

Water Supply Engineering
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Lecture - 14
Groundwater Intake

Hello everyone and welcome back. So we have been discussing about water abstraction and water intake systems this week and in just earlier class we did talk about various surface water intake. So how we withdraw water from the pond, lake, streams, reservoirs, so that we discussed in the earlier week and what are the various structures that are typically provided for withdrawing surface water.

This week we are going to discuss about the, not this week this class in fact, we are going to talk about the groundwater intake systems. So generally for several towns the if surface water is available that is the preferred sources but if no surface water is available, or inadequate amount of surface water is available then people have to rely on groundwater for meeting the required water demand.

This groundwater can be single source or can be used as an augmentation service for the surface water sources. So if let us say a river or canal or lake is available, but does not provide adequate amount of water to meet the entire demand of the city. So there are many places where partial withdrawal is done from the surface water sources and partial withdrawal is done from the groundwater sources.

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CONCEPTS COVERED

- Types of Groundwater Systems
- Specific Yield and Retention
- Storativity, Transmissivity, and Specific Storage
- Well Hydraulics
- Pumping from Confined and Unconfined Aquifers
- Study and Transient Pumping Conditions

So what we are going to discuss in this particular lecture is what are the different type of groundwater systems. Then what is the specific yield and retention? Then, we are going to talk about some of the features and properties of groundwater systems like what is the storativity, transmissivity and specific storage of the aquifer systems. We will discuss then well hydraulics specifically for pumping cases from confined and unconfined aquifer under study and transient pumping conditions.

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Groundwater Systems

- **Aquifer:** A formation that contains sufficient saturated permeable material to yield significant quantities of water. Aquifers are generally aerially extensive and may be overlain or underlain by a confining (impervious) bed.
- **Aquitard:** A saturated but poorly permeable stratum that impedes groundwater movement and does not yield water freely to wells but that may transmit appreciable water to or from adjacent aquifers. *Example - sandy clay*
- **Aquiclude:** A saturated but relatively impermeable material that does not yield appreciable quantities of water to wells. *Example - clay*
- **Aquifuge:** A relatively impermeable formation neither containing nor transmitting water. *Example - solid granite*

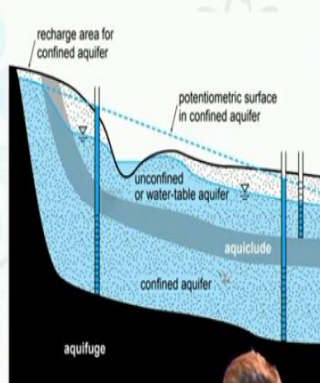


Image Source:
<https://www.idea.columbia.edu/~martin/hydro/epi/epi.htm>

So to begin with the groundwater formations are typically four type; aquifer, aquitard, aquiclude, and aquifuge. So aquifers are the formations that contain sufficient saturated permeable material which can yield significant quantities of water, okay. So it can store as well as yield significant quantities of water. So these are generally

aerially extensive and may overlain or basically underlain by a confining impervious bed, okay.

Then there are aquitard which are saturated but very poor permeability like sandy clay, okay. So they typically are not that preferred for withdrawing ground waters because of the poor permeability. Then we have aquiclude systems which remain saturated, which may remain saturated with the water. So they can store water, but they are more or less impermeable, so they does not typically transmit the water okay.

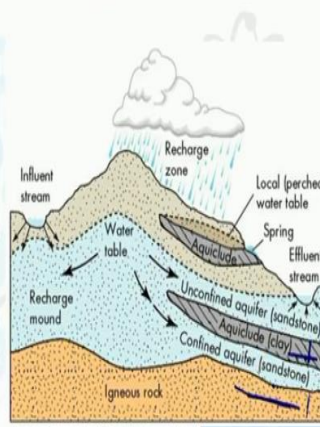
So they would not yield significant quantity or appreciable quantities of water to the wells; so for example, clay. And then aquifuge like solid granite or rock systems which are relatively impermeable, so they can neither store water neither contain water nor transmit water. So aquifuge and aquiclude typically forms impervious layers.

Whereas aquifer is the one which is most suitable formation for withdrawing groundwater because it can store and yield sufficient quantities of water and that is why these are the preferred system. So typically a groundwater formation may look like this. We will have impervious, aquifuge or rock kind of bottom there and then we may have a separating layer of aquiclude that kind of systems and then in between we may have waters stored in here.

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Aquifer Types

- **Confined (or pressure or artesian) Aquifer:** Bounded both above and below by impervious strata where groundwater is confined under pressure.
- **Unconfined (or phreatic) Aquifer:** Bounded by a free surface at the upper boundary, which is known as water table, where water is at atmospheric pressure.
- **Perched Aquifer:** Occurs whenever a groundwater body is separated from the main groundwater by a relatively impermeable stratum of small areal extent and by the zone of aeration above the main body of groundwater.
- **Leaky Aquifer:** Overlain or underlain by a semipervious aquitard, or semiconfining layer, these aquifers (confined or unconfined) loses or gains water through adjacent layers
- **Idealized Aquifer:** Assumed to be homogeneous and isotropic. Such idealized aquifers do not exist.



The diagram illustrates a cross-section of the ground showing various geological layers and groundwater features. From top to bottom, the layers are: a recharge zone where rain infiltrates; an unconfined aquifer (sandstone) with a water table; an aquiclude (sandstone) layer; a confined aquifer (sandstone) layer; and igneous rock at the base. Surface features include an influent stream, a local (perched) water table, a spring, and an effluent stream. A recharge mound is also shown. Arrows indicate the flow of water between these components.

Image Source: <https://gimadeeasy.weebly.com/groundwater-1.html>

Now this systems further as we say that aquifer are the more preferred system. Now aquifer also are different type. There are confined aquifer, unconfined aquifer, perched aquifer, leaky aquifer. So these are the different type of aquifers that we encounter in the subsurface. The confined aquifer which is also known as pressure or artesian aquifers are usually bounded both above and below by the impervious strata, okay.

So if we have impervious strata like you see here there is a layer of clay which is considered impervious and igneous rock which is impervious. So the bed in between this is the water that is like retained in the sandstone layer here is actually confined aquifer, okay. Because it is confined from both the ends. So that is why it is known as confined aquifer.

Generally water in here remain under pressure and that is why if we see the net head that is obtained so that like if we go to the previous slide we can see better here. So what happens that this is the confined aquifer and if you put a piezometer so although water level in this aquifer is here but that is because there is a impervious layer and water cannot pass over here.

So actually if we put a piezometer we will see that the pressure is much higher than the actually level of the water and it can actually be even higher at the ground surface so that can result in a artesian well kind of thing, okay. So we may actually get artesian or those kind of effects as a result of this pressure aquifers or that is why these are also known as artesian aquifers.

Unconfined aquifers, which are also known as phreatic aquifer or at times even known as water table aquifers. So these are generally bounded by free surface at the upper boundary okay. So like you see here, so here if this is our water table now, these aquifers are bounded by the bottom is of course bounded by the impervious strata but top they are just water table.

So there is no kind of confining layer at the top and these type of aquifer where pressure is actually equal to the atmospheric pressure, so these kind of aquifers are called unconfined aquifer. Then we have perched aquifer which occurs when there is

basically a groundwater body is separated from main groundwater body due to a relatively impervious stratum like if you see here you get a layer of aquiclude here. So some water is stored in here. It is actually not in the main groundwater body.

So some water is stored here and these are called, these kind of aquifers are called perched aquifer. These are good only for temporary withdrawal. So we can get some amount of water but because the formation is not very wide, not very extensive. So not much water is stored in here and if we withdraw this water soon this might actually turn dry.

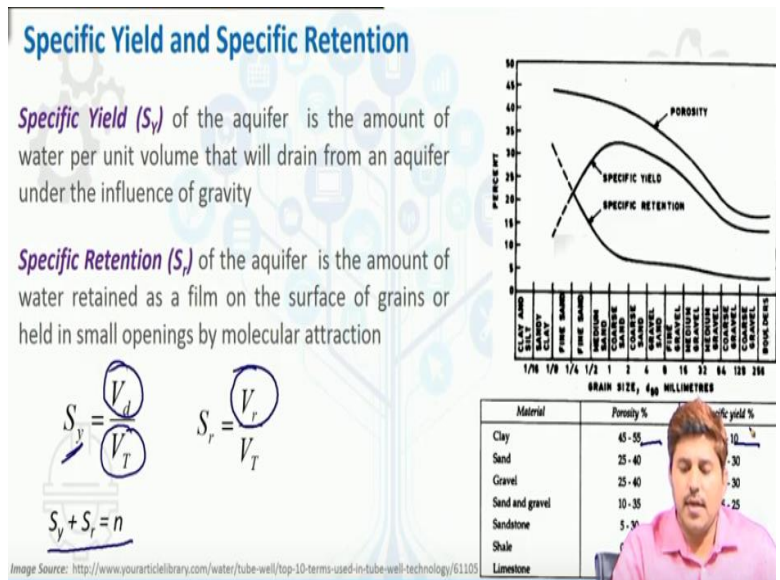
Then there are leaky aquifers which basically are either overlain or underlain by some impervious layer and on the other end it may actually have semi impervious layer like aquitard and those kind of thing. So what happens in the leaky aquifer, it will either lose or gain water through the adjacent layer. So if let us say this instead of aquiclude this has a layer of aquitard.

So that means this will actually start losing water over here. So the pressure will drop here. Similarly, if at the bottom and there is other type of structures it can lose water. So depending on this or for this particular aquifer if you see this is instead of aquiclude it is aquitard or say leaky material. So this aquifer will lose water whereas, this aquifer will gain water. So that is what is known as leaky aquifer, okay.

Now these are generic formation which we see in the field, but for analysis purposes many times we assume aquifer which are homogeneous and isotropic. This is the common assumption which is considered while analyzing the subsurface water flow and transport. But these kind of aquifers are known as idealized aquifer but in practice such idealized aquifers does not exist okay.

So it is just an assumption, but this is what usually people adapt when they go for analyzing the system, okay.

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Now two of the important properties of aquifer are the specific yield and specific retention. So a specific yield is actually the amount of sort of water that aquifer can release per unit volume okay when drain from an aquifer under the influence of gravity. So if we just allow under gravitational force aquifer to completely drain so how much water it can drain is known as specific yield okay.

So basically the amount of water, maximum water which can be drained out of an aquifer. Then there is a specific retention, is basically another aquifer property and this is the amount of water which will be retained as a film on the surface of the grains when basically the aquifer is subjected to be flowed under gravity or basically other way. So by the molecular attraction whatsoever amount of water that can be retained within the aquifer is known as specific retention.

So it is very difficult to get this portion of water, specific retention, and for withdrawal purpose that is in fact as good as dead volume of water, but it is the specific yield which serves the when we get into the withdrawal of the water from these aquifers system. So combination of specific yield and specific retention particularly in the saturated aquifer systems will give the total amount of water which is available in the aquifer which is going to be equal to the porosity of the water okay.

So specific yield essentially if you see the total volume of water is V_T , and the volume of water which can be drained is V_d . So the ratio of this is known as specific yield and similarly if the total volume of water which is retained in the system is V_r

and we divide it with the total volume of the water so we get the specific retention. And summation of these two will actually be equal to the porosity.

So if you see for the different particle size, grain size and different kind of material, if this is what is the porosity of the system, this is how we get the specific yield and specific retention curve for different materials, okay. So generally soils have good porosity and good specific yield also. Clay, although porosity is very high, but specific yield is not that much, okay. Sand has a better specific yield, okay.

Sand and gravel typically have a specific yield in the order of 10 to 30. So that way we can actually see which kind of formation is better for withdrawing water from an aquifer system.

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Storativity (Coefficient of Storage) and Specific Storage

Storativity (S):
The volume of water that an aquifer releases from or takes into storage per unit surface area per unit change in hydraulic head. Therefore, "S" equals the volume of water released from the aquifer when the piezometric surface declines a unit distance.

$S = \frac{\text{volume of water}}{\text{aquifer area}} \frac{\text{head change}}{\text{head change}}$
 $= \frac{L^3}{L^2 * L} = m^3/m^3 \text{ (dimensionless)}$

In confined aquifers, S ranges from 10^{-3} to 10^{-5}

Specific Storage (S_s):
The volume of water that an aquifer releases from or takes into storage per unit surface area per unit aquifer thickness per unit change in head

$S_s = \frac{\text{volume of water}}{\text{aquifer area}} \frac{\text{head change}}{\text{thickness} \cdot \text{head change}}$
 $= \frac{L^3}{L^2 * L * L} = m^3/m^4 = 1/m$

$S = S_s \cdot b$

The diagram shows a 3D perspective of a rectangular aquifer with a top surface area 'a' and a thickness 'b'.

The another important property of aquifer are storativity, which is also known as storage coefficient or coefficient of storage and specific storage. So storativity or the coefficient of storage is actually the volume of water that an aquifer releases or take into the storage. So it will either release are take into the storage under unit change of hydraulic head okay.

So basically it is equal to the volume of water which will be released from an aquifer when we drop down the piezometric surface by unit distance by one meter or so, okay. So this is typically denoted by S and is equal to the volume of water divided by

the aquifer area because we are talking about per unit surface area and per unit changing the hydraulic head.

So it will be divided by aquifer area and what is the head change in that. If we see the unit so this is volume of water. So maybe meter cube or that way. The area meter square and had change in meter. So this typically turns dimensionless, okay. So in confined aquifer this S ranges typically from 10^{-3} to 10^{-5} , okay. So that is the order that in which it ranges for most of the confined aquifers.

Then the specific storage is again similar property. Actually the volume of water that aquifer releases or take into the storage per unit surface area per unit aquifer thickness also. So specific storage is dependent on the thickness. So let us say if let us say this is our aquifer size okay, this is the unit area of aquifer that we are considering. So how much water this aquifer releases when we change the head by unit distance is actually the storativity.

But if it has a thickness of b and instead of looking at how much water it has released total water, we see how much water it has released per unit thickness also. So then if this is our unit thickness, so instead of taking this entire amount if we take only this amount, so then we call it as the specific storage okay.

So specific storage is essentially the volume of water per unit aquifer area divided means volume of water we need to divide with the aquifer area, aquifer thickness, as well as head change okay. So the unit is going to be per meter okay. And it will be the difference between storativity and specific storage is only of the aquifer thickness. Specific storage is per unit thickness of the aquifer whereas storativity does not basically depend on the aquifer thickness.

So storativity essentially because this is per unit thickness if we multiply this with the thickness of the aquifer b what we get is storativity. So this is the relation between the storativity and specific storage which is typically applicable in the systems.

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Storage in Confined and Unconfined Aquifers


In Confined Aquifers:
 When water is removed from a confined aquifer, hydraulic head (fluid pressure) decreases, and the granular skeleton of aquifer contracts.
 S_s in a confined aquifer reflects storage coming from **compression** of granular matrix and **expansion** of water

$$S_s = \rho_w g (\beta_p + n\beta_w)$$

ρ_w is density of water; g is gravitational constant (9.81 m/s²);
 n is porosity of aquifer, β_p is vertical compressibility of rock matrix; and β_w is compressibility of water (4.8x10⁻¹⁰ m²/N)

In Unconfined Aquifers:
 Main source of water is drainage of water from pores. In early stages, water comes from expansion of water and compression of matrix, however in later stage, water comes mainly from gravity drainage.

$$S = S_y + bS_{2d}$$



Now if we see the water that is stored in confined and unconfined aquifer, so in confined aquifer when basically we remove water from a confined aquifer, it is basically the hydraulic head which is fluid pressure that will decrease and as a result, the granular skeleton of the aquifer basically contracts and this releases water further okay.

So the specific storage in a confined aquifer typically reflects the storage coming from the compression of the granular matrix and expansion of the water okay. Whereas in unconfined aquifer again in the early stages in unconfined aquifer also water will come from the expansion of water and compression of the matrix, general matrix of the storage material, but in later stages it will typically come mainly from the gravity drainage okay.

So in unconfined aquifer the major storage is due to the gravity drainage particularly in the later stage. So in confined aquifer it is because of compression of granular matrix and expansion of water. So the specific storage may be given as may be given as $\rho_w g$. And then β_p and β_w are the two constants which are basically the compressibility of rock matrix and compressibility of water, okay and n is the porosity of the aquifer material.

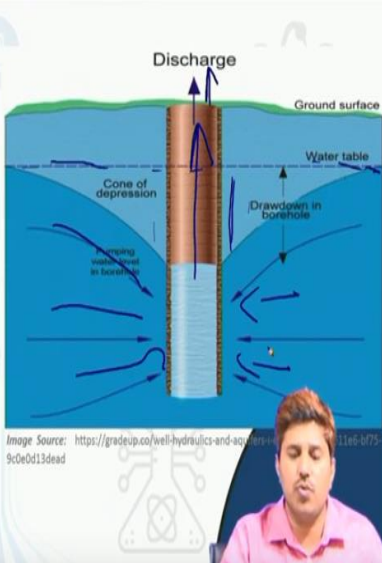
So that way we can get the specific storage in case of confined aquifer and in unconfined aquifer, basically the total storativity will come from the like specific

storage whatever we get and we have to add specific yield in there. So that is why that is how we can get the total storativity in the unconfined aquifer.

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Groundwater Wells

- Groundwater is collected through the use of well systems having well structure, pump and discharge pipes.
- Well usually consists of perforated casing that allows water to enter the well but prevents collapse of hole.
- When water is withdrawn, the a flow towards well is established to compensate the withdrawal.
- Because of head loss, piezometric surface adjacent to well is depressed; this is called 'cone of depression'.



The diagram illustrates a cross-section of a well in an unconfined aquifer. A vertical well casing is shown with a discharge pipe at the top. The ground surface is at the top, and the water table is shown as a dashed line. The cone of depression is the area around the well where the water table is lowered. The discharge pipe is labeled 'Discharge'. The water table is labeled 'Water table'. The cone of depression is labeled 'Cone of depression'. The drawdown in the well is labeled 'Drawdown in wellbore'. The diagram also shows the flow of water towards the well and the discharge of water from the well.

Image Source: <https://gradeup.co/well-hydraulics-and-aquifers/>

Now how we put, whenever we put a groundwater well for collecting groundwater, so groundwater is typically collected through well system and these well system will have a well structure, will have a pump, and will have discharge pipe. So these are the three major components of a well system, okay. This usually consist a perforated casing that allow water to enter the well but prevents collapse of the hole.

So that perforated casing is very important there, okay. Now what happens when water is withdrawn, there will be a flow towards the well which will be established to compensate the water withdrawal. So if we are discharging water from here there will be flow coming in basically towards the well in order to compensate that. And that is how basically we get a continuous connection.

However, what this leads to when we are basically pumping water so this leads to head loss and then piezometric surface adjacent to well will actually get depressed and this is called cone of depression, okay. So cone of depression will be basically like if this is our initial piezometric surface, so what happens when we start pumping water, the surface just near the well will be get more down.

And then as we move to the radially the distance the drawdown lowers, but this is slowly and progressively will form a base cone like this. And this is in three

dimension it is not just in two dimension. So a cone of depression is formed. And that kind of gives us an idea of how much drawdown is there okay.

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Well Hydraulics

- The knowledge of well hydraulics is needed to understand the processes in effect when one or more wells are pumping from an aquifer. This mainly considers the analysis of drawdown due to pumping with time and distance.
- When water is pumped from an aquifer, the pumpage creates a drawdown in the piezometric surface that induces hydraulic gradient toward the well. The induced flow moves horizontally toward the well. Drawdown at a given point is the distance by which the water level is lowered.
- Understanding of well hydraulics is essential to design a pumping strategy that is sufficient to furnish the adequate amounts of water.

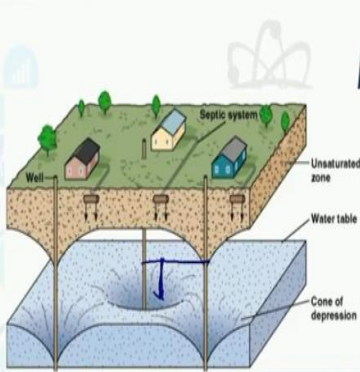



Image Source: <https://gradesup.co/well-hydraulics-and-groundwater-flow/>
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Darcy's equation

$$Q = -KA \frac{dh}{dx}$$


So what happens that in 3D you can see the cone of depression would actually be looking like this. Then from entire all the directions, it is actually will be forming a depression cone. Now it is very important to understand the concepts of well hydraulics if we want to analyze these systems and see how much depression we are going to get, how it is going to affect the groundwater systems.

And how effectively or how kind of environmentally friendly way we can abstract the amount of water needed. So when water is pumped from an aquifer, this pumpage will create a drawdown in the piezometric surface, okay which will be basically inducing the hydraulic gradients towards the well, okay. And because of this hydraulic gradients towards the well, the induced flow moves horizontally towards the well.

And drawdown which is typically like if this is say our initial surface. So at any particular point of time, what is the depth of the water table from the initial is known as drawdown. So the typical flow is analyzed using the basic Darcy's equation and then there are several variants available which are derived from this basic Darcy's equation in order to understand the behavior of groundwater flows under the different conditions.

And that is what is the importance of studying and understanding the well hydraulics.

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Steady Flow to Wells in Confined Aquifers

Assumptions:

- Steady state implies that the drawdown is a function of location only, $h = f(r)$.
- Homogeneous and isotropic medium.
- Flow to the well (in the vicinity) is radial, horizontal and laminar.
- The loss of head is proportional to the tangent of hydraulic gradient (dh/dx).

$Q = Aq = (2\pi rb)(-K \frac{dh}{dr}) = -2\pi r b K \frac{dh}{dr}$

Thiem equation
 $Q = \frac{2\pi K b (h_2 - h_1)}{\ln \frac{r_2}{r_1}} = \frac{2\pi T (s_1 - s_2)}{\ln \frac{r_2}{r_1}}$

Dupit's equation
 $Q = \frac{2\pi K b (H - h_w)}{\ln \frac{R}{r_w}}$

Where,
K = Hydraulic conductivity [L/T]
T = Transmissivity = Kb [L²/T]

Rate at which water is transmitted through a unit width of aquifer under a unit hydraulic gradient (Ease with which water moves through an aquifer)

So if we see the steady flow to wells in confined aquifer, now in confined aquifer as we discussed that it will be impervious layer both at the top and bottom and the water will be withdrawn from water will be withdrawn only from this part okay only from the aquifer thickness part which is actually b, aquifer thickness is b, okay. So initially let us say our water table was here.

But as cone of depression formed it has turned like this. Now we may have a couple of observation well. Say well one, well two, and correspondingly drawdown is s_1 and s_2 and the piezometric head in the well is h_1 and h_2 , okay from the same datum.

The radial distance of these observation well say r_1 and r_2 okay and the radius of the well is say R and the drawdown or the piezometric head in the well during the pumping, we are pumping at a discharge constant discharge is H and drawdown in the original well is s okay. So for analyzing such systems we take first several assumptions.

Since we are talking about study flows so study flow implies that drawdown is a function of location only okay. So the drawdown h is only a function of the location means radial distance and is not dependent on the time. It has already acquired steady stage that is what we are presuming here. The other assumptions are like the aquifer medium is homogeneous and isotropic, means the properties are same everywhere and in all the directions.

Flow to the well in the vicinity of well particularly is radial. So these flows are actually radial and horizontal and laminar and the loss of head is proportional to the tangent of the hydraulic gradient which is basically the Darcy's law. So we assume that Darcy's law is valid and we assume that discharge is constant. It is being pumped at a constant discharge.

So if Darcy's law is valid what we get is Q is equal to means the total discharge is area times the velocity, okay or what is equal to the K times dh by dr which is your Darcy's law okay. And then as a result what we, like we get is this expression okay. So Q is $-2\pi r b$, b is the thickness of the aquifer, K which is the hydraulic conductivity of the aquifer into dh by dr which is the hydraulic gradient, okay.

Now in the, if we solve this we get a solution which is given by Thiem and then with certain assumptions we get a Dupit equation as well okay. So Q is becomes actually $2\pi Kb (h_2 - h_1) \ln r_2$ by r_1 okay and this is in terms of head and if we convert it in terms of drawdown so we know that our drawdown s_1 is equal to $h - h_1$ okay.

So similarly like we can convert h_1 and h_2 in terms of drawdown and then we will get this equation. So this is the equation which is typically used for the study flow to wells in a confined aquifer okay. When we have two observations, two observation well, if we do not have observation well, so with Dupit assumptions that over a period of time like which says that there would be a point r distance where drawdown would be zero, okay.

So if we take that radial distance and consider drawdown as zero, so our h_2 becomes H and the next observation well instead of observation well we consider it to the our main well itself, okay. So in main well we have the height as h or h_w you can say. This is actually the same thing and r or r_w over here. So that way we can actually get the discharge value this way, okay.

Here K is the hydraulic conductivity. Now, this can also be converted to the transmissivity okay which is the rate at which water is transmitted. This is a very important property, the rate at which water is transmitted through the unit width of the

aquifer under unit hydraulic gradient, okay. So it basically reflects the ease with which water moves through and is actually equal to K into b , so hydraulic conductivity into b . So this Kb we can replace as T as you can see here or similarly this also can be replaced by T . So transmissivity can also be determined using this phenomena.

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Steady Flow to Wells in Unconfined Aquifers

Assumptions:

- Steady state implies that the drawdown is a function of location only, $h = f(r)$.
- Homogeneous and isotropic medium.
- Flow to the well (in the vicinity) is radial, horizontal and laminar.
- The loss of head is proportional to the tangent of hydraulic gradient (dh/dx).

The flow equation is similar for that of confined aquifers except we use h instead of b

$$Q = -2\pi r K h \frac{dh}{dr}$$

Thiem equation

$$Q = \frac{\pi K}{\ln \frac{r_2}{r_1}} (h_1^2 - h_2^2)$$

Dupit's equation

$$Q = \frac{\pi K}{\ln \frac{R}{r}} (H^2 - h^2)$$

The diagram illustrates a well in an unconfined aquifer. It shows the ground surface, the initial water table, the drawdown curve, and the cone of depression. Key parameters include the radius of the well (r_w), the radius of the observation wells (r_1, r_2), the total head (H), and the head at the well (h). The aquifer is shown above an impermeable strata.

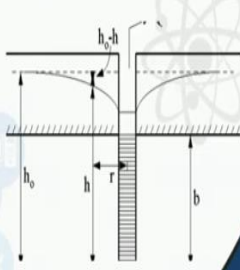
Then we have study flow to unconfined aquifer. Unconfined aquifer also all the assumptions are considered valid and in addition because in confined aquifer the thickness between the two confining layer b was taken as an aquifer thickness but in unconfined aquifer it is basically the piezometric head h because as we said that unconfined aquifer follows the water table.

So whatsoever is the piezometric head or free surface head is taken as the aquifer thickness. So instead of b in the same expression we will replace b with h . So what we get this is our major equation and when we solve this equations, so we get these expressions as Thiem's and Dupit's equations, okay. So which is actually valid for unconfined aquifer, under steady state of course.

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Transient Flow to Wells in Confined Aquifers

- Transient state implies that the drawdown is a function of location and time, $h = f(r,t)$.
- In reality, many aquifer tests will never reach the steady state (i.e., the cone of depression will continue to grow over time).
- Transient analysis can be used to infer aquifer properties, including storativity (not possible from steady measurements)



Theis or nonequilibrium equation

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t}$$

Where S is the aquifer storativity, T is the transmissivity, u is a dimensionless constant, t is the time since pumping starts, and r is the radial distance from the pumping well

Initial conditions: $h(r, t=0) = h_0$
 Boundary condition: $h(r \rightarrow \infty, t) = h_0$

$$h_0 - h = \frac{Q}{4\pi T} \int_0^\infty \frac{e^{-u}}{u} du \quad u = \frac{r^2 S}{4Tt}$$

The integral is known as the well function, $W(u)$

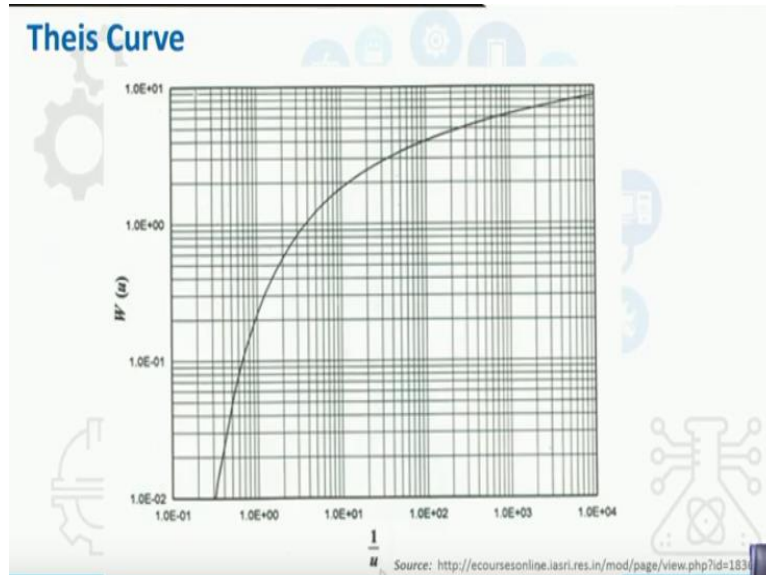
$$h_0 - h = \frac{Q}{4\pi T} W(u) \quad S = \frac{4Tut}{r^2}$$

Now, if we talk about transient flow to well in confined aquifers, so transient state implies that drawdown is a function of both, location as well as time okay. So in a steady state we said that the property was not varying with respect to time. The system has attained a steady state but in reality that is very like difficult to encounter in the field. Many aquifer test will never reach a steady state.

So the cone of depression will keep on growing over time if we keep on continue the pumping. So that means system is still under transient state because properties are changing with respect to time. So in the transient analysis, the Theis or non-equilibrium equation is used. So this is the basic equation and this is the initial condition at time $t = 0$ we have head is equal to h_0 and this is the boundary condition means r is equal to infinity the drawdown is 0 or $h = h_0$ okay.

So for this we get a solution of this equation like this and this here u is r square S by $4Tt$ and this integral part is typically known as well function here $W(u)$. So this equation can be reduced to as $W(u)$ and where we can get S from here. So these are the two equations, which are typically of use when we go for analysis of transient systems.

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How it is used. So there is a Theis curve which basically as just we saw that it is actually like the systems that we saw the $W(u)$ and u are the function. So we plot $1/u$ versus $W(u)$ and this gives us a curve which is known as Theis curve.

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Determination of Confined Aquifer Parameters: Theis Method

Step 1: Construct Theis Type Curve by plotting $W(u)$ and u on the log-log graph paper (or) obtain a copy of this curve from the literature.

$$h_o - h = \frac{Q}{4\pi T} W(u) \quad S = \frac{4Tut}{r^2}$$

Step 2: Plot field-data curve using observed values of **drawdown (s) versus r^2/t** on the log-log graph paper having the same scale as the Type Curve.

Step 3: **Superimpose the transparent field-data curve on the Type-Curve sheet**, keeping coordinate axes of the two graphs parallel to each other. Adjust the field-data curve until a best fit of field data points to the Type Curve.

Step 4: Select an arbitrary 'match point' on the Type Curve and note down the corresponding coordinates (s and r^2/t) from the field-data curve, and $W(u)$ and u from the Type Curve.

Step 5: Finally, substitute the values of these coordinates and the value of Q , in Theis equation to calculate T . Thereafter, substitute the values of the known variables in drawdown equation to obtain S .

Source: <http://ecoursesonline.iisri.res.in/mod/page/view.php?id=1836>
Groundwater Hydraulics by Daene C. McKinney

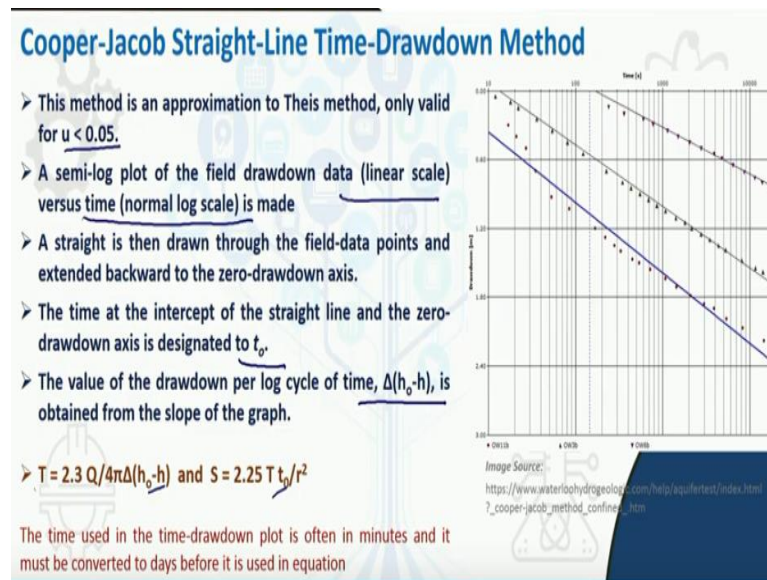
And then there is a method following which we can determine the aquifer properties. So if we want to determine the aquifer parameters, so what we can do? We can construct this type of curve or take this from the literature, then plot the field data using the observations of drawdown and r square by t , okay. So we plot s versus r square by t and then we have a $W(u)$ versus u curve okay.

And then we superimpose the transparent field data curve on this curve and then consider a match point, generally 1:1 match point is considered for better purpose. So

whatever data we are getting we have to see that it is actually following this type curve. And then from the match point, we get the value of s and r square by t and we get the value of $W(u)$ and u from type curve.

So all the values are known to us now, and then finally we substitute it to these equations and get the value of storativity and transmissivity, okay. So that is how we can use this.

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There are other simplified methods like Cooper-Jacob straight line time drawdown method, okay. So this method is an approximation of the basic Theis method and is only valid when value of u is less than 0.05, okay. So in this a semi-log plot is used between the drawdown and time. So this is the time of semi-log scale and drawdown is on the normal linear scale okay.

So drawdown data will be on linear scale and time will be on normal scale and then we plot linear this thing. The intercept that we get is designated as t_0 and the slope then from we get $\Delta h_0 - h$ which is obtained from the slope of the graph. So we know that t_0 and we know this value. So we can calculate storativity and transmissivity for the aquifer.

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Jacob Straight-Line Distance-Drawdown Method

- Can be used if drawdowns are measured at the same time in more than three wells.
- In this method drawdown is plotted on arithmetic scale as a function of the distance from the pumping well on the log scale.
- A straight line is then drawn through the data points and extended to the zero-drawdown axis.
- The intercept is the distance at which the pumping well is not affecting the water level and is designated r_0 .

$$\text{➤ } T = 2.3 Q / 2\pi\Delta(h_0 - h) \text{ and } S = 2.25 T t / r_0^2$$



Image Source:
https://www.waterloohydrogeology.com/help/aquifer-test/index.html?_cooper-jacob_method_confined_system

Similarly, there is a Jacob straight-line distance drawdown method. So earlier what we were seeing is the time drawdown method. Now, instead of time we can get a distance drawdown method as well. So let us say we have a distance. Again distances plot on a semi-log scale and drawdown is plot on a basically linear scale, normal scale and a straight line is drawn, okay.

The intercept is the distance at which the pumping well will not affect the water level and that will be designated as r_0 and then using these expressions, we can actually calculate the transmissivity and storativity.

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Transient Flow to Wells in Unconfined Aquifers: Neuman's Method

$$h_0 - h = \frac{Q}{4\pi T} W(u_A, u_B, \Gamma) \quad \text{Early Data} \quad h_0 - h = \frac{Q}{4\pi T} W(u_B, \Gamma) \quad \text{Late Data}$$

$$u_A = \frac{r^2 S}{4Tt} \quad u_B = \frac{r^2 S_y}{4Tt}$$

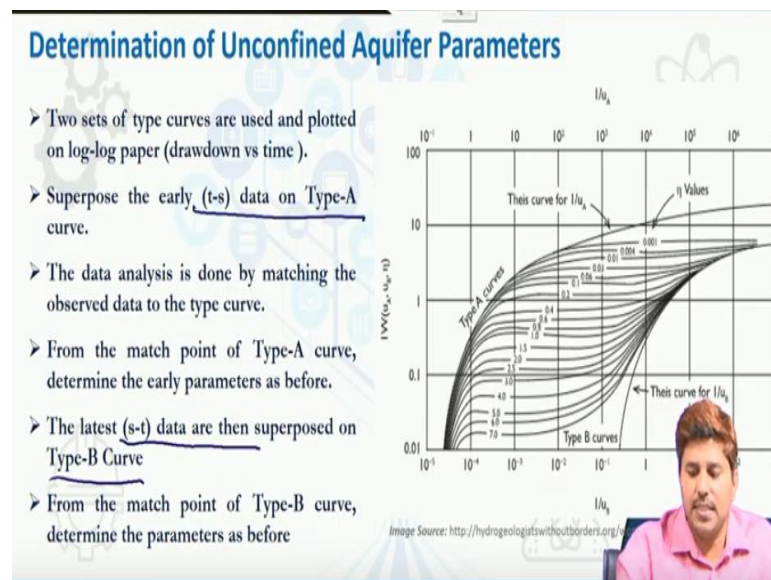
$$\Gamma = \frac{r^2 K_v}{b^2 K_h}$$

Where,
 S is the storativity (dimensionless)
 Sy is the specific yield (dimensionless)
 r radial distance from pumping well (m)
 b is the initial saturated thickness of aquifer (m)
 Kv is horizontal hydraulic conductivity (m/day)
 Kh is horizontal hydraulic conductivity (m/day)

The transient flow of basically to well in unconfined aquifers are more complicated, okay. So Neuman's method is typically used and this is not just given by one function.

There would be actually now two well functions, okay? So we have well function A, u_A and u_B both okay and it is basically this is the basic equation where the well function depends on u_A , u_B and τ . The early data can be plotted using this. The later data can be plotted using this where you u_A , u_B and τ are this and basically all other parameters and properties can be determined.

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So how we use this the Neuman's method for acquiring parameters. So we will have two set of type curves and that will be used and plotted on a log-log paper where is drawdown versus time curve will be plotted. We will superimpose early time drawdown data on basically our curve, on type A curve. And then data analysis is done matching the observed data to the type curve.

For later or latest time drawdown data that we have that will be superimposed on type B curve okay and similar analysis as we saw earlier may result in the our aquifer parameters for these as well.

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Recovery Test

- At the end of a pumping test, when pumping is stopped, water levels in pumping and observation wells usually begins to rise. This is referred to as the **recovery of groundwater levels**, while the measurements of drawdown below the original static water level (prior to pumping) are known as **residual drawdowns (s')**.
- If a well is pumped for a known period of time and then shut down, the drawdown thereafter will be identically the same as if the discharge had been continued and a hypothetical recharge well with the same flow were superposed on the discharging well at the instant the discharge is shut down. From the principle of superposition, Theis suggested residual drawdown as:

$$s' = \frac{+Q}{4\pi T} W(u) + \frac{-Q}{4\pi T} W(u') = \frac{Q}{4\pi T} [W(u) - W(u')] \quad u = \frac{r^2 S}{4Tt} \quad u' = \frac{r^2 S}{4Tt'}$$

t and t' are times since pumping started and stopped, respectively.

What happens when we are conducting these kind of well tests? So till we are doing pumping and particularly in the transient state, we see that drawdown is increasing and our basically cone of depression is increasing, okay. But when pumping is stopped, what we see that there is a possibility of recovery. So groundwater starts recovering because flow is anyway taking to the groundwater.

And if you are not pumping, so the cone of depression that was formed will start kind of recovering, okay. So this is basically recovery of groundwater levels and the measurement of the drawdowns that we get as opposed to the original static water level, which was prior to pumping are known as residual drawdowns. So when basically, if you see here, so this is the pumping period, the time when the pumping was going on and this is the recovery period.

So till this time, you see that the drawdown was increasing, and from this point when the pumping is stopped or pump is shut down, we can see that it is actually started recovering. So if a well is pumped for a known period of time and then shut down then drawdown thereafter will recover and for purpose of analysis, how it will recover it is basically the Theis.

The Theis has suggested kind of superimposition method for that. So they assume it is the same condition as if the discharge has been continued and a hypothetical recharge well, has also, with the same flow were also superimposed on that. So we still believe

that pump is not shut down, it is actually still pumping the water, okay. So you will see that the drawdown is actually still extending.

But, apart from that, we also have a recharge well, which is actually pumping in the same amount of water. So essentially the actual condition is that there is no discharge taking place. But what we assume that there is a discharge taking place and same amount of recharge is also being done. So the combination of this discharge and recharge nullifies any withdrawal of water and we consider that as a pump shut down case which is actually the case okay.

So when the pump is shut down there is no recharge or discharge is taking place, but for analysis purpose what we presume that still discharge is taking place and equal amount of recharge is also being done into that well. So with the simple Theis equation, what we can see that the residual drawdown would be actually this is due to the recharge well, this is due to the discharge well.

And net drawdown will be this where u and u' are this and t and t' are the time since pumping started and stopped respectively. So t is the time when pumping started, t' is the time from here and t' is the time when pumping is stopped. So t' is the time from this point forwards, okay.

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Recovery Test

➤ For u is less than 0.01, with small r and large t values, This equation can be simplified by Jacob and Cooper equation as:

$$s' = \frac{Q}{4\pi T} \left[\ln\left(\frac{2.25Tt}{r^2S}\right) - \ln\left(\frac{2.25Tt'}{r^2S}\right) \right]$$

$$s' = \frac{Q}{4\pi T} \ln\left(\frac{t}{t'}\right)$$

$$s' = \frac{2.303 Q}{4\pi T} \log\left(\frac{t}{t'}\right)$$

So, for $\Delta s'$, the residual drawdown per log cycle of t/t' , the transmissivity becomes:

$$T = \frac{2.303 Q}{4\pi \Delta s'}$$

Image Source: Schwartz and Zhang, Fundamentals of Geophysics, Wiley & Sons, New York, 2003

So this is how basically we get the recovery in the aquifers. For u less than 0.01 and when r and t values are r is small and t values are large, this Theis equation can be

simplified by Jacob and Cooper equation as well, as the Jacob and Cooper equation suggest. So as the earlier simplification we saw for unconfined aquifer. So similarly this also can be simplified and we can get drawdown as a function of t and t' over here, okay.

So we can plot the t by t' and the drawdown and it will get a straight line and then from that slope and intercept of this straight line, we can get the transmissivity of the system and then eventually we can compute the storativity as well. So using the recovery test also it is possible to compute the aquifer parameter. So that is the different type of systems that are used for withdrawing groundwater, okay.

There are many other features what we must see while we drain groundwater okay like what happens when there are more than one well. So how the well interferes to each other. Then what are the typical well losses. So some of this we will be discussing in the next class. So see you in the next class, thank you for joining.