

Urban Transportation Systems Planning
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Lecture - 21
Basic Considerations and Trip Distribution Matrices

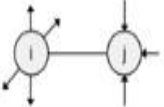

Welcome to module D Lecture 1. In this lecture, we shall discuss mainly about what is trip distribution? What are the basic considerations? And may the two types of trip distribution matrices that we handle in the context of trip distribution?

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Introduction

- Trip distribution: **Two known sets of trip ends** are connected together to form a trip matrix between known origins and destinations
- Trip distribution may be carried out with or without specifying the actual route and/or travel mode
- The number of trips is estimated for a particular period of time (average weekday or peak hour)

i \ j	1	2	3	4	P
1	t_{11}	t_{12}	t_{13}	t_{14}	p_1
2	t_{21}	t_{22}	t_{23}	t_{24}	p_2
3	t_{31}	t_{32}	t_{33}	t_{34}	p_3
4	t_{41}	t_{42}	t_{43}	t_{44}	p_4
A	a_1	a_2	a_3	a_4	T

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In trip distribution what we try to do is as follows. You can see in the right side, if you consider it as a matrix you have say 4 zones. For example, 1 2 3 4 and from each of this 4 zones some trips are being made to different other zones. But we do not know these cells, internal cells so far. What we know from the trip generation stage is the productions and attractions. That means we know the value of p_1 , p_2 , p_3 , p_4 and also, we want a_1 , a_2 , a_3 , a_4 .

But this internal cells t_{11} , t_{12} , t_{13} , t_{14} , similarly t_{21} , t_{22} , t_{23} , t_{24} all these cells we do not know. So, that trip just generation gave us the values of trip productions, that is, p_1 , p_2 , p_3 , p_4 and the trip attractions that is, a_1 , a_2 , a_3 , a_4 in general. Now in trip distribution stage, we want to find out these cells, internal cells which will match this productions and attractions. That is

what we said the two known sets of trip ends are connected together to form a trip matrix between known origins and destinations.

So, we know this 1 2 3 4 origins and destinations 1 2 3 4, we know how many are produced? How many are getting attracted? So, now we want to form a trip matrix that should include the cells as I said like t_{11} , t_{12} , t_{13} , t_{14} and so on. So, all these internal cells we would like to get in this trip distribution stage. Trip distribution may be carried out with or without specifying the actual route and or travel mode.

We do not need to mention the routes. We only want to say that from zone 2 to zone 4 so many trips are going to happen. We may or may not indicate at this stage that by which route this trip will happen or you know how different route wise the trips will get distributed. Similarly, we may or may not mention that by which mode these trips will be made. For example, you know 2 locations origins and destinations may be connected by cars one can you know travel by car one can travel by taxi one can travel by maybe public bus.

So, if we wish sometimes people also do that, they prepare this matrix mode wise that is also possible. But you can also prepare it this matrix without referring to specific mode, may be the total person travels that is what we are person trip that are being made that will be indicated in the matrix. Or as I said the similar matrix may be Trip Distribution matrix one by private port one by public transport say bus. That kind of matrixes also may be developed.

The number of trips is estimated for a particular period of time may be for average weekday, or may be for the peak hour that depends on the context. because there is always an implicit time unit. We are saying the productions attractions or we are saying from this origin so many people are making travel to that particular destination this there must be you know, implicit or explicit units for that.

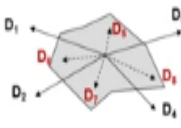
Sometimes we may mention that we mentioned it explicitly sometimes we may not mention it explicitly. But this time unit is very important. It may be that we are showing that distribution

during the peak hour or maybe during a typical weekday or depending during a typical weekend and so on depending on the context of the work.

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Introduction


- The essential problem in trip distribution is to determine the number of inter-zonal and intra-zonal trips from the estimated number of trips produced at and attracted to each zone




- Trip distribution is a function of
 - ✓ Socio-economic characteristics of the population
 - ✓ Type and extent of transportation facilities
 - ✓ Pattern of land-use, including the location and intensity

→ Inter-zonal Trip

→ Intra-zonal Trip





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Now the essential problem what we are trying to solve in this case is to determine the number of inter-zonal and intra-zonal trips from the estimated number of trips produced and attracted to each zone as I said in the previous slide. We know this p 1, p 2, p 3, p 4 similarly we know this a 1, a 2, a 3, a 4. So, we know that zone wise how much production is happening? How much attraction is happening? Or but we do not know the inter-zonal and intra-zonal trips that are happening.

Which will be intra-zonal trips? The diagonal element 1 to 1, 2 to 2, 3 to 3, 4 to 4 so all the diagonal elements are intra-zonal trips and remaining all other cells say 1 to 2 or 1 to 3 or 3 to 4 all these are inter-zonal trips one zone two another zone. So, you want to actually estimate or determine the number of inter-zonal and intra-zonal trips from this known number of productions and attractions. Now how these trips are likely to get distributed they are getting distributed or likely to get distributed based on several factors, say 1.

So, Socio-economic characteristics of population people are traveling from one zone to another zone. So, depends on how the distribution will happen? Depends on the socio-economic characteristics of population also depends on the type and extent of transportation facilities. How

these places are connected? Obviously, you can logically understand if two identical attractions one place is very well connected by public transport the other places not so well connected.

Obviously, the location which is well connected by public transport more people will travel to that destination. Similarly, it depends on the pattern of land use including the location and intensity. You remember that even we said all these factors are really important where the activities are located. What is the land use? What is the intensity of development? Say for example, more shopping opportunities there in one zone.

That will obviously attract more trips than you know, a similar destination where the less opportunity for shopping. So, more shopping trips will get attracted to the first zone, right as compared to the second zone. You can logically understand that the trip distribution depends on socio-economic characteristics of population. It depends on the type and extent of transportation facilities.

And also, the pattern of land use where which activities are located? Where exactly it is located? Where you know, because you said that the activity one activity or a shopping center located at in the heart of the CVD area and where you know, there are wide roads good parking facility visa is the same kind of shopping complex so shopping center located in the sower where the routes are mellow and you know vehicle cannot park on street, there is no obstinate facility and you know that kind of unplanned development.

Naturally, the trip attraction is not going to be same at two destinations. So, all these are really factors and one should bear those in mind.

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Introduction

Basic Considerations

- Inputs to trip distribution models
 - ✓ Trip generation outputs (productions and attractions for each zone)
 - ✓ Measures of travel impedance between each pair of zones
 - ✓ Socioeconomic characteristics of zones
- Outputs are trip tables, production zone to attraction zone
- Separate trip tables may be obtained by trip purpose



Let us come to the basic considerations of trip distribution. The first is what are the inputs that are required to trip distribution models in general? The very first thing is the trip generation outputs because you know that in four stage planning process trip distribution generally comes after trip generation. So, trip generation followed by trip distribution. So, obviously the productions and attractions for each zone which we have estimated or modelled in the trip generation stage, they are used as inputs to the trip distribution stage or trip distribution process.

Second, while we are trying to estimate this trip interchanges. How many are maybe 100 outputs produced at one location? But how those you know trips are getting is likely to get distributed? How the trips are likely to get distributed to different zones? Will depend on the measure of travel impedance between each pair of zones, as I say the travel impedance may include many things, we shall elaborate we shall have more discussion on this but let say the generally travel times.

If you know how well the transport connectivity is established? And how well the public transport system is operating? Or what is the travel time? What is the distance all such kind of you know, things will matter we can generally call them as measure of impedance between each pair of zones? So, we can generally consider them as a frictional zone, right? So, more the time or more the cost or more the distance generally we can consider that the friction is more.

So, if the friction is more obviously less people will go there or less number of people will go there. It also depends on the socio-economic characteristics of the zone. This aspect I will describe further later. You can understand it much better when we talk about the calibration of some of the models particularly the gravity model. So, in that context I can explain it better. But remember that the socio-economic characteristics of the zone even otherwise.

That is a you know, that is it one perspective one aspect of course. Other is also you can think say for example high income people, low income people their choice is not going to be the same even the same, you know, travel impedance is not going to impact exactly in the same manner to different socio-economic segments. May be if you know more high income people are saying and more people have got access to cars or private vehicles then whether the bus system is good or bad may not really matter much.

Still the trips can happen vis-a- vis is one zone where you will find probably the car ownership is low and people are more and more dependent on the public transport system. There how many trips will be made to a particular destination will definitely depend in a big way on the availability of public transport system. Now what are the outputs from this trip distribution process?

Output service you can basically as I said trip tables indicating production zone to attraction zone. So, how many trips are happening from one zone to another zone? In this case also, if we are considering the total travel person travel then we can get one matrix. If we are considering by different mode or different purpose then you can even get the trip travel by different modes or different purposes.

So, we may have one trip distribution for the public transport one trip distribution for the IPT mode say taxi three-wheeler one matrix for the private vehicle mode. You can get similarly if we wish we can also get theoretically the matrices separate matrices for separate trip purposes. Work trip matrix, educational trips separately also we can get. All these are possible.

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Introduction

Trip Distribution Matrices

- Two types of matrices
 - ✓ Production- Attraction (PA) matrix
 - ✓ Origin-Destination (OD) matrix
- The production-attraction format of the trip table should be converted into an origin-destination format to get **actual directions** of trips between TAZs



Now, when we are talking about trip distribution matrices most of the times, we shall talk about only OD matrix, Origin-Destination matrix. That is the matrix what we are going to use when we are talking about mode choice, when we are talking about traffic assignments subsequently most cases that is what is going to be more useful. But we also use something what is called production attraction matrix. In fact, production attraction matrix may be converted to origin destination matrix.

Production attraction format of the trip table generally needs to be converted into an origin destination format to get the actual directions of trip between the traffic analysis zones or the traffic zones. What is the basic difference between these two? One is as I say direction is not important not directional. The first one production and attraction production attraction matrix or the PA matrix while the origin destination matrix is directional.

Also, you can understand these basic differences very easily. If you consider that for each trip there is an origin there is a destination. Similarly, there is a point of production there is a point of attraction. Why these two are or how these two are different? That you can understand if you understand remember or recall I have discussed this in earlier lectures. If you; recall the difference between production attractions and origin distributions.

Say a person traveling from home to office and then a person just thinks travelling from office to home. Both cases the production will be at the home end. And the attraction will be at the destination end. Because we know that if there is home end either in origin or in destination the trip is always produced at the home end. So, the production will be at home end and attraction will be at the other end.

But if we say origin destination the first case the origin is home, destination is office. Second case, the origin is office, destination is home. But again, remind you I would like to remind you that in the second case the production is still at home attraction is still at office. So, when I take the production attraction, I will say both trips are produced at home end. So, when I am saying two, I am not considering the direction of movement.

Actually, both trips by definition are produced at home end and attracted at office end. But the direction of travel in both cases are not from home to office, one is home to office another is office to home. So, that difference is there. When we are talking about production attraction matrix we are thinking from production and attraction point of view not the direction of travel. So, maybe I will say both trips are produced at home end or a particular zone say zone 1.

But when we are talking about this origin destination matrix, then the direction is important. So, I want to know from zone 1 to zone 2 that means from home to office and office to home the direction is important. Because your mode choice when you want to understand that in peak hour how many people you travel like to travel in one direction because actual in terms of actual travel the direction is very important.

So, you would like to so peak hour in which direction or in the peak hour direction a peak direction how many people are trying to travel? And therefore how many buses will be required to serve the demand with a desired level of service maybe? Or when you are trying to see how the route cells, will happen you will see from this point to that point its directional. How that travel will happen? So, the capacity will depend whatever is the road capacity available directional road capacity compared to that.

What is the directional demand? Because you know that in the other direction which is not the direction of a peak direction, which is not the peak direction the problem may not be that severe. So, the problem will always be in a particular direction, where more travel is happening. Say in the morning time all the problems are for traffic which are CVD bound. In the evening it is this refers.

So, the direction is actually important in the context of both choices in the context of route choice and for all practical applications of this transport demand. So, it is directional demand that is more important. So, origin destination matrix is directional but production attraction matrix is not directional. It considers only from the perspective of production zone and perspective of attraction zones.


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Introduction

Production-Attraction (PA) matrix

- Has no directional meaning
- Does not reflect the real directions from origin to destination
- Represents the trip ends from production and attraction point of view of a zone

		Attraction zones				sum	
		1	j		n
Production zones	1	t_{11}		t_{1j}		t_{1n}	p_1
						
	i	t_{i1}		t_{ij}		t_{in}	p_i
						
	N	t_{N1}		t_{Nj}		t_{Nn}	p_N
sum	a_1		a_j		a_n	T	



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So, coming to the first production attraction matrix, as I said to you it has no directional meaning. If I am saying 100 trips are produced in zone 1 and attracted to zone 2. I really do not know whether all 100 trips are actually made from 1 to 2 or 50 are made from 1 to 2 and the 50 are made from 2 to 1 or 60 are made from 1 to 2 and 40 are made from 2 to 1 that we do not know. So, that the directional split or the direction dimension is missing.

So, it has no directional meaning does not reflect the real direction from origin to derive a destination. So, I am saying between 1 and 2, 100 trips are produced, right? So, I am saying that

we really do not know the real directions from origin to destination. It represents the trip ends only from production and attraction point of view of a zone. And again, I would like to remind you I would like to repeat that it is happening because origin destination and production attractions are not same always, right?

Let us say for non-home base strips the origin destination and production attractions are similar. So, in the same meaning one can use it. But for home base strip it is not so. Even if a trip is from office to home, we will stay origin is office destination is home which is actually important because that is what the directional part is included in that description. But when we look at production attraction point of view, we will stay still that trip is produced at home end and attracted to office.

But that does not indicate the direction of travel whether it is from home to office or office from home that information is not included in the production attraction matrix. So, that is what is shown here.

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Introduction

Origin-Destination (OD) Matrix

- Required for **directional traffic assignment**

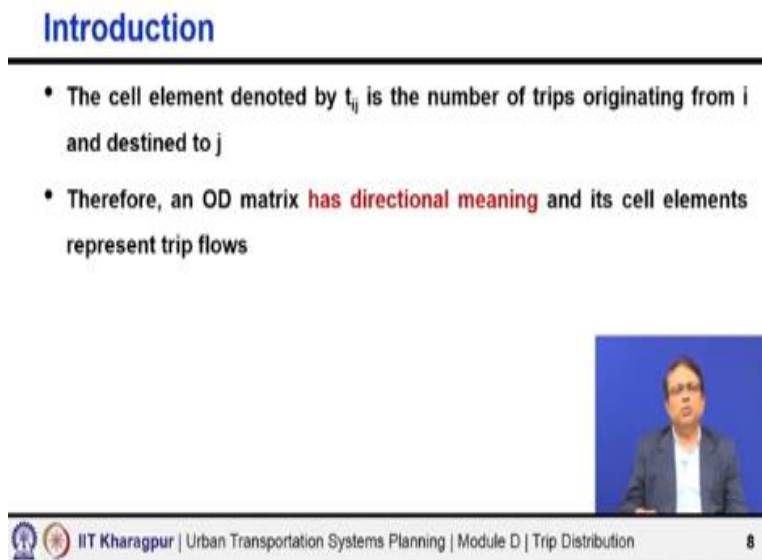
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When you come to the origin-destination matrix, it is actually directional. If I am saying 1 to 2, that means I mean that the direction of travel is from 1 to 2. If I am saying 1 to 4, I actually mean the direction of travel is from 1 to 4. All numbers what I am indicating in that matrix that many trips are actually happening from this origin to this destination. So, if I am saying an element t_{ij} .

That many numbers of trips are directional trips are happening from zone i to zone j and that is what is most cases we require.


So, in majority of the subsequent discussion, lectures and also in other modules we shall time again refer to this, origin destination matrix will always talk about OD matrix. But remember that there is also something what is called production attraction matrix? And why and how the production attraction matrix is different from the origin destination matrix or the OD matrix.

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Introduction

- The cell element denoted by t_{ij} is the number of trips originating from i and destined to j
- Therefore, an OD matrix **has directional meaning** and its cell elements represent trip flows



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Here as I said in the OD matrix the cell element denoted by t_{ij} is denoted by the number of trips originating from zone i and destinating to zone j . Therefore, the OD matrix had directional meaning which is important and which is necessary because the problem of transport is actually the direction is very very important, right? In the peak direction, you will find everything is so much of you know congested and so much delays happening but in the optic direction, maybe everything is very nice.

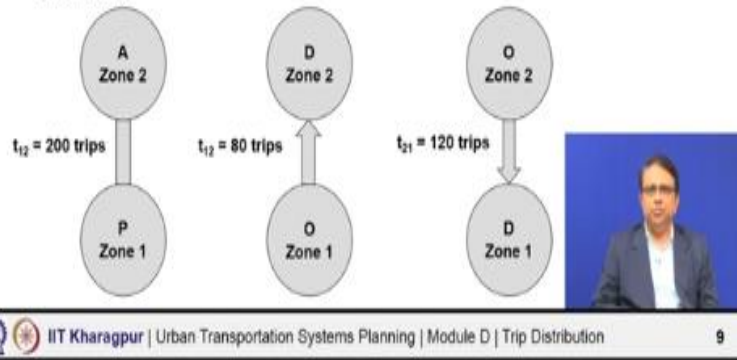
So, we need to understand the directional aspect, right? So, that is why the OD matrix is so important.

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Introduction

Conversion of P-A Matrix to O-D Matrix

- For directional traffic assignment, the PA matrix must be converted to an OD matrix



How to convert P-A matrix to OD matrix? As I said, the production attraction matrix is not directional, whereas origin destination matrix is directional. And our ultimate goal is to get directional matrix. So, we want finally the origin destination matrix, why? I explained it earlier that we want to know in the peak direction of travel how many people are traveling? May be accordingly we shall see that how many buses are required?

Or in the peak direction how many vehicles are travelling? And then we would like to see that where the bottleneck or the road capacity is becoming, you know, in adequate or where the capacity augmentation of the road is really essential. There could be many many applications. But we need to know the directional travel. That is very important. Now, the question is how to what is really the fundamental difference and how to get the directional travel from the production attraction matrix? That means how to get the OD matrix from the PA- matrix.

Let us take this example. You see the left side I have shown A and P. A means attractions P is production. So, zone 1 is taken as production and zone 2 is taken as attraction. So, with zone 1 as production and zone 2 as attraction, we have total 200 trips. That means for all those 200 trips the production is in zone 1 and attraction is in zone 2, Fine? No confusion? But then does it mean that all 200 trips are happening from P or zone 1 to zone 2 that we do not know that we have not said.

May be out of those some trips are actually happening from 1 to 2 in that direction and some are happening from 2 to 1 in that direction. That is what is shown here in the next 2 sketch. You can see we are expressing here as O and D. That means here we are saying 80 trips are happening with zone 1 as origin and zone 2 as destination. That means the direction of travel is from zone 1 to zone 2; that is the 80 trips.

Next one, we said that zone 2 is O that means origin zone 1 is D that means destination. And 120 directional trips are happening. That means 120 trips are actually happening from zone 2 to zone 1. But all this 80 + 120, 200 trip all these trips. For all these trips zone 1 is the location of the home. So, when we express the production attraction matrix, we said that all 200 trips are getting produced in zone 1 and getting attracted to zone 2.

So, the first one is non-directional and the other two when we come to the O-D matrix. Then the numbers indicate directional trips clearly from that origin to that destination in the in that direction.

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Introduction


- The general formulae for finding the cell elements of an OD matrix, from the cell elements of PA matrix, are

$$t_{ij} = \lambda t_{ij} + (1 - \lambda)t_{ji}$$

$$t_{ji} = \lambda t_{ji} + (1 - \lambda)t_{ij}$$

Where,

λ = Proportion of trips originating from the production zone



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Now how to generally get the OD matrix from the cell elements of PA matrix? That is said here if you want to get t_{ij} for the OD matrix. Then we can take the t_{ij} which is shown in the right side equation of the production attraction matrix into lambda. Lambda may be the proportion of trips, which is actually originating in zone i and travelling to zone j plus when you take that t_{ji} in

the production attraction matrix then same the lambda proportion is happening from j to i. Now from i to j then how much will happen? 1 minus lambda.

Lambda is the proportion which can take value between 0 to 1 so, 1 minus lambda into t_{ji} . So, if you add both these components then you can get the directional travel in the O-D matrix t_{ij} element. That is number of trips happening from i to j. Similarly to get the t_{ji} we can get it lambda of t_{ji} this right side t_{ji} is from the production attraction matrix plus 1 minus lambda into t_{ij} .

t_{ij} is lambda into t_{ij} is happening in the direction of i to j that is why they were included in the first equation. So, 1 minus lambda in t_{ij} is actually happening in the reverse direction that means from j to i. So, that is included in the second component.

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Introduction

Example


Given P-A Matrix


Production Zones	Attraction zones			
	1	2	3	Total
1	60	60	40	160
2	20	80	40	140
3	20	70	50	140
Total	100	210	130	440

Let the proportion of trips originating from the production zone be 0.4

O-D Matrix

Origin Zones	Destination zones			
	1	2	3	Total
1	60	$24+12 = 36$	$16+12 = 28$	124
2	$8+36 = 44$	80	$16+42 = 58$	182
3	$8+24 = 32$	$28+24 = 52$	50	134
Total	136	168	136	440




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I can explain this further in my next example, here you can see that I have shown here the first one is a PA matrix and then trying to show you how you can get the OD matrix from this PA matrix. Let us take one cell or one particular origin destination or production attraction to explain how the calculations can be made. Say for example, if you consider here in the PA matrix 1 to 2 is 60. That means for all this 60 trips production is in zone 1 and attraction is in zone 2. I repeat production is in zone 1 and attraction is in zone 2.

Similarly, in the same PA matrix 2 to 1 indicates that the production is in 2 and attraction is in 1. So, for all these 20 trips the household or the home end is at zone 2 and for all these 60 trips 1 to 2 the home end is 1. Now if we assume that any you know trips like this 40% of the trips are actually being made from the zone of production to zone of attraction in that direction. And 60% is being made in the reverse direction. Then out of these 60 trips where the home end is 1 and the destination or the attraction end is 2 how many of those are actually travelling with origin 1 and destination 2.

That means from 1 to 2 in that direction obviously 40% of those. So, 40% of 60 so 24 that is there in the OD matrix if you see 1 to 2 cell this is 24. But then this 12 is coming from where look at this 20, 2 to 1. So, this 2 to 1 out of this 20 again 40% will happen from 2 to 1 in that direction. So, the remaining 60% is happening what? In the reverse direction reverse direction means what 1 to 2.

So, 60% of this 20, so that is 12 total we are getting 36. If I come to these 1 2 to 1 cells of this O-D matrix then 8 how we get? We have got 20 numbers in the P-A matrix. So, 40% of those 20 actually happening 2 to 1. So, there production is at 2 attraction is it 1 origin is also 2 destination is 1 + 36. How we get the 36? 36 is 60% of these 1 2 cells of P-A matrix, that means 60% of 6.6 into 6, that is 36.

Now how this 36 strips? The production is in zone 1 attraction is in zone 2. But travel is being made with origin as 2 and destination as 1. So, then in the directional OD matrix what will be my cell value for 1 to 2? $24 + 12 = 36$ and what will be my cell value for the directional travel from 2 to 1? $8 + 36 = 44$. That is the way similarly all other cells one can calculate. So, given PA matrix if you want also know if you know the proportion of trips originating from the production zone. If that is known then you can actually get the origin destination matrix.

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Summary

- Trip Distribution Process
- Basic considerations
 - ✓ Inputs
 - ✓ Outputs
- Types of Trip Distribution matrices
 - ✓ P-A Matrix
 - ✓ O-D Matrix
- Conversion of P-A Matrix to O-D Matrix



In summary I would say that we discussed here in this lecture about the trip distribution process what we try to do and what we try to get? What are my inputs to this trip distribution process? What will be my outputs? Then the two types of trip matrices that we handle in this trip distribution context. One is the production attraction matrix another is the origin destination matrix.

And we also said that how we can convert the PA matrix to OD matrix. That means production attraction matrix if it is known and percentage of trips actually originating at the production point is known then we can get the origin destination matrix. So, with this I close this lecture thank you so much.