane.

Similarly, if we change the plane, if we have a another different planes suppose this is the plane; then a different normal vector define this plane ok. So, at what is the cutting plane that plane is defined by this normal vector ok. Now, so at this at any point though the point we can keeping the point same, we can have infinite number of points.

So, in this case two things are important; at what point we are we are at what point and along the point what plane, we have taken into account. If we change the plane, then at the same point we can have different the traction that we are going to define; then we can have different traction at the same plane at the same point if we change the plane. So, 2 things are very important; one is what is the point at which you are calculating the traction and the second thing is what is the plane you are considering?

The plane has to pass through the point, but again there are many planes can be can be considered passing to the point, but which particular plane we are considering that is important and that is defined by this normal vector n ok. Now, consider small area say a small area on this around this point consider a small area and consider small area.

Suppose this area is δA δA ; this is δA finite area small area δA ok. Now, if when these two will interact on this surface, we have a force like the previous one; we have a distributed forces and couple through which these 2 body is this 2 body can interact with each other right.

Suppose, they are this is suppose F ; this is $F \delta A$ and similarly this is δF , this is δF ok. Now, remember one thing if it is δF force and this should be in opposite direction. So, it is minus δ , but we are already showing this is in opposite direction that is why the minus sign is not written here ok.

Now, similarly we can have a couple as well. Suppose that couple is or it will use some suppose that couple is at this point is δM , δM . Similarly, we can have couple at this point is δM δM ok. Now, this is very important here. What our assumption is now these 2 object; object number 1 and object number 2, the part 1 and part 2.

Now, part 1 and part 2 will interact with each other through ΔF and ΔM which is defined on a small area around small area surrounding this point, at this at this point. Consider at this point, but a finite area ΔA ok. Now, what happens that when ΔA tends to 0, now you as you remember the definition of stress we defined? The definition of stress was force by area, force by area.

But that was not the complete definition of stress. The definition of stress is force by area with limit area tends to 0 that was the definition of stress right and that definition you studied in your class 2 as well. So, now, in order to define stress, we need to consider the area which is very small ok. Now, when ΔA tends to 0, when this ΔA tends to 0, this ΔM , ΔM this couple, the surface couple this also vanishes ok.

So, now when ΔA tends to 0, then these couple vanishes. So, then the interaction between this 2 object you know or the 2 part can be defined only through ΔF right. So, if we define a quantity. So, T_n ; for instance T like this which is limit ΔA tends to 0; ΔF by ΔA .

Let us define a quantity like this which is which is essentially in dimension wise same as stress force by area with area tends to 0. Now, this point is very important. This is we said that this interaction only through only through ΔF , this is possible when ΔA tends to tends to 0 right. Then, we can have this the effect of the couple the M is called Surface couple. M is called surface couple and this one is Surface force. This is Surface force ok.

Now, because of this surface couple, we can have we can have we can also have stress which is called couple stress ok. Now, in this case when ΔA tends to 0 as ΔM vanishes, the couple stress also vanishes. So, then in that case the interaction between these 2 body is only through ΔF . Now, if we define a stress like quantity say T as ΔF by ΔA with the limit ΔA tends to 0, then use n here; use n here just to say that these forces and area on a plane which is normal to n which is defined by normal vector n .

Then, this T_n is called, this T_n is called Traction or Traction vector. Traction vector because force is a vector ok. Now, but at this point one thing you can check, we can you can note it, but there are some continuum Theory continuum say advanced solid

mechanics. So, there are Continuum Theory which do take him to consider do they take into account this effect of couple stress.

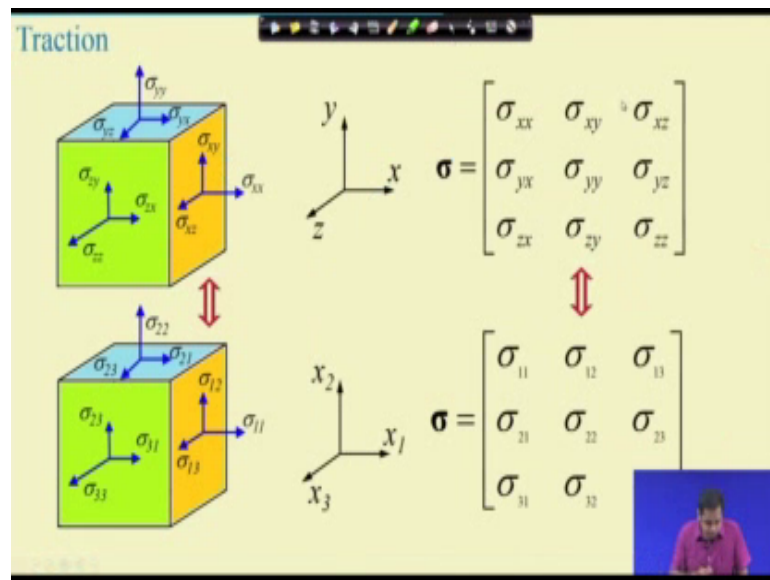
But within the classical continuum mechanics and also in the in the in the context of this subject, these we do not consider ok. Now, that is essentially causes stress principle is. Now, so this is Traction vector. So, Traction vector is essentially traction is dimension wise it is also a force; but that this is defined, force can be the stress when we wrote in a tensor form stress is always at a point which as which can be written which has stress components in three-dimensions.

But, when we talk about traction, traction just not a point traction ah also another important addition to the addition to the definition is the at which plane were calculating the traction and when. So, traction is equal to so this is traction ok. Now, once we have understood what is traction, let us try to find out let us see how to determine interaction.

You see suppose at this point, at this points suppose this is the point of consideration, at this point we have a stress σ right σ ; but then traction is dimension wise it is a stress, but it is not a tensor it is not a second order tensor it is a first order tensor or a vector ok. Because this stress is projected, this stress at a point is projected on this plane and when we project it that projection gives you traction.

So, Traction is a stress like quantity, it is a vector ok. Now, maybe when we compute this traction when we find out the relation between traction and the stress tensor at this point probably things will be clear. So, this is a Traction. So, traction is through which different parts of the body interact with each other. Now, let us move on and then see how to determine traction ok.

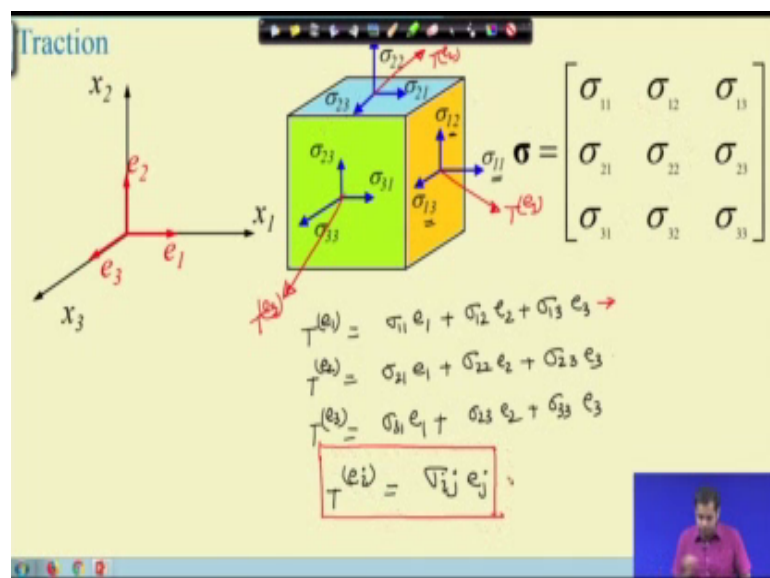
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But, before that let us let us simplify our let us simplify our notation, you see now onwards instead of writing sigma x sigma x y and so on, we will use sigma 11 sigma 12 which will be it would be easier to write. So, sigma xx is this, so, this representation of stress, now become this and this stress tensor is this.

But the way it is represented the remain same for instance sigma 22 is it is a sigma it is a stress component on a plane which is defined which is normal to direction 1 and then, the direction of the stress is 2 and this that definition remains same ok.

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Now, consider. So, this is the this is our stress tensor at a given point and $x_1 \times x_2 \times x_3$ are the coordinate axis. Now, consider $e_1 e_2 e_3$ are the are the basis vector in $x_1 \times x_2$ and x_3 directions ok. You look at the that look at that this T if we have, if we defined say T_1 or T_{e_1} also you can say T_{e_1} , T_{e_1} is essentially what T_{e_1} , we have σ_{11} σ_{12} and σ_{13} right.

So, T_{e_1} is essentially your σ_{11} σ_{11} which is in e_1 direction and then, plus σ_{12} which is in e_2 direction and then, plus σ_{13} which is in e_3 direction right.

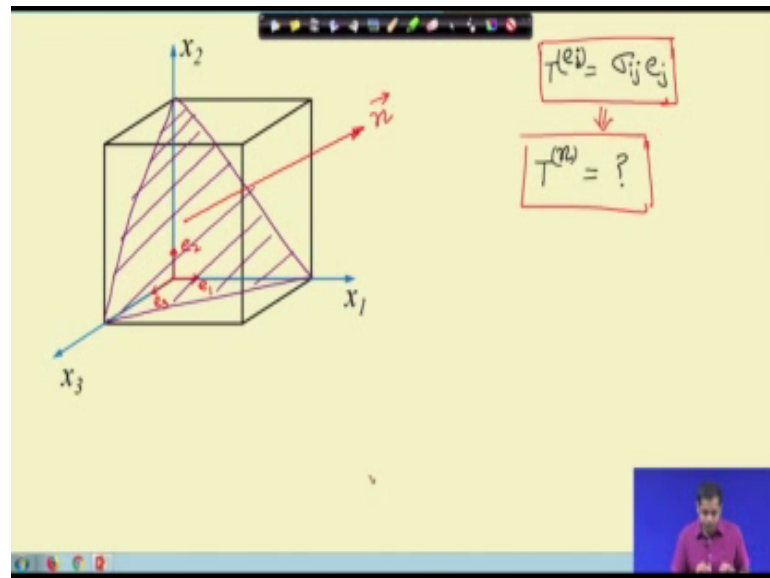
Similarly, T_{e_2} , we can write T_{e_2} which is σ_{21} e_1 direction e_1 plus σ_{22} e_2 plus σ_{23} e_3 . So, if it is the it is the plane it is the plane so, essentially your T_1 is T_1 is T_1 is on this plane. This is your T_{e_1} and then, we have T_2 on this plane T_2 on this plane which is T_{e_2} and then, T_3 will be on this plane, on this plane which is T_{e_3} ok.

So, component of T_{e_1} essentially are written in terms of this basis vector as this ok. Now, similarly we have if we write T_{e_3} T_{e_3} , the components will be e_3 in x direction in one direction x_1 direction is a σ_{31} . So, this is σ_{31} e_1 , this is how we represent and then, σ_{23} e_2 and then plus σ_{33} e_3 ok.

Now, if I have to write this in indicial form then T_{e_i} , $e_i e_i$ could be i , i varies from 1 to 3. So, this could be this will be equal to equal to your σ_{ij} e_j . This is very important definition ok. So, this gives you traction, this gives you traction on this plane; this gives you traction on plane defined by defined by e_2 and this gives you traction on plane defined by e_3 .

So, T_{e_1} , T_{e_2} and T_{e_3} are the traction on different planes which are defined by e_1 , e_2 , e_3 respectively ok. Now, this directly comes from the representation of stress right. So, this definition this expression will be using shortly; this indicial notation.

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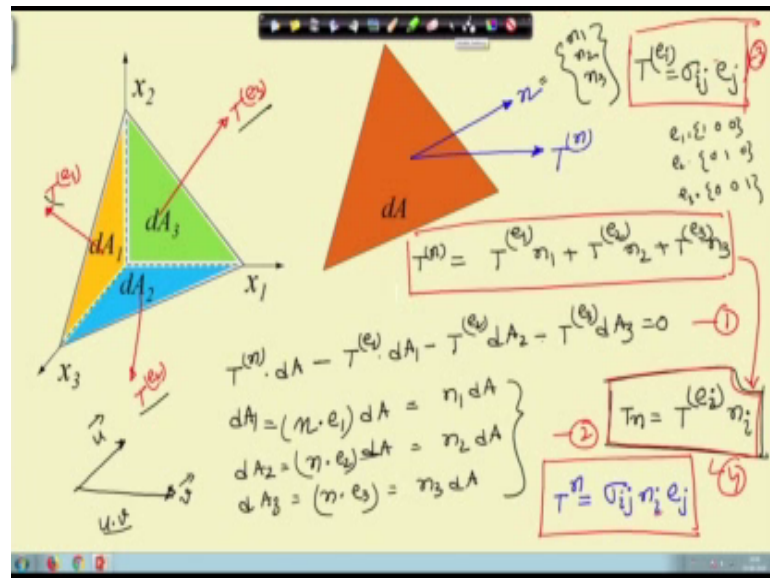
Now, let us let us move on take this is the three dimensional that is a small volume element that we considered.

And we also know that this is your this is e_1 ; this is e_1 and this is e_3 and this is e_2 , this is e_2 right and we also know that traction on this plane is $T e_1$; traction on this plane is $T e_2$ and traction on this plane is $T e_3$, that is now we defined. Now, let us now in order to compute traction.

Now, what we want to know is we know what is $T e_1$, $T e_2$, $T e_3$. We know that $T e_i$ is equal to $\sigma_{ij} e_j$ that is we know, that comes directly from the stress state of stress at this point ok. Now, what we are interested now is suppose consider a plane, consider a plane say cut consider a plane like this. Cut a plane like this, this is the plane ok.

And suppose, this plane is defined any plane is defined by a track by a unit normal. Suppose, this plane is defined by a normal n ok; normal vector n . Now at a point this is very small volume. So, this area is also small. Now what we are interested now is we are interested to find out what is $T n$; that is our objective. But what information we have? We have this information. So, from this how to determine this? So, we know traction on 3 planes and with that information how to find out traction on an any be arbitrary plane ok. Now, let us see that ok.

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Now, this is the this is all this is the plane we consider. Now, suppose this area is dA_1 , dA_1 is the area which is in x_2, x_3 plane and then, this area is dA_2 which is an x_1, x_2 plane means this plane is normal to a normal to e_2 direction x_2 direction and dA_3 is the plane which is in x_1, x_2, x_3 direction.

And then, and then the area which is this is the area this area this is dA ok. Now, what we have is now on this plane, suppose this plane is now normal is n and we want to calculate what is the traction on this plane ok? Suppose, this is T_n ok. Traction on this plane.

But what information now we have is we have this information, if we see on this plane we have we know T_{e_1} , T_{e_1} on this plane, we know. T_{e_2} and on this plane we know T_{e_3} ok. Just now, we wrote and T_{ij} is equal to this T_{e_i} is equal to $\sigma_{ij} e_j$ that is we know ok. Now, if it is in equilibrium, then what are the forces we have? Now we have this, this 3 forces; these 3 stresses which can be converted into forces. This 3, T_{e_1} , T_{e_2} , T_{e_3} on 3 3 mutually perpendicular plane and then, on this inclined plane we have this traction n .

Now, then if we have to write the equilibrium condition, then equilibrium condition says that that total force; force on this plane will be traction into traction vector on this plane into dA which is the area of this. Now limit remember, we are not writing ΔA anymore because we have assumed this area is very small that is why. So, that is why the differential form, we are writing it.

So, dA_1 that should be equal to my its minus T_{e_1} into dA_1 minus T_{e_2} into dA_2 it is just simple force balance. It should be minus; then minus T_{e_3} into dA_3 that should be equal to 0; that is so static, static equilibrium condition ok. Say this is equation number 1; this is equation number 1.

Now, let us find out that if we do we have any relation between dA_1 , dA_2 , dA_3 and dA ? Yes, we do have. Let us see if you remember, then you studied dot product between 2 vectors; inner product between 2 vectors. Suppose, we have a vector. We have a vector say u here and another vector say v another vector say v .

So, if we take the inner product $u \cdot v$; $u \cdot v$ essentially gives you the projection of one onto the another ok. So, when we talk about the projection, then what happens your dimension reduces. So, vector when we take the dot product between 2 2 product 2 vector which is the projection of one vector onto the another. So, essentially it becomes a scalar.

So, similarly if we write, if we take if we say write dot product between say n a dot product use different ok; dot product between n and e_1 , $n \cdot e_1$ they all are vectors. So, I am not explicitly writing the vector $n \cdot e_1$, $n \cdot e_1$ is essentially what? $N \cdot e_1$ is essentially the projection of n on e_1 .

Now, if we multiply this by dA , dA the area of this; then these essentially gives you projection of projection of this area on a plane which is defined by e_1 which is essentially dA_1 . So, dA_1 is essentially projection of dA on this plane which is obtained by just simply taking the dot product. So, this is equal to then $dA_1 = dA \cdot e_1$ ok. Now, suppose n_1 is equal to n_1 is a vector which is n_1, n_2 and n_3 and then e_1 in this case e_1 is what? E_1 is equal to e_1 is equal to say $1, 0$ and then, e_2 is equal to you can check $0, 1, 0$ and e_3 is equal to $0, 0, 1$.

So, $n_1 \cdot e_1$ gives me if it dot product between this and this gives you $dA_1 = dA \cdot n_1 \cdot e_1$ into dA ok. Now, similarly dA_2 can be obtained as n_2 taking dot product e_2 into dA which gives you $n_2 \cdot dA$ and then, dA_3 similarly n_3 into e_3 is equal to $n_3 \cdot dA$. Now with this, with this if I substitute this dA_1 , dA_2 , dA_3 in this expression; what we get is what we get is essentially this.

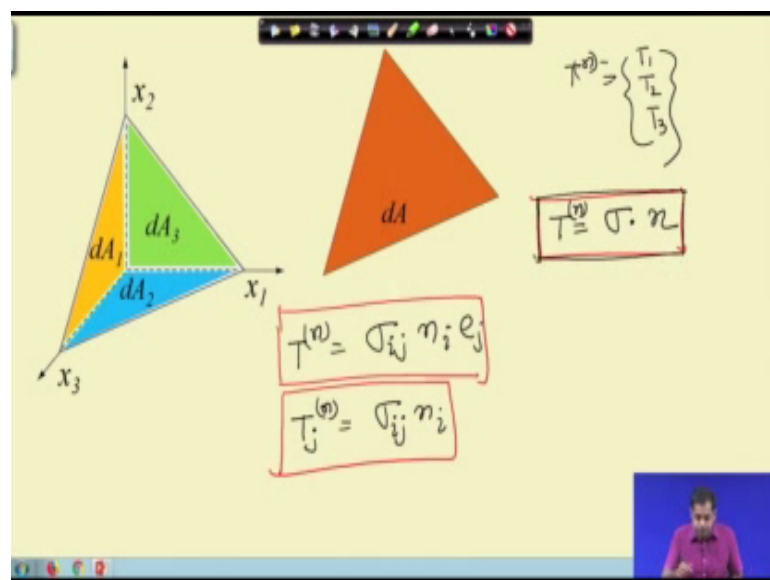
If we substitute this, then we can write it ok, we can write it here. So, from this we get is we get $T \cdot n$; $T \cdot n$, then equal to $T \cdot e_1 n_1$ plus $T \cdot e_2 n_2$ plus $T \cdot e_3 n_3$; you check this is from equation number 1 to equation number 2.

Now, can we write this in initial form? If we write this in initial form then, what we get is $T \cdot n$ is equal to then from this, from this what we get is $T \cdot n$ is equal to $T \cdot e_i n_i$. So, this is we are almost done. So, we have 2 thing here; one is $T \cdot n$ is equal to $T \cdot n$ is written in terms of $T \cdot i$ which is $T \cdot i n_i$, then we have T_i which is written in terms of sigma is a this.

So, if I take this is equation number 3 and this is equation number 4, if we substitute this thing in this expression, what we get is this. $T \cdot n$ is equal to $\sigma_{ij} n_j e_i$. This is very important expression.

You see, so if we have, if we know sigma ij which is defined with respect to respect to coordinate system which is that coordinate system is defined with respect to the basis vector e_1, e_2, e_3, e_j and then, if we take a plane any arbitrary plane which is defined by vector n_i with a normal vector n ; then traction on this plane can be obtained by this now which is.

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So, traction is now let us write it here. So, traction is written as $T \cdot n$ is written at $\sigma_{ij} n_j$ and then n_i into e_i that is what just now we wrote. Now traction $T \cdot n$ is a vector. So, if T

n is equal to T n is equal to if T n is equal to $T_1 T_2 T_3$; then $T_j T_j$ can be written as T_j , the j th component of this can be written as $\sigma_{ij} n_i$; this is the j th component. So, this is the entire vector initial notation in the vector and the initial notation of the each component of the vector.

Now, if I have to write in the if I have to write in matrix form, then we can write that T is equal to T is equal to you see what is essentially this? This if you remember the dot product, this inner product; this is essentially the inner product between σ and n . So, it is inner product between σ and n ok, this is important.

Now, you see if we try to, if we just now as I said that the physical interpretation of inner product is essentially the projection of one into the other. So, if we take σ is the state of stress at a point and if we have a plane which is defined by n ; then, if you take σ dot n the inner product between σ and then, essentially what it gives you? It gives you projection of σ on that plane ok.

So, whenever we have a projection a dimension decreases. So, σ is a second order tensor. When we have inner product with a first order tensor, the resultant is a first order tensor the traction which is traction vector ok, traction vector; so this is traction vector. So, this is this is very important. This is how we can compute traction at a point ok.

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σ

$$T^n = \sigma \cdot n$$

$$T_n = T^{(n)} \cdot n$$

$$\sigma_n = \sigma_{ij} n_i n_j$$

$$\sigma_s = \sqrt{\sum \{T^{(ij)}\}^2 - \sigma_n^2}$$

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Now, with this just one small thing before we close we have stress at a point this is our point this is the point, this is the point stress is defined as σ at that point and then, we have a knob we have a plane which is the area of this area we have a plane which is defined by normal vector n and then, we take a small area dA and then, just now we discussed the traction is traction can be optics traction is $\sigma \cdot n$ or it or it is $\sigma \cdot n$.

Now, then what this is this is traction which is $\sigma \cdot n$. Now, this is essentially a vector. Now if I take a if I take a dot product between this vector and n ; if I take a dot product between T_n $T_n \cdot n$ dot product between T_n and n . Again, dot product between 2 vector inner product 2 vector is the position of one onto the other.

So, inner product between T_n and n gives you projection of T_n on n projection of T_n on n is equal to this n . So, which is the T_n not now please note one important thing, T_n not necessarily in the direction of n . T_n is acting on a plane at that particular point, but it direction could be anything; a direction is not same as they did at the normal direction n .

So, if I take the dot product between these two; then we get the normal component of T_n which is perpendicular to this plane which is σ_n . So, σ_n which is we can say the normal stress. Normal stress is essentially $T_n \cdot n$ ok. Now, if I write in the initial form this becomes that T_n is equal to $\sigma_{ij} n_j$ a $\sigma_{ij} n_j$ and then the dot product become n_j .

So, σ_n is equal to this ok. So, this is the this is the normal stress on this plane; normal stress on this plane ok. Now, what is the now another thing what is the dimension of this σ_n ? Dimension of the σ_n is the dimension of T_n is the first order tensor, n is a first order tensor; inner product between 2 first order tensor gives you a scalar.

So, dimension of σ_n is a scalar. So, it is just the normal stress on this plane. So, if I have to get the shear stress on this plane, suppose this is this is the shear stress σ_s . This is normal σ_n . This is σ_s is the shear stress. So, σ_s will be σ_x will be so if we use this T_n is the resultant force. T_n this is square minus σ_n square. This gives the normal stress on this.

Now, this you can write an initial indicial form ok. So, what let us just briefly summarize what we have learnt; what we have discussed today. We have discussed what is traction and then, physically try to understand what is the meaning of traction and then, if we know this stress at a point, how the traction can be related can be obtained from this stress of a point and with that we have understood that traction is essentially if you know the stress at a point and consider a plane passing through the point, then traction is essentially the projection of that stress onto that plane at that particular point ok.

Now, and that traction is tacked and since it is the projection of that stress, it can be obtained by the inner product between σ and n . We also defined that once you know the traction in a plane, then the normal and shear component on that plane can be written and again using the using the projection of the traction.

Another important thing here please note that traction whenever we call stress, it is stress at a point, if we identify the point, fix the point this stress is defined at that point can we represent at that point. But when we talk about traction, it is just not the fixing the point on that point which plane we are considering that is also important because at the same point we can have infinite number of planes. But, if you have different plane, the traction on different plane will be different.

Though, the state of stress at that particular point is same, but since the planes are different, the projection of that stress on different planes will be different and that is why traction will be different on different plane ok. So, we stop here today. Next class, we discuss what is, the last first 3 classes, we discussed the different concept of stress, different representation and different projection of states and what is the physical meaning of that projection.

Next class, we discuss what is its strain; how when a material undergoes deformation; what are the possible modes of deformation and from the deformation how to how to define strain and what are the different kinds of strain; can we have can we write this strain in a in a tensor form the way we have written, we have we have expressed stress; can we find out the symmetry in that tensor, the way we found out symmetry in stress tensor. Those things we will discuss in the next class ok so next class.

Thank you.