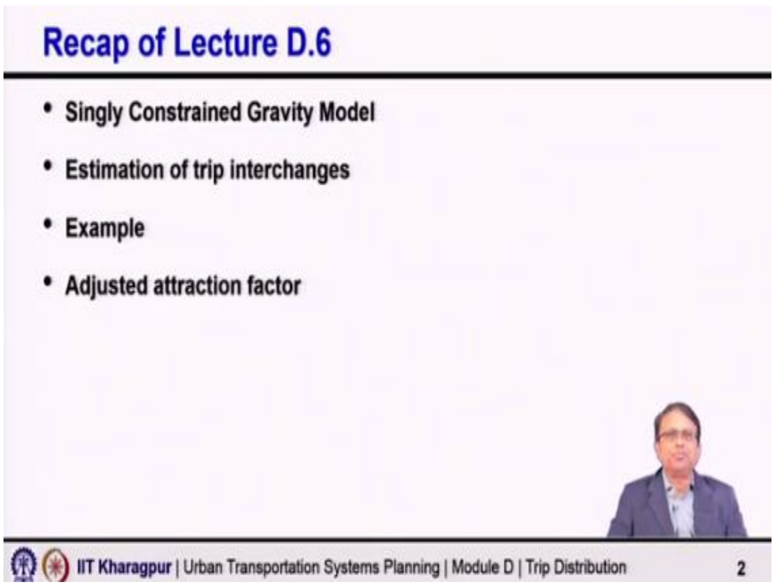
Urban Transportation Systems Planning
Prof. Bhargab Maitra
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture - 27
Bureau of Public Roads Calibration Procedure (BPR)

Welcome to module D lecture 7. In this lecture we shall discuss about Bureau of Public Roads calibration procedure. Or what is commonly known as BPR calibration procedure for the friction factors.

(Refer Slide Time: 00:31)



In the previous lecture, lecture 6 we discussed in details about Singly Constrained Gravity Model. Why we call them singly constraint? In mention that it could be production constraint or it could be attraction constraints. Then we said how stepwise we develop such models and apply them? Then two can example also to demonstrate how such kind of models can be applied to get the trip in interchanges.

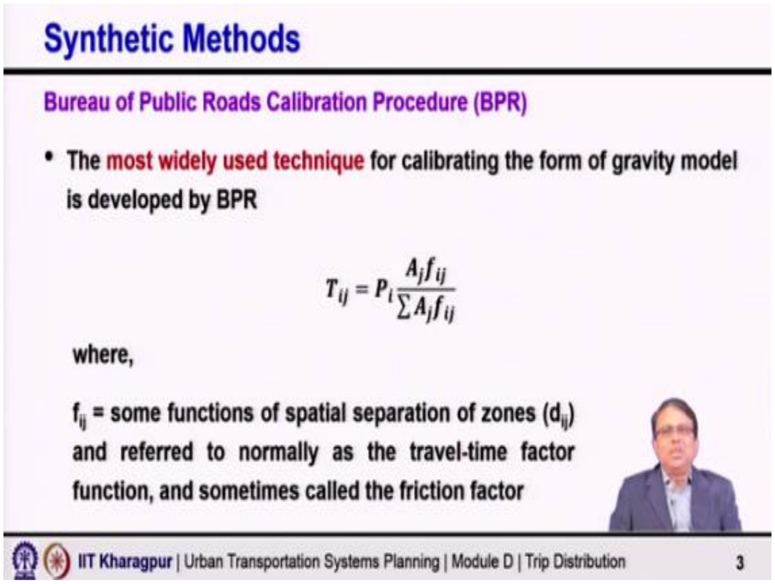
And also, to get the trip interchanges which will auto which will match the row totals and the column totals. That means which will match the productions and attractions both constraints and as we use singly constrained gravity model. It automatically ensured matching of one end. We use production constrained model. So, the production end matched but the attraction ends did not match automatically.

So, we explained to you how externally we can do iteration and balancing so that the attraction ends also match closely. That was explained in the, all was explained in the previous lecture. But if you remember the example, we took we said someone has developed this friction factor values for you. We took an example; productions, attractions and also the travel time distribution. And then we give a corresponding you know, f ij values corresponding to the travel time. And we said somebody has developed this friction factor values or friction factor curves for you.

Now in this lecture that is the point where we want to focus how to develop this friction factor values or friction factor curve? That means time versus f ij value. How to develop that? Somebody has to develop then only one can apply. So, here we shall focus on that part how to develop the friction factor values from the corresponding trip travel times. We shall follow a procedure what is known as BPR calibration technique popularly known.

And that following that BPR calibration technique we shall explain how you know, this can be applied to develop the f ij values.

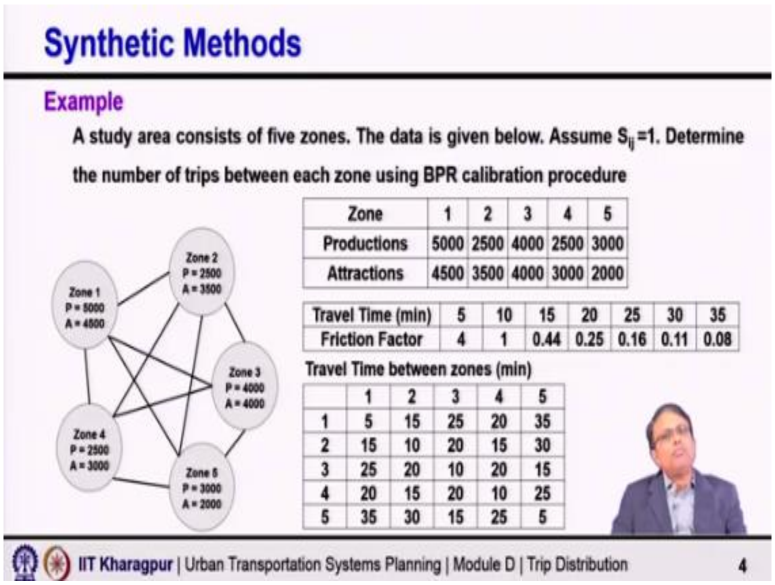
(Refer Slide Time: 03:27)



So, what we do these BPR calibration procedure? It is the most widely used technique for calibrating the gravity model and what we are essentially doing we are actually using again here a singly constraint gravity model inside. So, you can see this equation you are quite familiar with this equation by this time. T ij = P i, a fraction of P i will go to a particular zone j. How much fraction will go? A j f ij divided by sum over A j f ij sum over all j.

That is the fraction that will go to a particular destination j. So, this is the basic model that will be used inside even during the calibration.

(Refer Slide Time: 04:29)



Let us take an example and with this example, we shall explain the steps because if I explain only the steps without an example, it may be difficult for you to understand and connect all the steps. But so, I try to take an example and then go stepwise with reference to this example. So, that, things become very clear to you. Let us consider a study area with 5 zones. The data is given here again zone to zone adjustment factor initially we are assuming as 1.

We are not using any zone to zone adjust factor now. And let us say that zone 1, 2, 3, 4, 5, there are 5 zones. The productions are given for example 5000, 2500 in zone 2, in 3 4000, in zone 4 2500, in zone 5 3000. Similarly, the attractions for all these 5 zones are also given 4500 3500 4000 3000 and 2000 respectively. Then the, Travel time matrix you know I have adjusted the tables like this.

So, after this production attractions come to the travel time between zones. So, how this are connected? Different zones are connected. And what is the Travel time for each pair of zones?

So, if you want to go 1 to 2 15 minute 1 to 3 25 minute 1 to 4 20 minute, this is an example. So, I have taken the travel times in multiple of 5. But this is nothing like you cannot take other value you can always take any value.

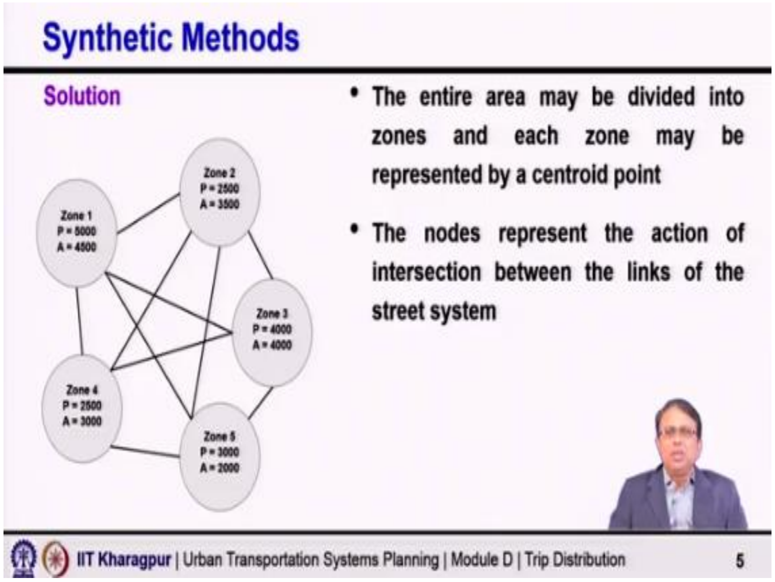
In this example, I have just taken travel time values which are in multiple of 5 minutes. But as I said any value any range is fine. If you take a big city if you say Calcutta or if you take Mumbai or you know any other city the travel time will vary maybe vary from 5, 10 minutes to even 120 minutes 2 hour even more it can happen. The peak hour it may even take 2 hours. So, the travel time range could be quite big. But here it is just an example because my sole purpose is to explain you clearly the steps how we develop.

So, I have taken an example, which is relatively simple. So, my travel time values are there. So, you can see here what are the discrete travel time values? 5, 10, 15, 20, 25, 30 up to 35. So, finally, for each of these travel time values I need to find out the corresponding f ij value that is what is my objective that is what the calibration we want to do. I have assumed they are some values. Let us say for 5 minute it is 4.

You know already that logically higher the values less of travel time lesser should be the values for friction factor. So, that trend is maintained. But who told before? Nobody is just my assumption instead of 4 you could take 40 also does not matter. Take any value. Even if you suppose you have forgotten that you know, the higher the value lower will be the friction factor higher the value of travel time that lower will be the friction factor values.

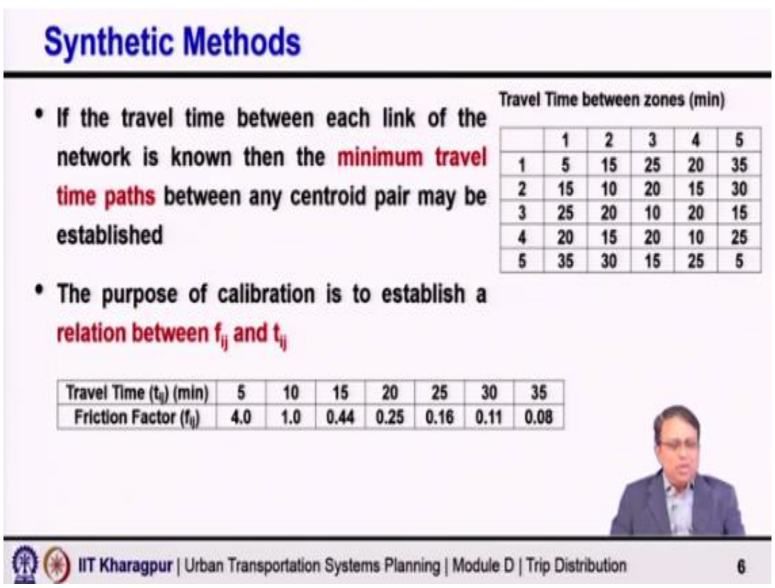
If you have forgotten, you have taken say reverse. Let us say 5 minutes, you have taken 0.08, and then 10 minute 0.011, and then finally 35 minute you take 4. You have taken just a reverse pattern. Still does not matter. Finally, everything will be shaped up. That is what is the beauty of this calibration technique or even you take equal value. I do not know 5, 5, 5, 5, 5, 5 all 5 minute all friction factors are 5 or friction factors are 10 or 1 anything you like you take it does not matter you assume any value you like. Then stepwise we can go and we can get the final curve.

(Refer Slide Time: 09:39)



How we get it? So, the stepwise if I have to say the first step is the entire area may be divided into zones and its zones may be represented by a centroid the nodes representing the action of intersection between the links and the straight system. So, these are all the basic things which anyhow you have to do if you are actually trying to calibrate a friction factor curve or friction factor values.

(Refer Slide Time: 10:07)

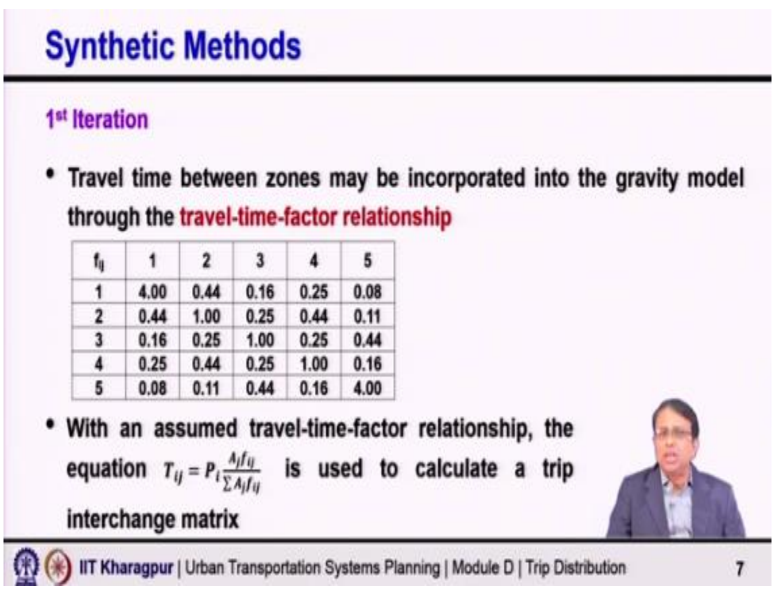


Then as I said the travel times are known. How travel times are known? Again, it is a network so there will be multiple connections multiple paths. So, you have to probably get the minimum travel time path or the shortest path you have to get. And those travel time you will put in the particular cell. Say for example 3 to 4 if I go if I want to go then maybe 20-minute time that is along the minimum path.

So, I know this is the minimum travel time. These are the practical I am saying practical aspects because here I have given the time. But what these times values indicate? They are the actually zone to zone along the shortest path, that is what I want to mean. Now as I said, you know, what is the range of travel time? Or what are the discrete values in this case? You know, that 5, 10, 15, 20, 25, 30 and 35 are the discrete values in this case.

You have to make an assumption of the corresponding f ij value. I said clearly. Take any value you like take all equal value, no problem. Take higher value for lower travel time and lower value for higher travel time, which is logical that is also fine. If you take just reverse you take lower value for lower travel time and higher friction factor value for higher travel time that is also fine it does not matter. Assume something, once you have done it then the next step. What is the next step?

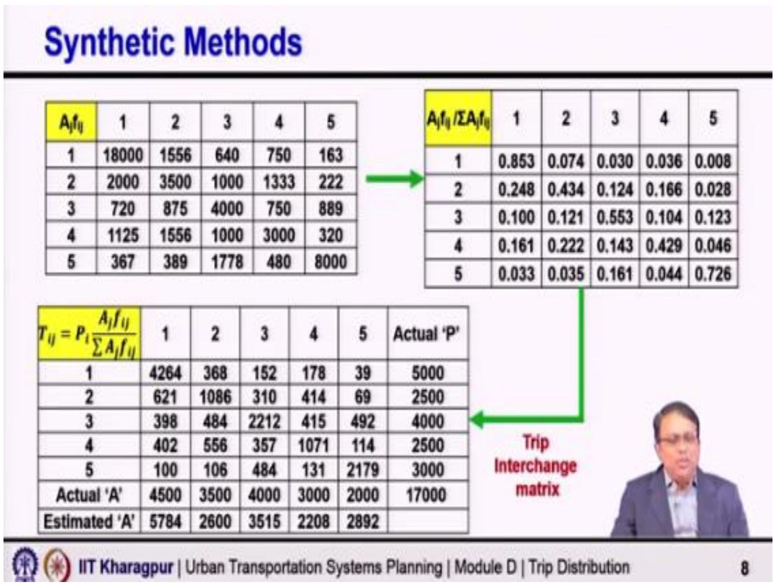
(Refer Slide Time: 12:07)



The next step is basically I am going for the first Iteration. What I will do first? I know the travel time intra-zonal and inter-zonal travel time from the given matrix. So, I shall actually you know, take that those values and then take the corresponding f ij value. For say example 1 to 1 travel time is 5 minutes, so that means by f ij value is 4. 1 to 2 15 minutes that is why that means by f ij the value is 0.44.

So, I make a matrix of f ij from the known travel time I now convert those travel time and get a matrix which is basically f ij matrix. Then assume travel time with this assume travel time factor relationship. We now go for application of a single constraint gravity model with this friction factor values which are assumed certain factor values for different travel time uses a singly constrained model and go for the trip distribution. So, that is step 2.

(Refer Slide Time: 13:46)



I have shown it here. The first table on the left the green arrow shows how one table to another table to then 3 tables are shown. So, we start from the left top table. Then go to the right top table and then from that we get the finally the table which is shown in the bottom. So, the first one what we calculate we know a j value attractions of each zone 1, 2, 3, 4, 5. All these zones the attractions are known so attractions are given here.

Zone 1 attraction 4500, zone 2 3500, zone 3 4000, zone 4 3000, zone 5 2000. And then we know also this f ij values. Take the zone 1 first so 1 to 1, 1 to 2, 1 to 3, 1 to 4, 1 to 5. So, all are a j means attraction of zone 1 attraction of zone 2, second case; third cell attraction of zone 3, fourth cell attraction of zone 4, fifth cell attraction of zone 5 but then f 1 1, f 1 2, f 1 3, f 1 4, f 1 5. How we get this f values?

We know the travel time and from that we have also calculated or formulated this friction factor tables so f 1 1 is 4, f 1 2 is 0.44, f 1 3 is 0.16, f 1 4 is 0.25 and f 1 5 is 0.08. How we got this value? Again to remind you we got these values because we know that actual travel time. That is what is the given here and we have assumed certain values of friction factor corresponding to various travel time.

So, as per the given travel time and as per the assumed friction factor values, we know what are the f ij values for the specific pair of zones and accordingly we can calculate this A j f ij. So, for zone 1, zone 2, zone 3, zone 4 and zone 5 to all the destination zone 1, destination zone 2, destination zone 3, and destination zone 4 and destinations zone 5. So, all these values you can calculate. Once you know these values you can also calculate sum over a j f ij each row just add the total. That will give you for all destinations zone j.

What is the total? That means in the first case 18000 + 1556 + 640 + 750 + 163 that is the total sum over a j f ij then each cell like that I can do it for 1, 2, 3, 4, 5. All cases for every zone I can calculate some over a j f ij. Then each cell divided by the sum of the total for this corresponding row. So, a j f ij divided by sum over a j f ij, those values are shown in the second table on the right corner right top.

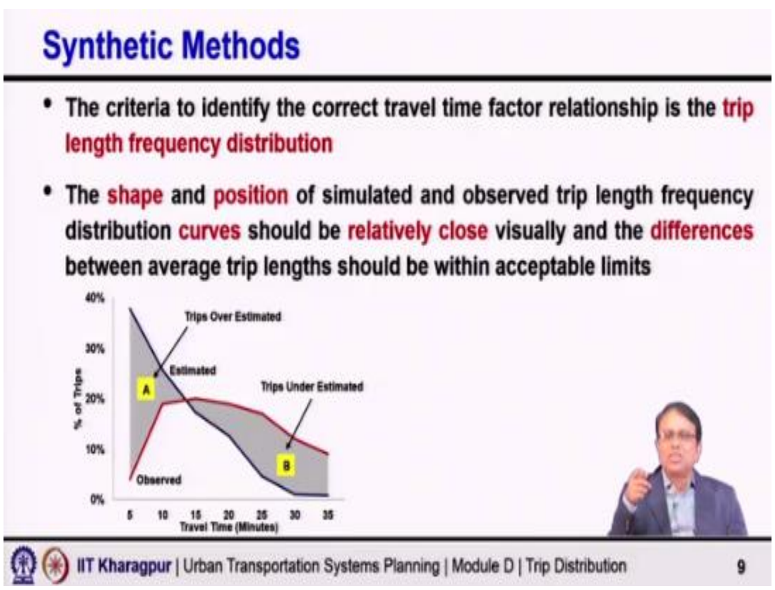
Once you have done that then for each origin zone, like zone 1 I know what is the value of p i. How do I know that? That is again given the production values are given. Each zone the productions are given for 1 5000, 2 2500 like that. So, let us say zone one 5000. So, 5000 into 1 1 is 0.853. So, 5000 into 0.853 that gives you 4264. 5000 into 0.074 that gives you 360. 5000 into 0.030 that gives you 1652 like that you get 178 and then you get the 415.

All these values are known and if you sum it up, you are seeing that you are getting 5000. Again, no wonder because you have used a production constrained gravity model. Obviously, the row total cells will match. Column total as usual did not match exactly but we will not apply the balancing now at this stage. So, we go ahead with that. Now If our assumed friction factor values are correct. I repeat If our assumed friction factor values with respect to different travel times are correct.

Then this distribution grossly what we got should match with my observed distribution? How we compare these distributions? What we will do? We will calculate now the trip length frequencies. What does it matter? That means we are not going to compare zone to zone values we cannot do that. Because what is our objective? Why we are not doing it? Now tell me the answer what we why we are doing it not doing the cell to cell comparison now.

Because our basic purpose at this stage is the calibration of friction factor time this vis-a-vis f ij value that is what is our target. Are we able to properly calibrate that curve? Are we get really suitable values of f ij corresponding to every travel time value? So, what we will do? We will calculate trip length frequencies with respect to each travel time. That means in the network we know that I have some 5-minute travel I have some 10-minute travel we have some 15-minute travel, 20-minute travel, 25-minute, 30 minute and 35 minutes. In this example.

(Refer Slide Time: 21:12)



So, with each of those travel time how many total trips are being made as per my model estimates? They may be happening, multiple pairs. So, maybe 10 minutes, maybe not just one pair but multiple pairs. If they are as a 2 pairs where the trips distribution is happening with 10-

minute travel. I will simply add those 2 values. Because I want to get total number of trips with 5-minute, total number of trips with 10 minutes, total number of trips with 15 minute and so on.

Then this total number I want to divide it by the total number of trips all travel times. So, then what I will get. I will get percentage of trips. Total percentage of trips or percentage of total trips happening with 5-minute travel, percentage of total trips happening with 10-minute travel, percentage of total trips happening with 15 minutes travel and so on so forth. So, that is what we want to get.

What we can call that? That we can call trip length frequency distribution; trip length in terms of travel time of the trip. So, we have certain trip length in terms of travel time and the corresponding frequency percentage of total travel that is happening with that travel time. Now if I have really taken the values correctly, which actually, working or reflecting the real travel then by observed trip length frequency.

That means what is my observed travel with 5-minute, 10-minute, 15-minute, 20-minute percentage of total travel made with all each of this travel time values. As we have observed and as we have got the values from the model, they should exactly match. They should give a very close match. So, what we are doing here. The criteria is to identify the correct travel time factor relationship is the trip length frequency distribution.

So, you are comparing this total length frequency one as we have obtained from the gravity model output and as we have obtained from our field observations. Again, I am saying what is my x? My x is the travel time of the trip length. What is my y? Percentage of total traps happening with that travel time. Then I have got one blue line one red line shown here. What they indicate? one is based on the gravity model one is based on the observed value.

So, shape and position of simulated, simulated means what you have got from the gravity model output as just have shown in the previous slide. And observed trip length frequency distribution curve should be relatively close visually once I plot them, they should appear to me, yes there is a reasonable match reasonably good match with these two curves. And the differences between average trip lengths or average frequency rather the percentage of trips within acceptable limit.

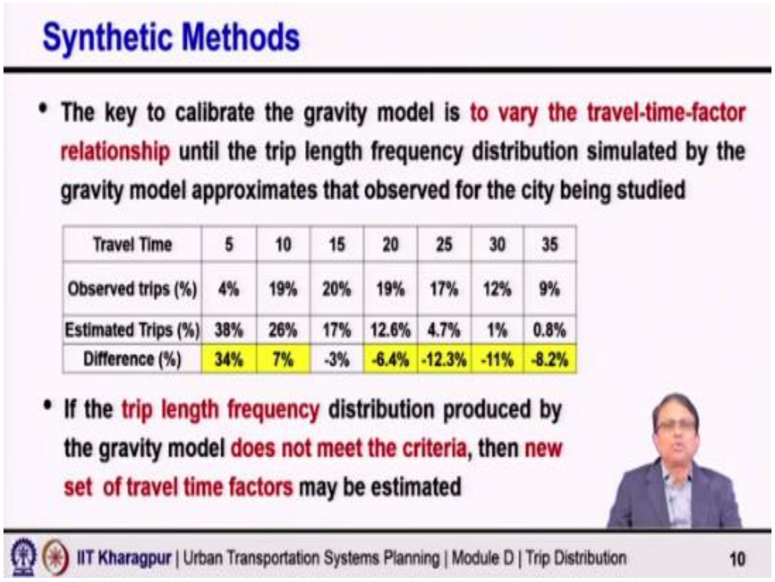
So, two curves basically should match. Here you can see this graph one is the blue line which is the estimated and the red line is actually observed. You can clearly say for 5 minute travel whatever my gravity model is saying percentage of trips, it is actually overestimated. Very high percentage nearly 40% trips are happening with that. But in reality, only less than 10% is happening similarly for 10 minute travel again my model overestimated.

But on the other hand, for higher travel time or trip length say for 30 minutes, 35 minute, the models are underestimated. So, now that clearly shows the friction factor values corresponding to different travel times, what I have assumed those values are not correct. So, you have to modify them. How to modify? How we can modify? We can modify based on looking at this graph or looking at the actual values.

Say friction factor values are used as a multiplier. So, if my model values are higher for any particular travel time the corresponding percentage here if it is higher that means I have used a higher multiplier value as a f ij I need to actually reduce it. You have multiplied with some value. And you find that total value is percentage is higher. So, that means you have actually used a higher multiplier higher value of the for the multiplied f ij.

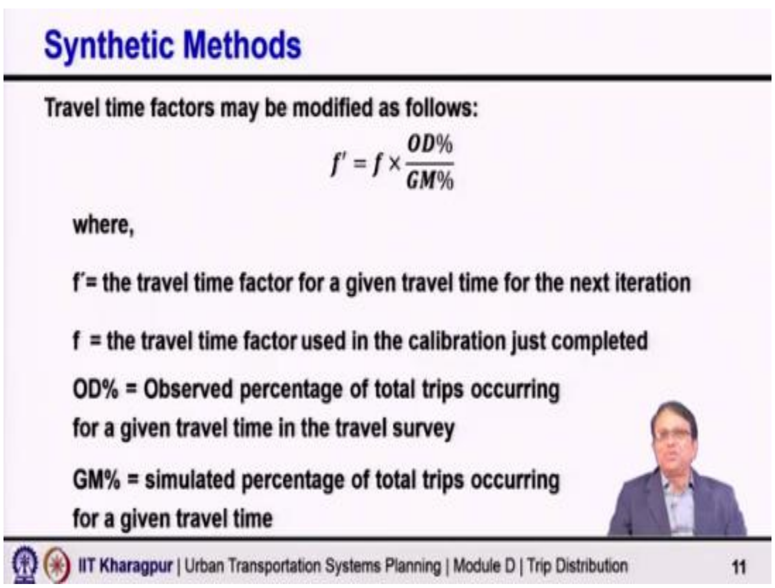
If you have got lesser, that means you have used a lesser value. Then you should increase it. If you have used if you find your over estimating the model trips are over estimated then you have used a higher value you should actually reduce it. So, those adjustments are to be done.

(Refer Slide Time: 28:13)



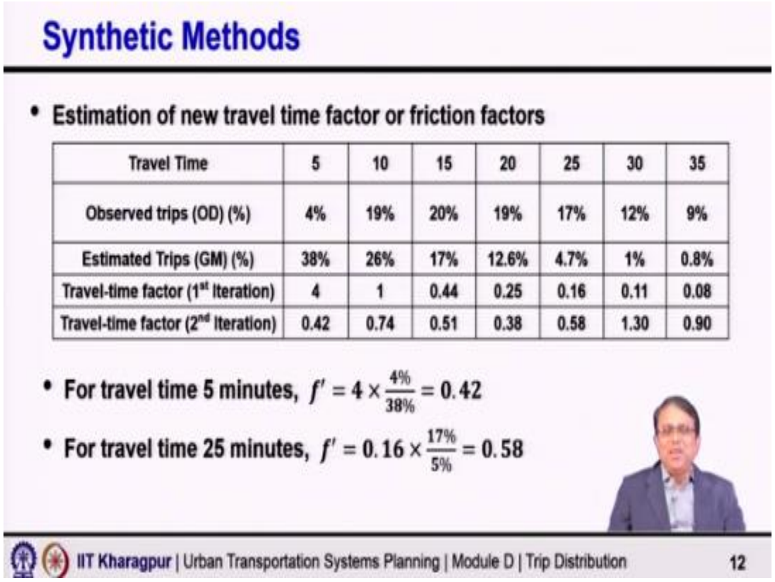
How those are to be done? Before you do that, I have shown it here. Let us say that you know the model value and we know also the observed trip lines. So, you clearly see there is a big difference visually also this slight showed you that they are not matching and here in tables also actual values also show that they are not matching. Somewhere you have overestimated somewhere we have underestimated. So, we need to modify the friction factor values.

(Refer Slide Time: 28:47)



How we modify? As I tell you, this is the formula. My friction factor value for a given travel time, if it was f and then the new value should be f dash equal to f into observed percentage divided by as the percentage you got from the gravity model. As I say if gravity model has overestimated, gravity model value is more that means if I evaluate should be reduced. That is what will happen? If the gravity model has underestimated that means I have used lesser value. I should use the higher value of f ij. So, accordingly this will give you higher than F value for the F dash on the modified friction factor value.

(Refer Slide Time: 29:45)



That way let us modify it. So, these are this slide shows that what is my observed trip, what is my estimated trip from gravity model and what was the travel time factor used in the first iteration. And then how using this formula modified friction factor equal to friction factor, which was used earlier in the last iteration and by using that what observed gravity model percentage you got and what is your actual observed percentage for that travel time.

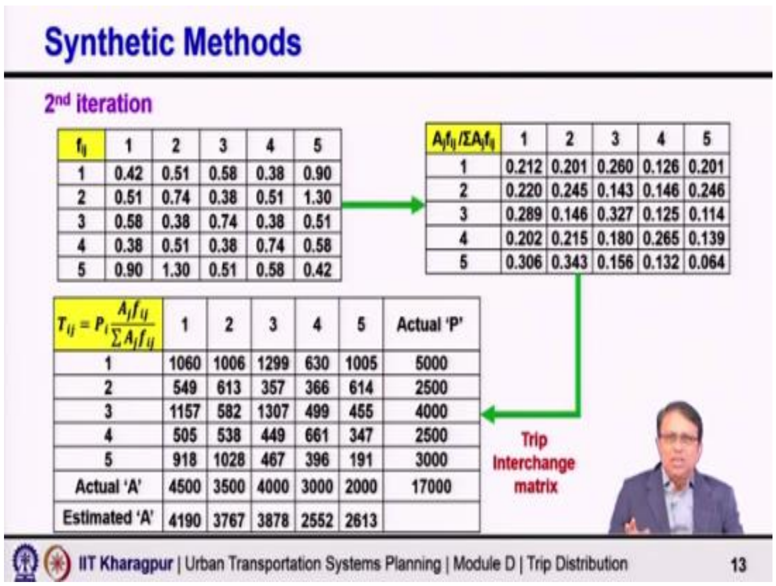
You can compare and then accordingly modify it. So, exactly following that we modified the value of f ij. I have shown you here how would the modification is being done? As I said, if my gravity model given me higher value than the observed then f ij value I have used this higher I should reduce it. And if my gravity model has given me lesser value then observed then I should use the higher multiplier.

So, my f ij value should increase exactly. You see it here you estimated value in the gravity model is 38% and your travel time actual observed value is 4%. So, you used actually value of 4.

So, the 4 values very high we need to reduce it. So, what we will do 4 multiplied by what we actually got observed 4% divided by gravity models is 38%. So, now you should use the value of 0.42. We are shaping up.

So, with whatever travel time whatever is the condition accordingly, the values will be modified. Now you have a modified value as you have shown.

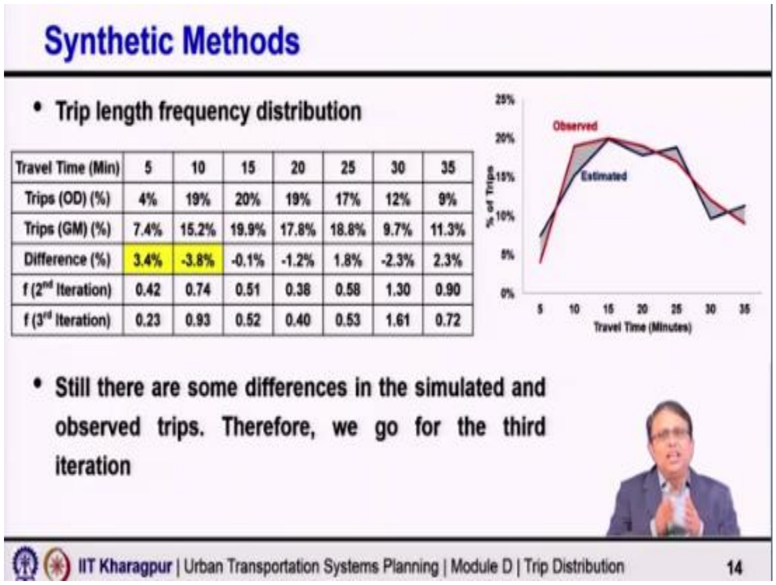
(Refer Slide Time: 31:45)



With this you go again for the second iteration. Again, you produce three tables. First you calculate f ij value for each cell with respect to each i. So, row 1 then all the j values row 2 all the j values and then sum it over. A j f ij then A j f ij divided by sum over A j f ij then again multiply it like p i into A j f ij by sum over A j f ij. So, take the respective productions of each row multiply it by the corresponding cell value what you have got in table 2 you get the thing here again, the productions will match automatically because it is a singly constrained model.

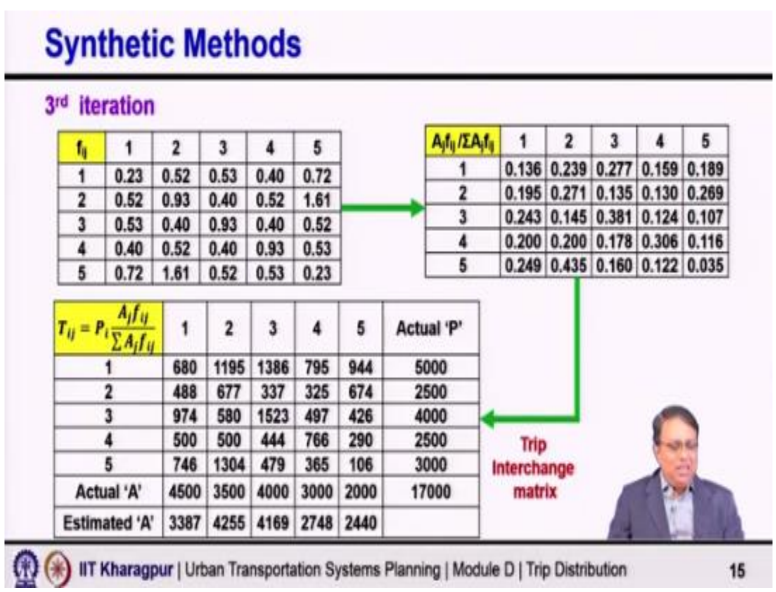
Again, you calculate how many trips with 5 minutes, how many trips with 10 minute and so on. And then what is that percentage of trips considering the total trips? How much percentage of total trip? Now the model is predicting with the revised f ij value with respect to various travel time.

(Refer Slide Time: 32:51)



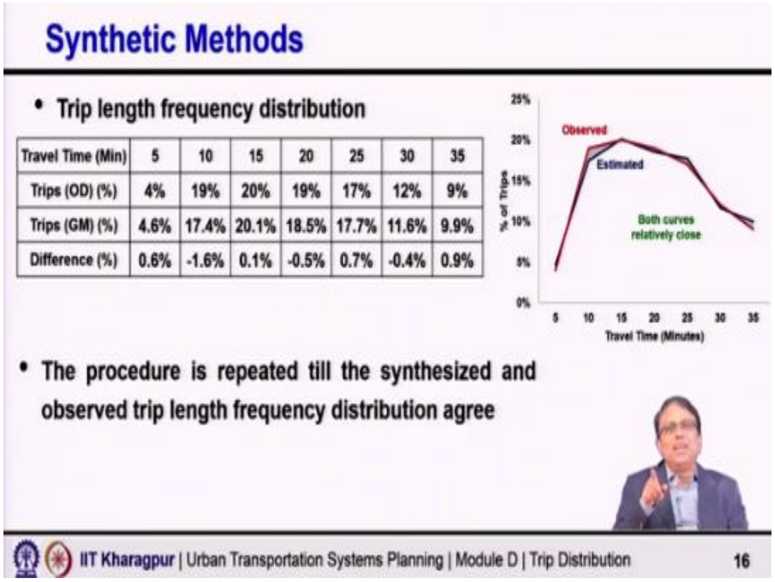
Again, you compare you find in this iteration. You are realizing that now you are getting a slightly better fit. It is even much better as compared to what you got in iteration 1. But still it is not matching probably you expect even it should match more closely. Go for one more iteration. So, again, modify the value of f. So, I have shown here what if we used in the second iteration and then with those f what percentage of trips with respect to each travel time, we got from our gravity model and what percentage of trip is actually observed based on that again I find out the value.

(Refer Slide Time: 33:49)



And then go for iterations, go for iteration 3. Calculate it again. Again, all the steps are similar.

(Refer Slide Time: 33:57)



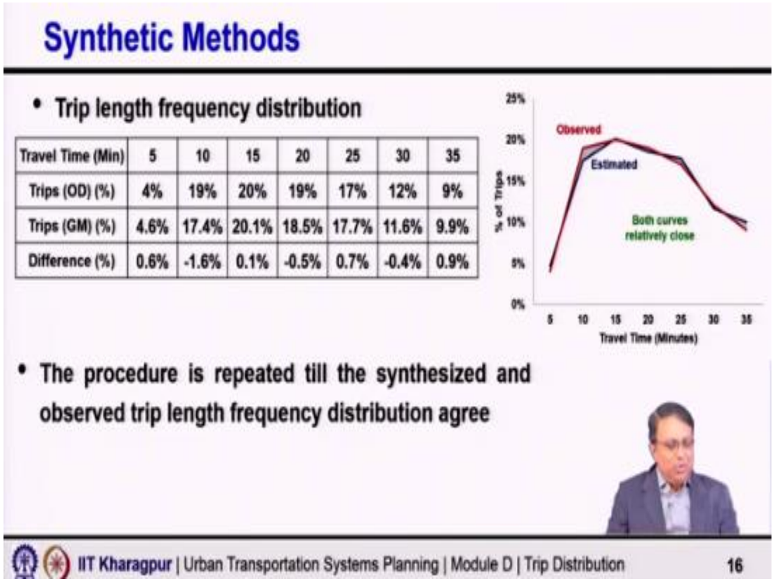
And now again, you see you find even now more or less it has matched. So, I stop here my iteration if you are still not happy, continue. So, finally, it will exactly match. That means how the distance is acting as a deterrence sorry, the travel time in this case because we have taken it as travel time. So, how the travel time is acting as a deterrence? f ij the value is exactly capturing that, so that the trip length frequency.

The frequency of travel with respect to each trip length as you observe; in the reality or in the field and as you are getting from the model are exactly matching. Now in a bigger network, let us say you need a big city you are working. So, travel time may vary from you know, 5 minute to maybe 120 minute, 130 to 140 minutes. So, for every 5 minutes, you will do or every minute you will do there will be many not always the values will be in 5 minute multiple.

What I will do in that case? In that case, what is possible probably you create several groups may be 0 to 5 minutes, 5 to 10 minutes, 10 to 15 minutes every 5 minute I am grouping. You can do it every 10 minutes. So, like that create different groups and put all travel together in that group and represented the group with a mid-value. So, suppose I have every 10 minutes I have created a group every for with a difference of 10 minutes, then 0 to 10 minutes represented by a midpoint of 5 minute.

10 to 20 minutes represented by a midpoint of 15 minute. Then you consider 5 minute, 15 minute, 25 minute like that up to maybe whatever is the upper range you consider. And then you actually follow the same thing, take the midpoint. But once you get the curve now even any intermediate value, if I have got 8 minutes, what is my friction factor value? I can get it from the curve you have established it.

(Refer Slide Time: 36:27)

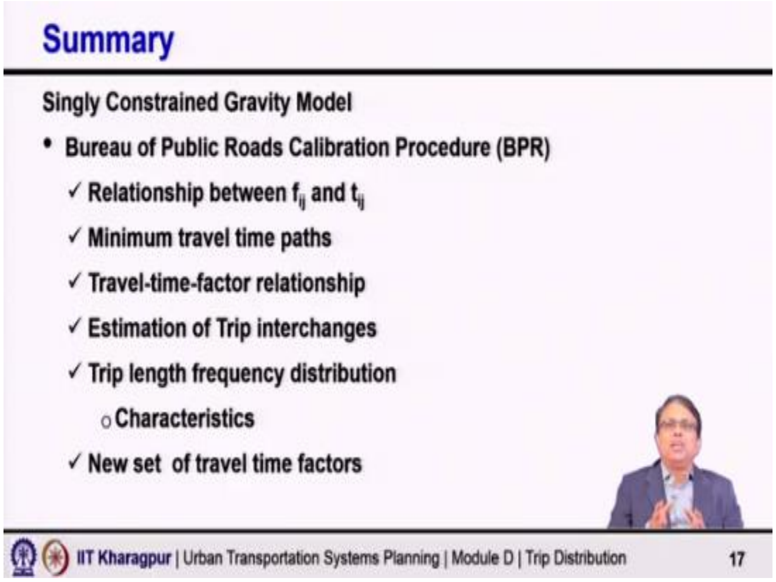


Now one more thing is important here which I have not discussed I mentioned briefly zone to zone adjustment factor. See now we are matching the total, 2 minute travel, 5 minute travel, total 10 minute travel, total 15 minute travel. But this total is coming from multiple pairs. Maybe 15 minute travel is happening in three different pairs three different t ij values, combinedly giving me this total 15 minute travel. I have matched the total 15 minute travel.

I have not matched each and individual cell. Suppose it is coming from three cells collectively this match. Individual three cells as obtained from the gravity model also gives me a trip distribution table and field also, I know the, itself value. So, cell to cell may be not matching. The total with 15 minute travel total 10-minute travel total 20 minute travel matching. But within 15 or within 20 or within 10 minute trouble multiple cells maybe they are those cells are not matching.

So, now here you use zone to zone adjustment factor or f ij value what are used in the initially in the gravity model. So, now you can use some factor to make some cross kind of adjustment and agreement to each and every cell within the total. So, the total is matching now, it is zone to zone socioeconomic adjustment factor you can do to match those.

(Refer Slide Time: 38:14)



So, what we discussed here? We discussed in details about the BPR calibration procedure. We said that what is the relationship between f and f ij t ij? Then finding out the minimum travel path, the travel time factor relationship, estimation of trip interchange, all the steps we said and how we can get the new set of value and how finally it should match closely. The observed and the model tripping frequency should match and that is what is my calibrated values and if I join all those values that gives me a curve. So, with this I close this lecture, thank you so much.