Urban Transportation Systems Planning
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Lecture - 22
Methods for Trip Distribution, Uniform Growth Factor Method and Average Growth Factor Method

Welcome to module D lecture 2. In this lecture we shall discuss about various methods for trip distribution and then two specific approaches or methods which are used under growth factor techniques. That is uniform growth factor method and average growth factor method. Those two techniques also we shall discuss.

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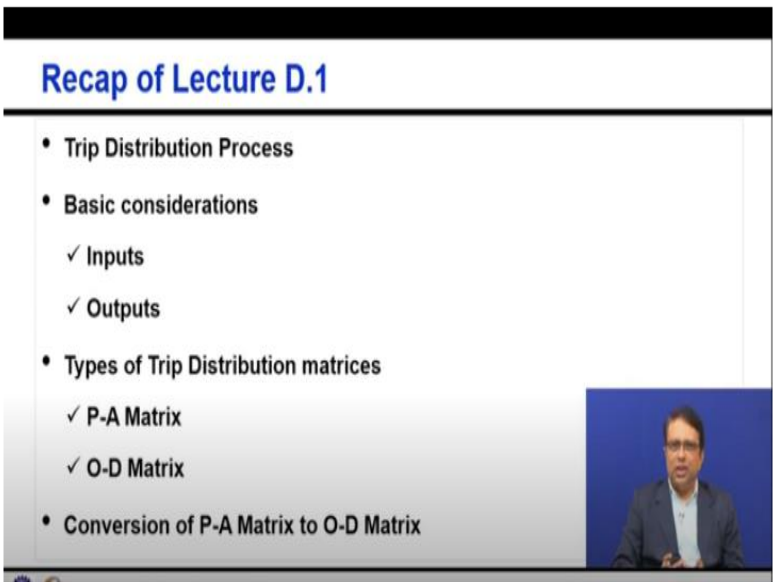

In our lecture first lecture on trip distribution we talked about why and what we do in the trip distribution process, what are the inputs to this process, what are the outputs from the trip distribution stage and then specific two types of matrices which we handle in the trip distribution context. One is the production attraction matrix, one is the origin destination matrix and how these are different.

To remind you again, the P-A matrix is not directional the total production and attraction that is what we are saying zone wise. So, how many productions are happening in zone 1 and which are with respect to travel from 1 and 2 between 1 and 2. But the O-D matrix we give specific directional travel. If we say 1 to 2 that means specifically it is from 1 and 2, 2. That is direction towards zone 2.

That is basically fundamental difference between this P-A matrix and O-D matrix. And we also explained how we can convert P-A matrix to the O-D matrix if we know the necessary inputs.

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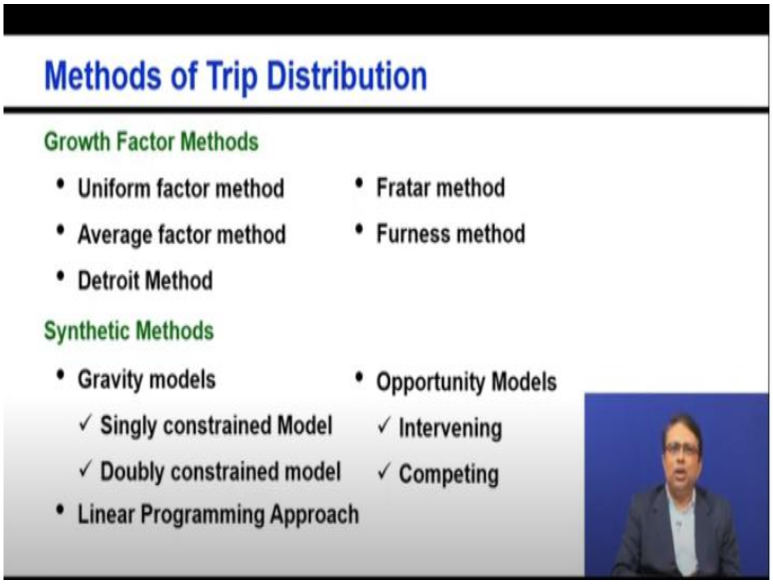

With this background today, let us discuss about various methods of trip distribution. In this slide, you can see so many methods so, many models which are mentioned here. Actually all the works or all the models or methods can be classified into two broad groups. One we can call it as growth factor models and in other categories of model could be called as synthetic model. Growth factor models are simpler, maybe used very effectively to update.

You know, short term update of the matrix, trip distribution matrix which are already there for short update when there is no significant change in the transportation network. And synthetic models are more rigorous, more logical. Reasons are inbuilt inside and they can be used even to estimate the individual cells. Now there are again within that there are gravity models, there are opportunity models, there are linear programming approaches.

So, many things can be used and can come all can come under synthetic model. In fact, if you talk about gravity model gravity model is not just one model. There are so many, again models have been developed by researchers who are several decades which all may be called as gravity model, so again it is a category of model. Again opportunity model, we can consider it as a category of model.

Within that we have intervening opportunity, we have competing opportunity and so on. So, in this module trip distribution, we shall discuss about all of these models. So, to start with we shall take up the growth factor models first, and then we shall go to synthetic models. So, first, let us take it the growth factor method.

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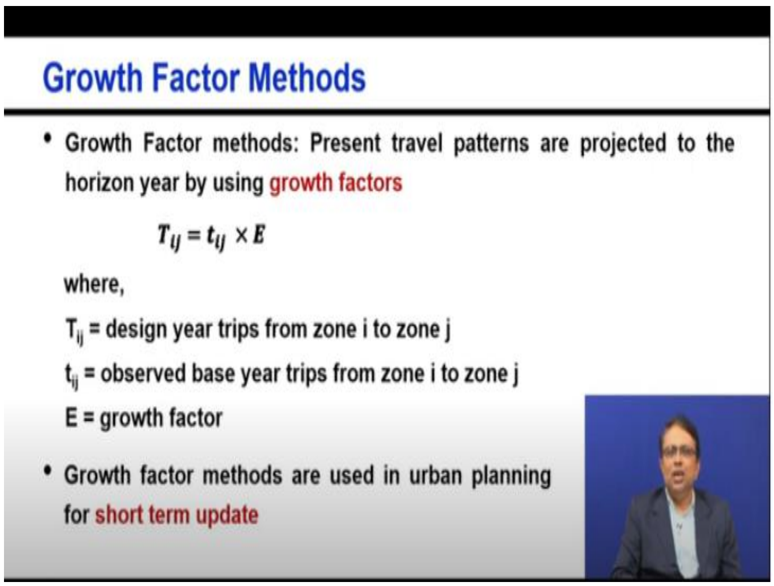

In growth factor method what is fundamentally done is as follows. The present travel patterns are projected to the horizon year that is the future. So, we know the present travel. Present everything is known. That is the fundamental assumption in growth factor models. Then by using growth factors we try to get the projected value in the future or in the horizon year or in the design year. So, T ij if I say that future value how many travel will happen from zone i to zone j?

Directional travel maybe we will say that T ij equal = t ij, t ij indicates how many observed base year trips from zone i to zone j multiplied by the factor which is called E. E is the growth factor. Now this growth factor we have again say that there is available several models which are all put under growth factor methods. How do they how they are different from each other? They are different in terms of how we are calculating that E or how we are applying that E, growth factor.

There one model growth factor based method or model is different from another growth factor based model. But every case this t ij that is the present or the base year element t ij element is known to us and we are applying a factor which is called may be growth factor to that to get our future value of that cell that is T ij. So, T represent future value, t ij represent the present value. And the present value is multiplied by growth factor E.

Now all these growth factor methods are used in urban planning for short term update. We shall discuss more about the applicability. Each approach the growth factor methods have their own advantages as well as their own limitations. Synthetic models have their own advantages but not that always we would like to use synthetic models. There are so many cases where you cannot use synthetic models.

Or where you need not use synthetic model, you can use simply a growth factor based model to do your work. That is what is specifically mentioned here in the last point. They are used in urban planning particularly useful for short term update.

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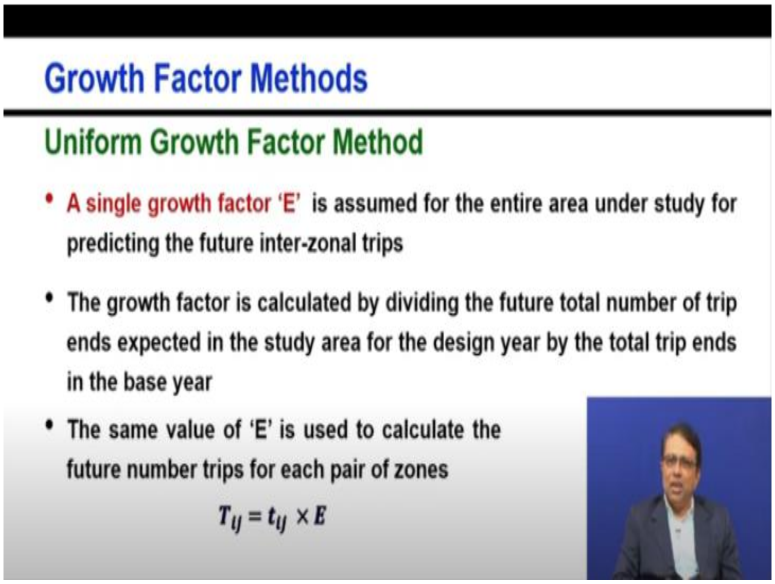

Now there are several methods or several models which have been developed. All may be called growth factor based methods because they are fundamentally the same. You have the t ij known in the base year or the current year. And we want to find out T ij value for the horizon or design year using the knowledge of t ij that is the current value and by multiplying that with the growth factor.

So, fundamentally all growth factor models are same in uniform growth factor method a single growth factor E for all the elements in that matrix. We use only one value of E. And that one value is applied to each cell of the matrix to update t ij to t ij. So, the E value is same for all the cells. That is why it is called uniform growth factor. That every cell we are using are multiplying with the same growth factor value. So, that is qualities as uniform growth factor method.

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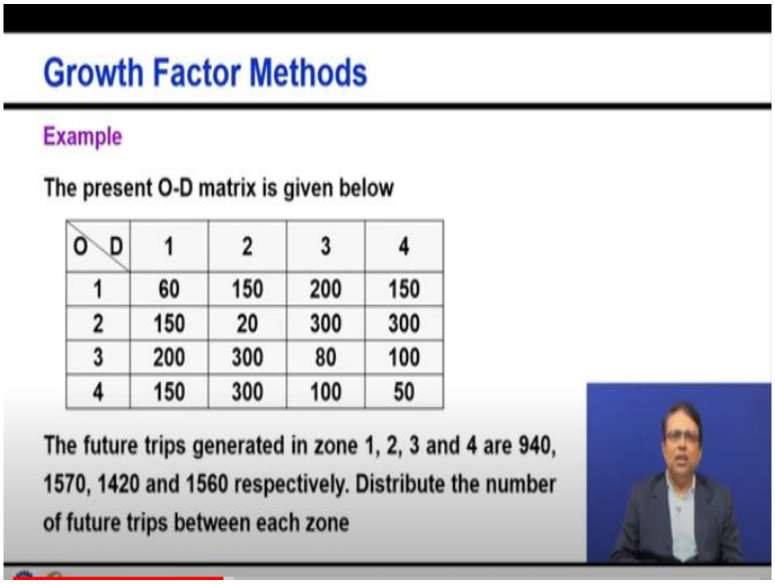

Let us take an example to understand it. Let us say the present O-D matrix is given. Like 1 to 1 60, 1 to 2 150, 1 to 3 200, 1 to 4 150 and like that so, each cell now these cell values are t ij. And we are saying that the future trips generated from 1, 2, 3,4 are this matrix in the whole matrix you can see that t ij equal to t ji. So, the productions and attractions for each zone can be obtained and for each zone the production and attraction will be the same because t ij equal to t ji.

So, the present one we can get what is the production for zone 1, 60 plus 150 plus 200 plus 150. What is the attraction for zone 2? Let us say 150 plus 20 plus 300 plus 300. So, like that the present productions and present attraction from for 1, 2, 3, 4 is 4 zones are known. What is also given the future generated trips in zone 1, 2, 3, 4 are 940, 1570, 14201, 1560. Now obviously attraction taking that production attraction for any zone the production attraction is seen here in this example.

So, the attractions also are likely to be the same 940, 1570, 1420 and 1560. Now, we would like to get these future trips using the uniform growth factor method.

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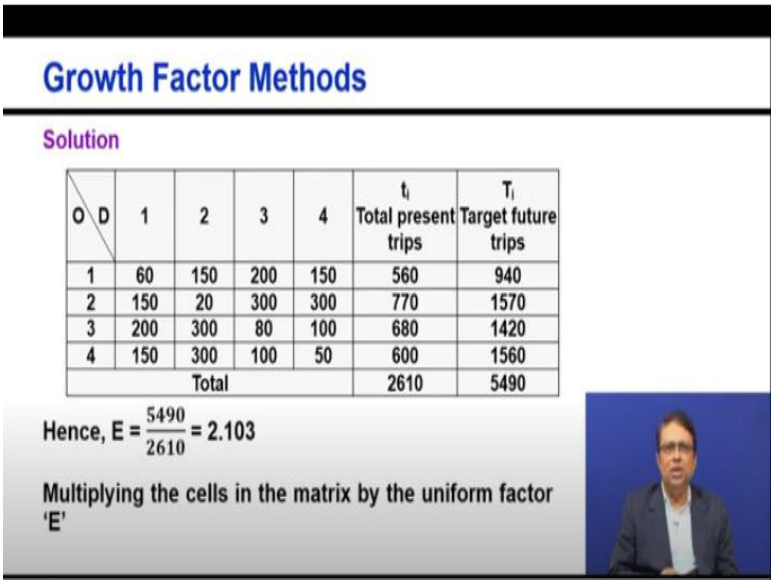

How to do that? So, the same matrix is shown here, you can see 1, 2, 3, 4. 4 zones and the original 4 zones in the destination and what we have done? We have calculated the total present productions and attractions are not shown but they will again be the same. So, the present productions are 560, 770, 680 and 600. And we know then total how many trips are happening in the whole study area consisting of four zones. 560 + 770 + 680 + 600.

So, we have 2610 trips. 2610 trips are happening now. The future productions are also given in this example. So, go back to the previous one. You can see 940, 1570, 1420 and 1560. So, those values are put here. So, we know that the future total trips in the study area will be addition of all these numbers which comes out to be 5490. So, the present total trips 2610, future total trips, 5490. So, what will be my growth factor?

My growth factor will be simply 5490 divided by 2610, so, 2.103 so this is my growth factor. And since this is uniform factor method what we will do? We will multiply each cell by this growth factor of 2.103. Then as per this model we will get the future values.

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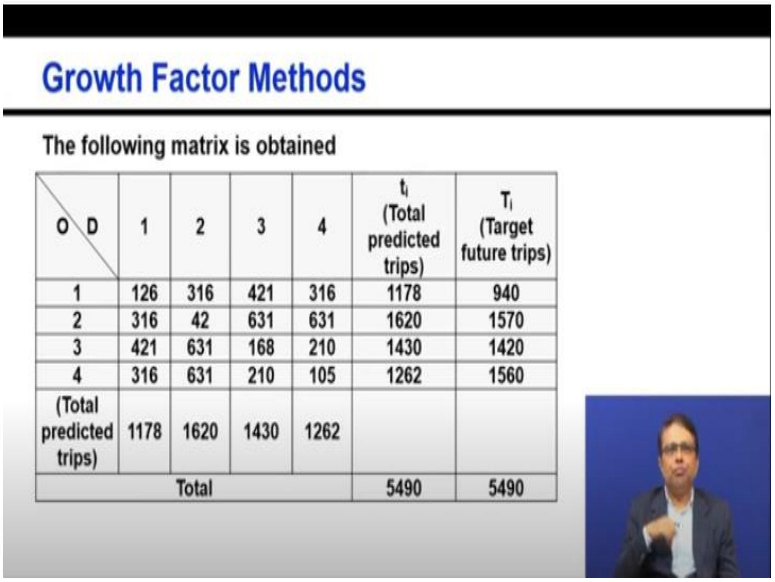

O \ D	1	2	3	4	t_i (Total predicted trips)	T_i (Target future trips)
1	126	316	421	316	1178	940
2	316	42	631	631	1620	1570
3	421	631	168	210	1430	1420
4	316	631	210	105	1262	1560
(Total predicted trips)	1178	1620	1430	1262		
Total					5490	5490

Exactly that is done. You will see the values are calculated and obviously say for zone 1 if you see interestingly now the total number of trips will match 5490. That is what our target and what we predicted also 5490. Because the growth factor total trips are upgraded. So, the total trips will match. But then you look at each zone. What you are getting? It is not very encouraging. We are expected to get supposed to get our target was to get 940 for zone 1. We got 1178.

Similarly for zone 4 our target was 1560. We got 1262 and the remaining also some differences. And as I said here in this case t ij and t ji are same for zone. Take a zone. You will find that productions and attractions are the same for zone 1, 1178 production, 1178 attraction. Zone 2, 1620 productions, 1620 attractions this is because t ij equal to t ji. Both elements are the same. So, what we find from these, that we are able to upgrade the total trips as per our requirements.

So, somehow we get a matrix where the total number of trips are matching or to the total number of trips is matching. But if we say origin and destination wise or zone wise rather zone wise if we see that whatever what we actually should get and what we actually got by applying uniform factor method the difference is very significant. There is a gross kind of mismatch. So, these need to be improved further.

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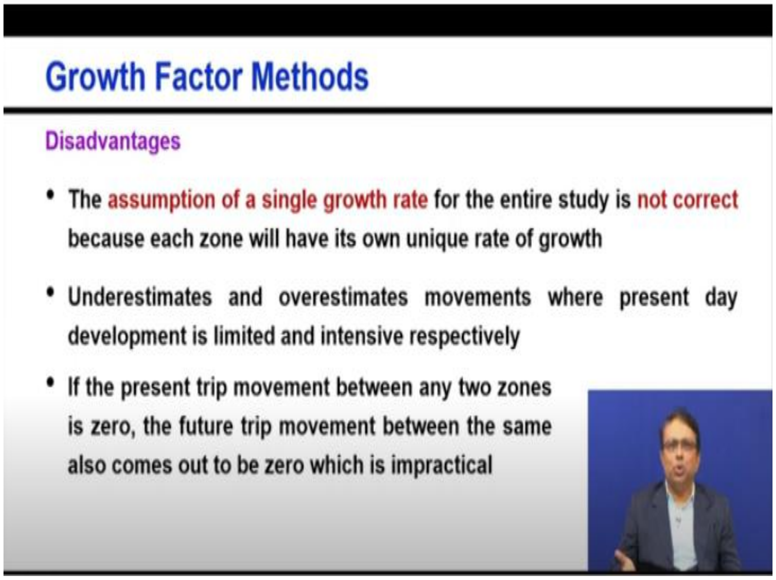

So, as we said what are our disadvantages? You can say that the assumption of a single growth rate for the entire study is not correct. It cannot be correct because different zones are there and they all have unique rate of growth. If you see the present trip production is 560 for zone 1 and the future is 940. So, what is the growth? We can calculate it 940 divided by 560. Similarly for zone 2 you can see the growth 1570 by 770.

Zone 3 1420 by 680 and zone 4 1560 divided by 600. So, you will clearly see that the growths are not the same in all zones. It cannot happen. Different zones in urban areas, the growth in terms of productions in terms of attractions across different zones growths cannot be the same. But what we assumed here, one growth factor for every cell. For all cells one growth factor. So, what happened?

The total number of trips matched but row wise production attractions did not give a good match. Because we did not recognize that each zone has its own growth. Second, it underestimates and overestimates movements where present day development is limited and intensive respectively because you are using a multiplier. Of course this is a common issue probably with all growth factor models.

Since you are using, in this case the same multiplier, then the lower values will remain lower. The higher value remains higher because if a zone is producing very low then on that number we are doing using a multiplier. So, obviously the future also will still be less. And here uniform growth factor so all the cells are getting multiplied by say whatever, was the value 2.1. So, every cell gets multiplied by 2.1.

So, low values also get multiplied by 2.1, high values are multiplied by 2.1. So, low will remain low, high will remain high. And if third point, if any of this cell is 0, then whatever you get is also 0 in the future. Because you are multiplying 0 with something so always it will be 0. This is particularly a common problem for all the growth factor based methods.

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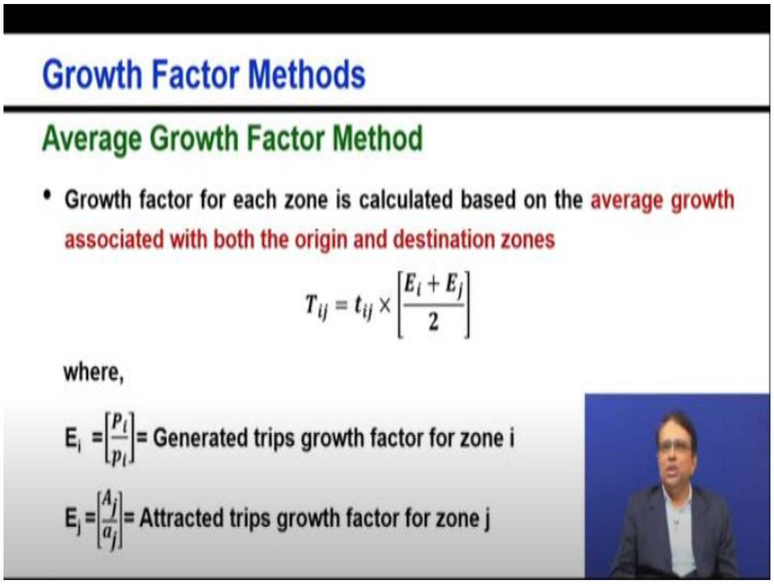

Now to overcome this then next came, a different model or a method what is called as average growth factor method. So, what was the issue? The issue was earlier uniform factor method that we did not consider the growth of different zones, we did not recognize that. Every cell we multiplied by the same growth factor. So, obviously my error was more. Here what we are doing? We are calculating separate growth factors for separate or cells.

Each cell has a separate growth factor. Why each cells the separate growth factor? Now the cell is T ij. So, I have got two texts. One is i another is j. So, any cell belongs to or is a part of a column which is j belongs to part of row which is I from zone to zone. So, there are two tags i, j. And the trip is happening between i and j. So, what it does? It takes the average growth of i and j. So, you can see here the T ij, future t ij value that is future year or design year or horizon year t ij value is t ij. What is happening now?

Multiply by E i plus E j, E i is the growth of zone i, E j growth of zone j by 2. That means we are taking the average of that. So, every cell it has got 2. One is i another is j. So, whatever is happening in i, whatever is happening in j both growths we consider and take the average of that. And so this cell growth will happen as per that growth. So, every cell i, j combinations will be different. So, the growth factor also will be different.

So, that way it is better and much more logical than the uniform growth factor method. So, that we have said here. What is E i? P i by p i but that means productions expected productions in zone i future row total and the present row total is p i. Similarly growth of zone j, A j attraction of zone j in the future divided by the present attraction of zone j that is a j.

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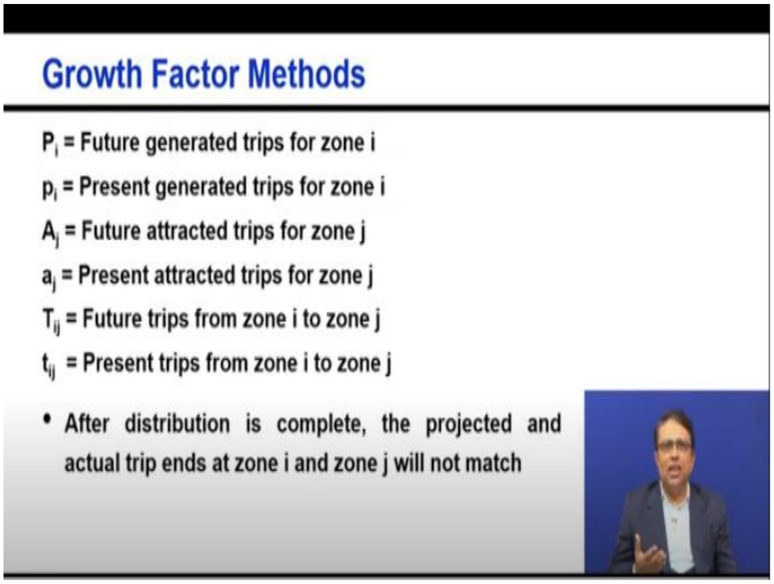

And what I said that P i is the future generated trip in zone i, small p i is the present generated trip in zone i, A j is the future attracted trips for zone j, a j is the present attracted trips for zone j. T ij is the future trips from zone i to zone j and t ij is the present trips from zone i to zone j. Now once to complete one distribution, you will still find that the projected and actual trip ends may not match, in fact, will not match.

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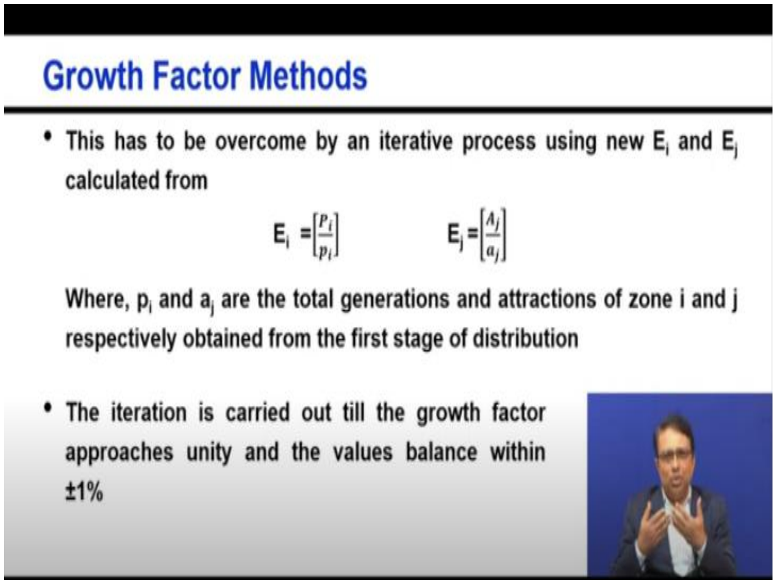

Then what is that we have to do? We can go for one more iteration. Whatever will be your new matrix after first iteration, you assume that that is your input matrix. And repeat exactly the same procedure what you adopted initially. Go for one more iteration. So, like that iteratively if you do you will find the target and actual estimated productions and attractions in the future will match very closely.

And if they match closely then you know that your internal cell distributions are probably reasonable because they give close match to your targeted row total and targeted column total.

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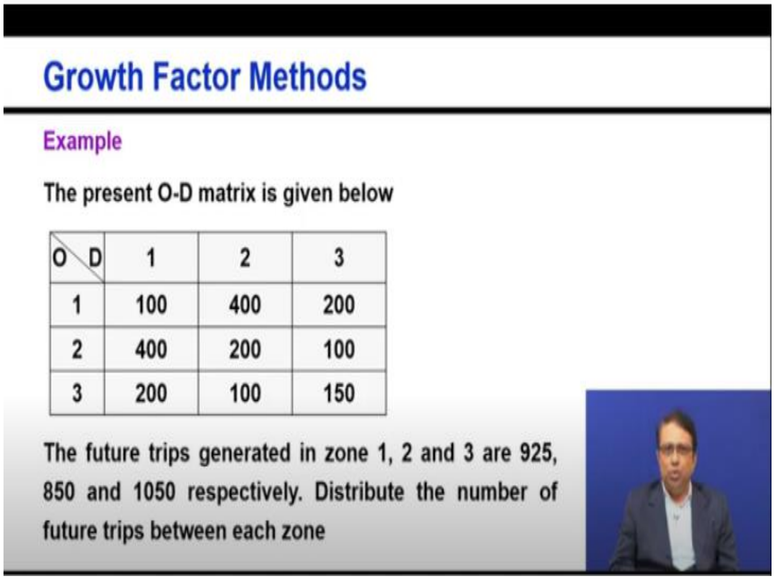

Let us take an example here. 100, 400, 200 those are the cells 1 to 1, 1 to 2, 1 to 3. Series also T ij equal to T j1 because you see 1 to 2 400, 2 to 1 is also 400, 1 to 3 is 200, 3 to 1 is also 200. So, here also the productions and attractions for a zone are the same. Now the future trips generated from zone 1, 2, 3 are known to be 925, 850 and 1050. And we want to distribute the trips and get the cell.

Its 9 values we want to get in the future which will give us a better match of row totals and the column totals with respect to zone 1, 2, 3. That is what is our target using the average growth factor method.

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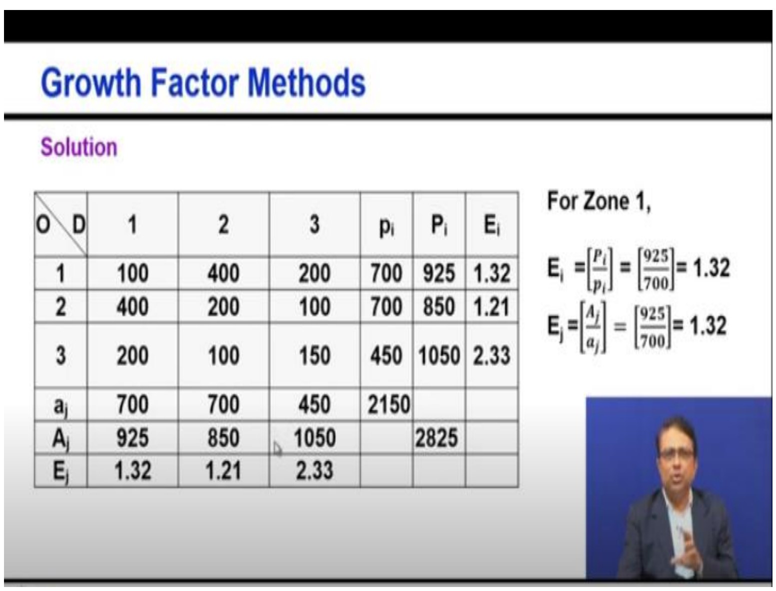

So, we do it here we start, what we do? We find out the growth factor of zone i, it is 1.32, zone 2, 1.21, zone 3, 2.33. How we got this one? We took the future of productions of zone 1, 925 divided by 700 then we get 1.32. Similarly you will get for attraction end also because for each zone production and attraction values are the same. So, here also it is column total if you see present is 700. Our target is 925.

So, I get a growth factor of 1.32. Like that each growth cell, each zone you can get the growth factor. So, we got three growth factors for the productions of ends, three growth factors for the attraction ends. Of course, they are similar here for each zone. The growth for the origin and growth at the destination for productions and attractions are the same.

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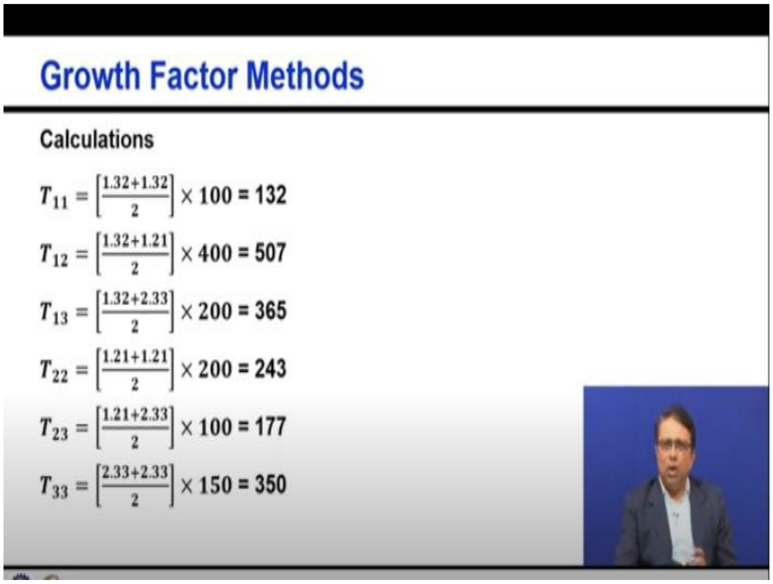

Now what we will do? We know say T 11. 1 1 means one end is also one and the other end is also one. So, origin and the growth is 1.32, destination end is 1.32. So, 1.32 + 1.32 by 2 into 100, 100 was original value. So, there remain 100 into 1.32. So, you get total value 132. Similarly T 12 growth of zone 2 is 1.21. 1 is 1.32. So, average of 1.32 and 1.21 into 400. So, you are getting 507. So, like that you take each cell, see i what is j, what is the growth at the production of i, what is the growth at the attraction end of j, take the average growth and multiply that cell respective cell with that growth to get the future values.

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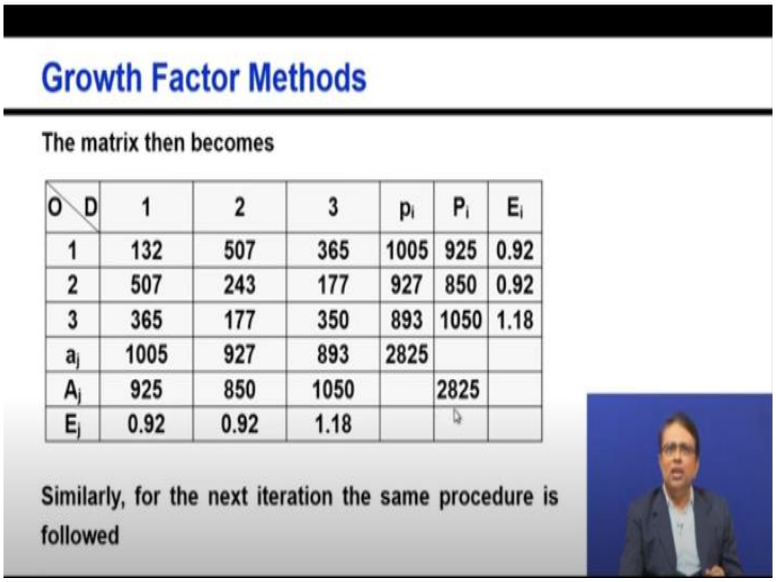

So, like that we got it. So, what we can say here, here the total target trips in the future are matching as usual. This match we also got in the uniform factor method. This is not a surprise. This is not something better what we got here. Anyhow there also we got it uniform factor method as well. But look at this value. We got 1005 whereas our target was 925. Zone 2 we get 927 whereas we wanted to get 850. Zone 3 we got 893 whereas we wanted to get 1050.

So, still there is a mismatch. And in uniform growth factor method we could not do anything else. We had to leave it. Here we can still try to make it better. So, we have to go for the next iteration. So, what we will do? We will say as you started with iteration with an initial matrix and then you have a target total and the growth factor at the origin and the distribution and then you update it. Now stop assume that this is your starting matrix.

That means your 1005, 927, 893 and the internal cells whatever you get is this whole thing is your starting point. And you want to modify this cell so that for zone 1 you get total production as 925 so with the growth factor of 0.92. Second case instead of 927 you want to get 850 so the growth factor of 0.92. Third case you got 893 you want to get 1050 so with the growth factor of 1.18.

So, these are your new growth factors for the next iteration. As you got growth factor here 1.32, 1.21 and 2.33 instead of that now you got your growth factor as 0.92, 0.92 and 1.18. So, you will go to a second iteration and as you go to the second iteration, you get again a matrix. Just see what is your E value, growth factor values? Now, obviously you can decide your acceptable error limit.

I do not expect this to be exactly 1 and maybe 0.99 is ok for me. Someone else 0.95 or 0.96 to 1.04 is fine for me. Somebody may say well 0.99 to 1.01. You said your limit. But then you need more number of iterations. And you will see that all these cell values will get adjusted slowly in such a manner that your target total and what you will get from your updated matrix for each zone will match more closely.

So, obviously now what you got as 0.92, 0.92 and 1.18, if you go to the next iteration you find each of these values will move may be closer to 1. They may not be exactly 1. But as you go for more and more iteration, it will move towards 1. And then depending on what is acceptable for you decide an error limit. Every planning estimate will have some error, so if 5% error is ok or 10% error is ok you decide whatever is your acceptable error limit.

Accordingly you decide and that many, iterations probably will be required or more and more iterations will be required to get the results as per your expectation.

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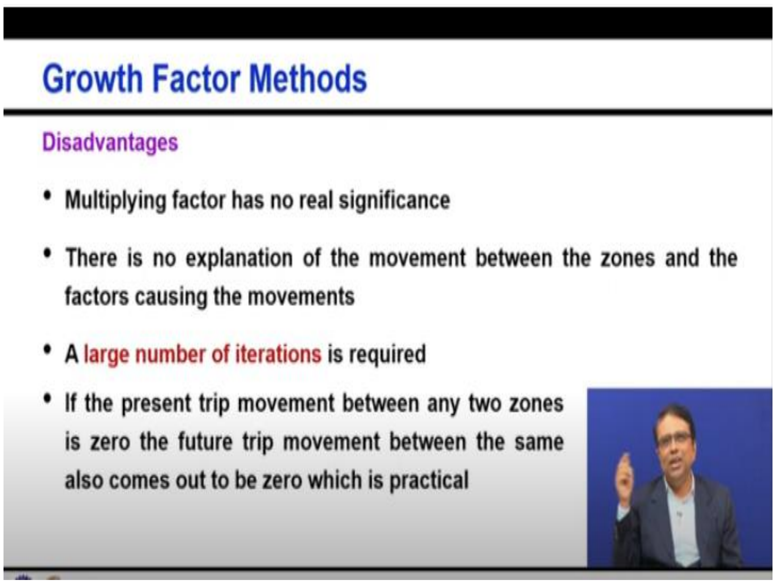

So, what are the disadvantages? Disadvantage here is again multiplying factor has no real significance. Of course, we duly consider more logical factor we use. Because earlier just one factor for every zone whereas know that each zone has different growth. And you talk about a cell then i and j both ends are important. So, growth at i and growth at j both will influence. So, that consideration is there.

But there is no real significance. It is just a factor. That is what we used it. What is the physical meaning of that? That is probably not much, nothing much can be said. There is no explanation of the movement between zones and the factors causing movement. We know that how many trips would happen will depend on actually that properties of the transport network, travel time, travel cost and connectivity. All these are very very important.

So, we do not consider all any of those. Simply some number game, we are doing some balancing, some factoring, so that we get you know our row total and column total. Whatever we are getting from the modified matrix are matching closely with our target total. So, nothing beyond. Sometimes you may require a large number of iterations. If you are depending on your error margin and also the same problem is here.

If any cell initially is 0 whatever you do finally end up the game. You will get again 0. Because you are multiplying a factor on that original value. So, if the original value is 0 everything is 0.

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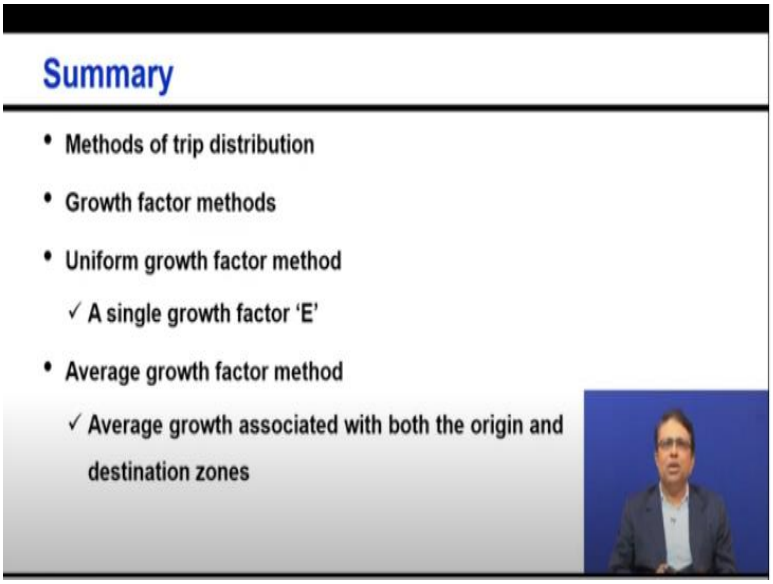

So, to summarize, we discussed in this lecture about two broad methods of trip distribution. Growth factor based and synthetic method. We did not discuss it in detail. But we just mentioned that there are two broad categories or two broad approaches for modeling. Then we discussed what is really we do growth factor based method. Present cell we take, multiply it by a factor E or a growth factor to get the future cell or in the horizon year or in the design year.

Different methods vary or different methods actually calculate this and apply this growth factor in different manners. That is why they are different methods. But all actually apply a factor to a given or the present cell to get the future of the design year or the horizon year cell. And then we discussed two specific methods: uniform growth factor method and average growth factor method and explained to you how they are applied with some examples. So, with this I close this lecture, thank you so much.