Urban Transportation Systems Planning

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Lecture - 28
Doubly Constrained Gravity Model

Welcome to module D, lecture 8. In this lecture, we shall discuss about doubly constrained gravity model.

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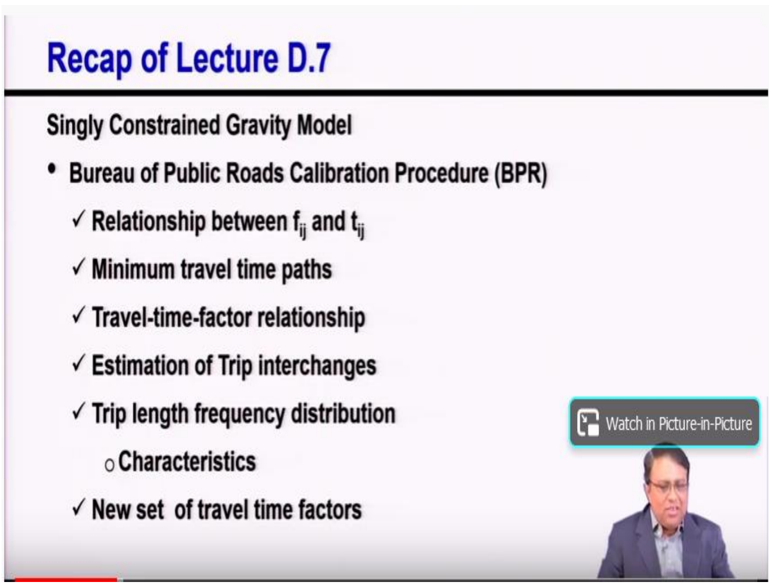

In lecture 7, we discussed in details about the calibration of friction factors using BPR method. I would like to remind you once again what we did in the calibration procedure. Basically, we tried to match the trip length frequency distributions as observed from the field and as obtained from the gravity model. So, in gravity model, before we actually calculated the t ij values the trip interchange values using gravity model, we assume some values of f ij with respect to different travel times.

And then using those f ij's, we calculated the trip distribution and then what we did 5 minute; how many total trips? 10 minute; how many total trips, 15 minute; how many total trips, like that we calculated then we express this total trip into percentage of the total trips made. So, then how much percentage of total trips is made for 5 minute travel, how much percentage of total trips is made using 10 minute travel.

How much percentage is made using 15 minute travel something like this for different travel time or the different trip lengths. These are the of graph frequencies obtained from gravity model and we also match this with the frequencies as we observed from the field data. And we try to match them closely by going through several iterations and modifying the assumed f ij value based on the kind of mismatch.

Whether the gravity model is predicting higher or the observed values are higher with respect to that travel time. And then finally what it gave us? It gave us the way that travel time is acting as deterrence, is captured nicely by this f ij values. And we find the percentage of trips made with different triplets of different travel time, is actually negating capture nicely by this f ij values in a singly constrained gravity model.

And the values are close to what it is observed in the field or observed values in the field. So, that is the match, gross level match. But as I said, the total 5 minute traveler matching, so that time how it is acting as deterrence is grossly dis- matching. But then if total 10 minute travel is matching and if travel time is the only factor, influencing this trip distribution, then the multiple cells which have the same travel time. Those cell values automatically should match because my overall gross percentages or frequencies are matching.

So, if travel time is the only factor then individual cells automatically match. But then we may find that although the total 5 minute or total 10 minute or total 20 minute travel the percentage of total trips with that travel time is matching, but then that might be total travel, 10 minute travel might be coming from 3 or 4 different cells. Now this 3 4 different cells, the total is matching with the observed one but cell to cell, still not matching.

That means there is something beyond the travel time or beyond this friction factor. That is basically the composition of people, the social economic distribution of people and different journeys is different. So, some additional adjustment factor we used, what we told is as zone to zone adjustment factor to match this even individual cells in a reasonable manner. That is the role of socioeconomic adjustment factor.

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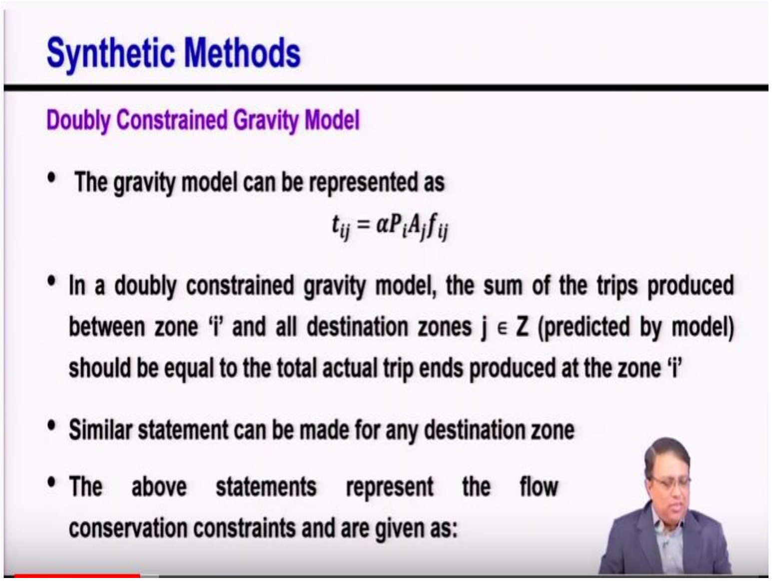

Now with that today, we shall discuss about Doubly Constrained Model, and as I said earlier, what is doubly constrained model? The both ends should match automatically based on the model formulation. Like the way, I am writing the t ij equal to something if i sum it over j, i should get P i, if i sum it over i, i should get A j. Then both ends are matching automatically by the model itself.

Model itself will ensure both ends are matching. Singly constraint only one end was matching the other end was not matching. We had to do something externally. But here no need to do something externally. The model form itself will ensure that. So, that is what I said. General form is alpha into P i, A j, f ij, that is the general form. The above statements represent the flow conservation constraints and are given as follows.

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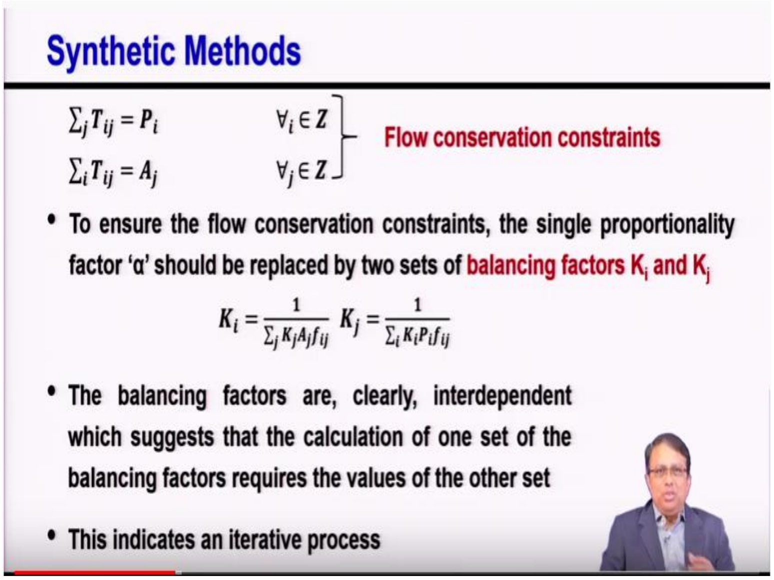

Normally as i said, the two constraints means this one t ij sum over g equal to P i sum over t ij sum over i = A j. Now to ensure this normally in a in a doubly constraint model, there are two balancing factors what we use. They are called K i and K j. So, what is the value of K i, K i = 1 by sum over K j, A j, f ij and sum over what J. Similarly K j = 1 by sum over K i, P i, f ij sum over i. You can clearly see this is something very interesting.

The K i j value depends on K j value, K j value depends on k i value. So, what it indicates? It indicates that you need to go for an iterative process, to you know, get stable values of K i and K j. When we can say the values have stabilized or stable values are stable? Going from one iteration to another iteration, the K i and K j values are really not changing significantly or the change is very very negligible. So, that is what is doubly constrained model.

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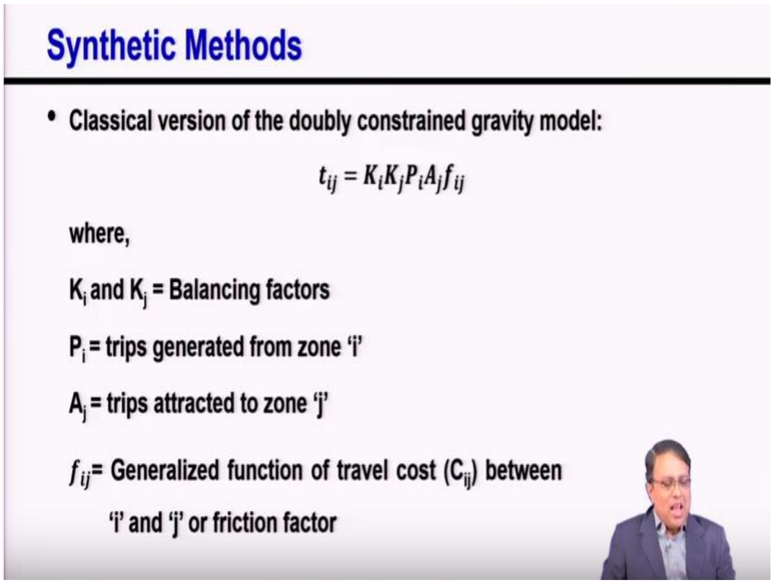

Here I am showing the classical version of a doubly constrained gravity model, t ij = K i, K j as I said these 2 factor are balancing factor, K i, K j, P i, A j and f ij. What is f ij? f ij is the you know, the friction factor whatever you say or we can call is a generalized function of travel cost.

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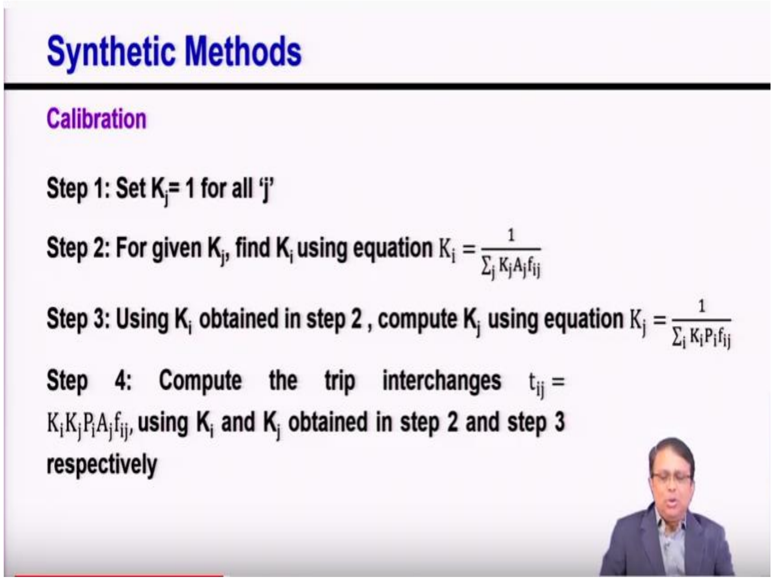

And how to get this f ij value? You can so many ways it can come. One possible way is you can take it as a function of the c ij, the cost of travel. The generalized cost of travel that is one possibility. Now how to do the calibration? Calibration, we can do in steps. The way fast step is, we assume that all K j values for all j the K j is 1 or unit that is by starting point. Then go to step 2 for this; assumed value of k j as unity, now you calculate the value of K i for every i.

How we calculate as I said, K i = 1 by sum over K j, A j, f ij sum it over all j. Now, obviously K j values are 1. But then with that 1 value of K i, K j you can now calculate the value of K i. So, you have now got a set of k i values. Now with this K i, in step 3, now, you go back and calculate the value of K j, Iterative procedure. So, how you will go and calculate the value of K j? K j = 1 by sum over K i, P i, f ij, and sum it over i,

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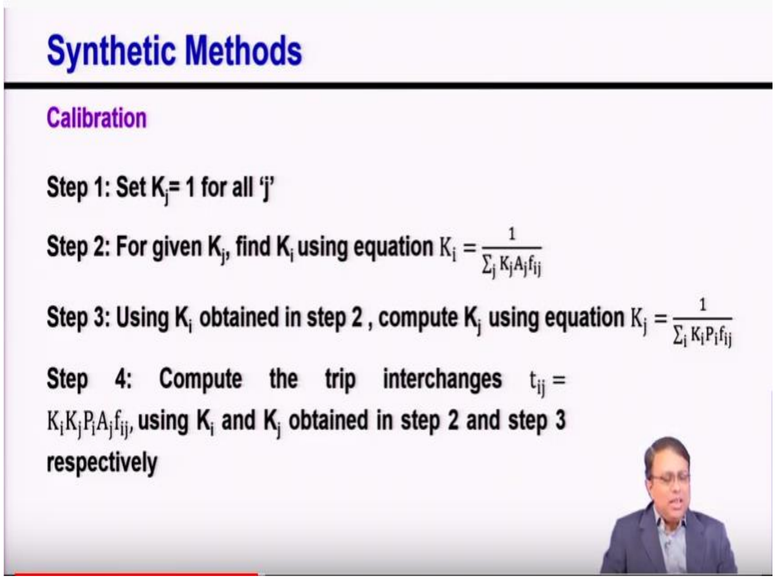

If f ij is the friction factor as I told that you know, sometimes you can use f ij and here also I have said, if f ij is the friction factor or the generalized function. And so many ways you can make it as a function of cost you can make it as function of time, but is basically the friction factor values. Now what we did we first assume K j as 1 for all j. Then with those K j's we calculated all values of K i.

That means value of K i for all i. Then again with those sets of values of K i, we went back and calculated value of K j for all j. And after this, we can even go back now and apply this model doubly constraint model because we have a value set of value of K i we have got a set of value of K j. Then we can calculate K i, K j, P i, A j, f ij and using this doubly constraint model we can calculate the value of t ij.

We can do that you have known value of K i for all i, you have known value of K j all j, you have given value of P i for all i, you have given value of A j all j and you also know the f ij values as I explained in the beginning of this lecture and also in the previous lecture that how we can develop a f ij values. So, similarly you have also calculated the value of f ij. So, all unknown I can calculate t ij values.

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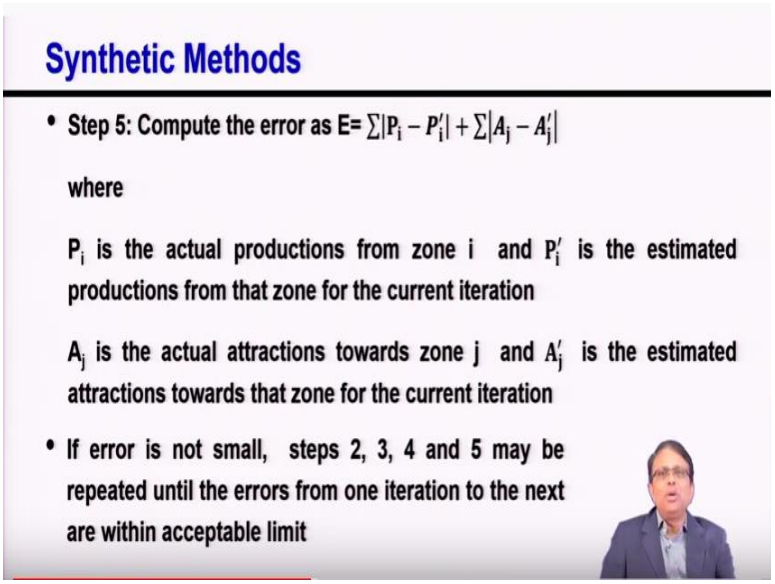

Then, I can also calculate in step 5, what is my error? That means for every i I have actual production and I have an estimated production value from the model like t ij sum over all j i will get a P i. So, what is the difference in the production for in zone? Similarly, what is the difference in the attraction in each zone? The absolute values we can take because you know, some may be positive some may be negative. So, we need to take the absolute values.

So, that gives me what is my total error. What is the total mismatch? Now if this mismatch is not very significant and if you as a modeler, you are happy then you stop here, no problem. But in all possibilities with this Iteration, wherever you have reached till now you will find that there are certain errors may be at the production end or the attraction end. There will be errors and this errors are probably you are still not happy.

So, if that is so, you want to know. I still should get a better match at both ends, then what we have to do? We have to go for further iteration. Now, why this error has come? You have a f ij value does not change. Time or cost is given correspondingly somebody has calibrated the f ij values. So, you cannot you are not changing anyhow in the context specific case given problem, you are not changing the f ij values so f ij will not change, P i will not change.

A j will not change. Given P i given A j will not change. So, what will change? What will influence the t ij values? It is only K i and K j, the balancing factors. So, balancing factor that means still you have not got the correct values of the balancing factors. So, if you there is an error here, that is coming actually because of this balancing factor K i and K j. So, you have to go back to that area. So, what we will do? We will again go back to step 2.

That means we have certain values of K j with that we will again calculate K i. With that those values of K i will again go to step 3, calculate again the values of K j. And then you can again come back to step 4 do the trip distribution and then you can again check what is your error value? If you are satisfied fine; otherwise again go back do one more, iteration. And, because finally our aim is that we want our the production ends and the attraction ends to match as per our expectation or within the permissible error or within acceptable error rather,

So, till we are satisfied every time we come back and we calculate if we are not satisfied, again we will go back; will make one more round of iteration. We change, calculate the given the K j last set whatever was by K j value, l calculate K i value. And then with those carry revised K i value I will again go and calculate K j value. And like that I will keep on iterating and sometimes things will converge. This is one possible way, you can do it.

The other thing as I told, what needs to be adjusted here is only K i and K j. The remaining thing once you apply see this doubly constraint model, I go back and show you K i, K j, P i, A j, f ij. So, P i does not change, A j does not change, f ij does not change. So, what changes? Only thing changes is K i and K j. So, it is all about balancing this or making this balancing factor stable. So, another possibility what you can do, I need not go for step 4 and step 5 again.

Probably I did not go back. I will start with step 1 starting K j’s 1, then go to and calculate in step 2 K i values. Then with that go to step 3 calculate the K j values. Again I will go back to step 2 with those calculate K i values, come back to step 3, calculate K j values. So, step 2 and step 3 I will keep on repeating. How long? Till the time I find that K i and K j values are not changing so much. That means you can always keep an error or keep in limit of the change in the values.

So, whatever you got the K j values in previous iteration, now in the new iteration your K j values are not so different from the previous iteration. So, you can always for every zone j, you can actually compare what I got in the previous iteration, K 1, K 2, K 3, K 4 for all the zones what are my K values?, and what I have got now. What is my difference? So, naturally when the difference is not that much, that means your K i, K j values have stabilized.

Even if you go for one more iteration or two more iteration the values are not going to change significantly. Then with that you directly come to step 4 and directly come to them step 5 and check that whether you know, your errors are within acceptable limit. And in all possibility if you have really stabilized, the values are stable for K i and K j, the balancing factors. Then you are expected to get a reasonable error or within acceptable limit of error in your step 5.

So, step 4 and step 5 you can omit also and you straightaway you can iterate it. Or as I said, earlier come up to step 5 if you are not happy with the final result, go back to step to 2, 3, 4, 5. Again, you are not happy go back to 2, 3, 4, 5, that is another way you can do. Both are possible, you know, if you see different books and other materials you will find both ways people have addressed, you know this doubly constraint model.

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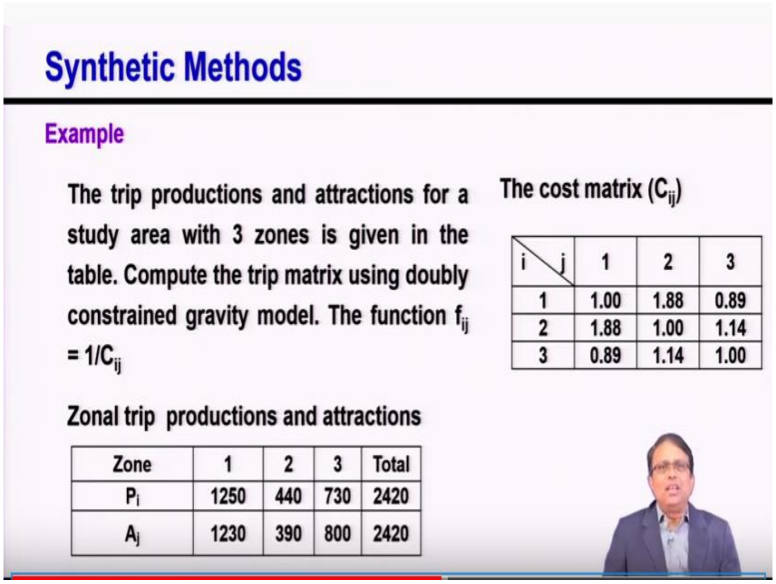

i \ j	1	2	3
1	1.00	1.88	0.89
2	1.88	1.00	1.14
3	0.89	1.14	1.00

Zone	1	2	3	Total
P_i	1250	440	730	2420
A_j	1230	390	800	2420

Now, let me take an example. Where I have said that I have only three zones, again a classroom example, the productions and attractions are giving and we can see their matching, the total productions from the study area is 2420 or total attraction is also 2420. So, they are matching closely, there is no issue in that. And then in this case, instead of distance suppose the friction factors have been calculated based on C ij value and it is assumed that friction factor is 1 by C ij.

This is just an assumption. So, instead of giving separate friction factor, in this problem it is assumed that f ij is 1 by c ij. Although you know, by this time how to calibrate f ij value you can always calibrate it. But in this example, instead of giving directly the f ij values for different corresponding cost or corresponding time one can assume that f ij is 1 by C ij which is again logical, because you know, the cost will be higher, the multiplier will be less f ij will be less. Cost is lesser the multiple f ij value will be higher. So, 1 by C ij is taken that way.

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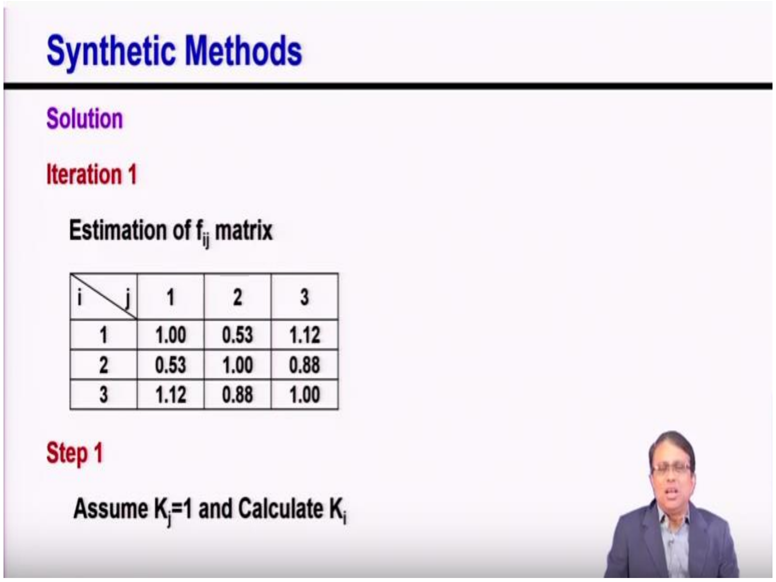

i \ j	1	2	3
1	1.00	0.53	1.12
2	0.53	1.00	0.88
3	1.12	0.88	1.00

So, with this, we go for the first iteration. What we get? We get the value of you know, as I said that this is the f ij matrix, if my costs are this, say C ij values are this, then 1 by C ij is my friction factor values. So, I have got the f ij matrix. I got assumed now K j all K j = 1 and with that we want to calculate K i that is what I have shown here.

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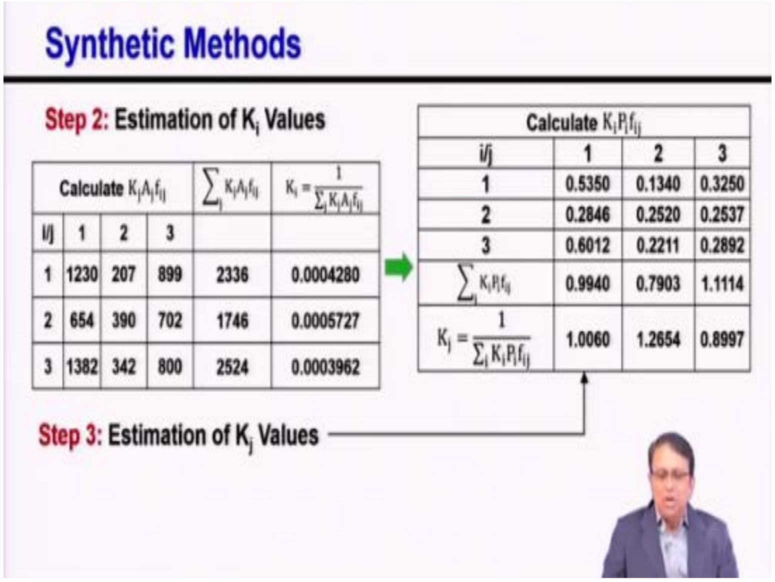

Calculate $K_jA_jf_{ij}$				$\sum_j K_jA_jf_{ij}$	$K_i = \frac{1}{\sum_j K_jA_jf_{ij}}$
i/j	1	2	3		
1	1230	207	899	2336	0.0004280
2	654	390	702	1746	0.0005727
3	1382	342	800	2524	0.0003962

Calculate $K_iP_if_{ij}$			
i/j	1	2	3
1	0.5350	0.1340	0.3250
2	0.2846	0.2520	0.2537
3	0.6012	0.2211	0.2892
$\sum_i K_iP_if_{ij}$	0.9940	0.7903	1.1114
$K_j = \frac{1}{\sum_i K_iP_if_{ij}}$	1.0060	1.2654	0.8997

You can see in step 2, what we have done? We have taken the zone's product high as 1 2 3 in 3 rows, and j as 1 2 3 in 3 columns and then calculated this value, K j, A j, f ij. K j is assumed as 1 A j we know the attraction of zone each column that is 1, 2, 3 and then corresponding f ij value will 1 1, 1 2, 1 3, what is the cost value and then what is the corresponding f ij value? We put that and then sum over K j, A j, f ij sum over all j, that means sum over all columns and then we also calculate there what is the value of K i equal to 1 by sum over K j, A j f ij.

So, whatever you have got in this sum over K j, A j, f ij, sum over all j take 1 by that you will get this value of K i. Now with this K i value, now I go on the right side as I say in step 3, but the table I have shown it on the right. So, we can calculate now the value of K j. How we do? Again, I say my i and j we put the value now K i, P i, f ij. So, my column wise, I calculate the value. I calculate the column total that is K i, P i, f ij sum over i.

So, you have the total and then you calculate the corresponding K j is for that zone, each of this zone destination zone. What is the thing one by sum over that whatever is calculated is shown here. So, like that you can do step 2 and step 3.

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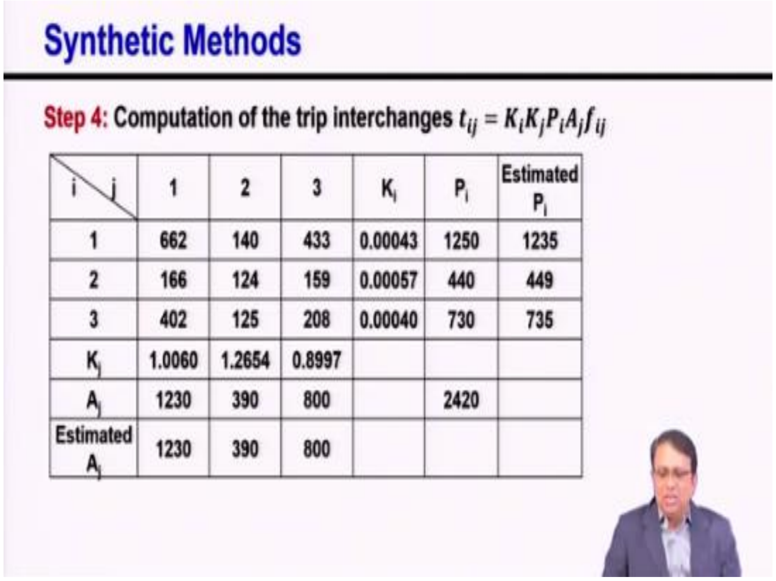

With that, now I apply this t ij, K i, K j, I have all values of K i, I have all values of K j. I have all values of P i, I have all values of A j. So, for all i equal to 1 to n, j equal to 1 to n, I know K i, K j, P i, A j and also the f ij value. So, simply apply it. I calculate the trip distribution matrix. Then I show how much I have got. And what was my estimate? So, both are known.

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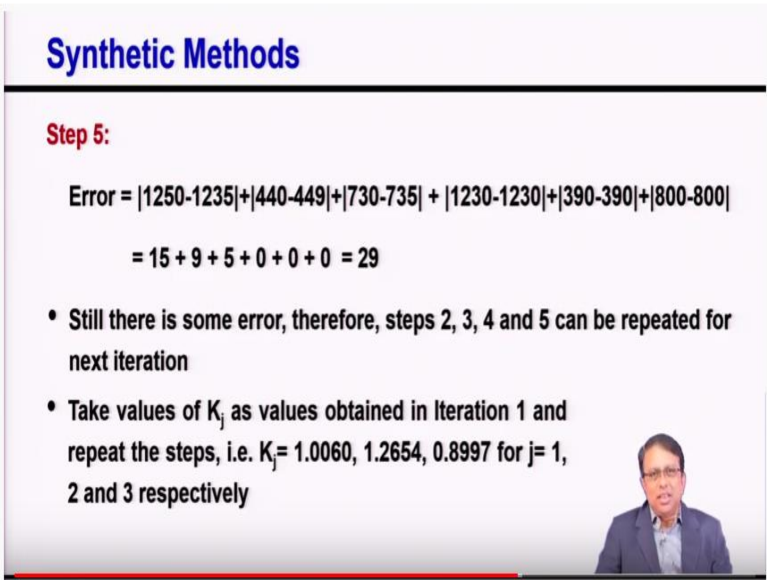

So, you know the P i is 1250, attraction is 1230, for zone 1 1250, 1230. Now, you got estimated is 1235 production instead of 1250 and attraction has matched. Because A j values final we have used that. So, that is you know that will balance. So, that is our matching. So, now we are not still happy because we find that we are calculating the error and we find that there is still an error and we are not happy.

So, what we want to do now? We want to again, you know go back. Still there are errors. So, we are repeating step 2, 3, 4, 5 with this case. We are going back again calculating with that K j value, what are the K i values? With that K i values, what is the K j value and then come back?

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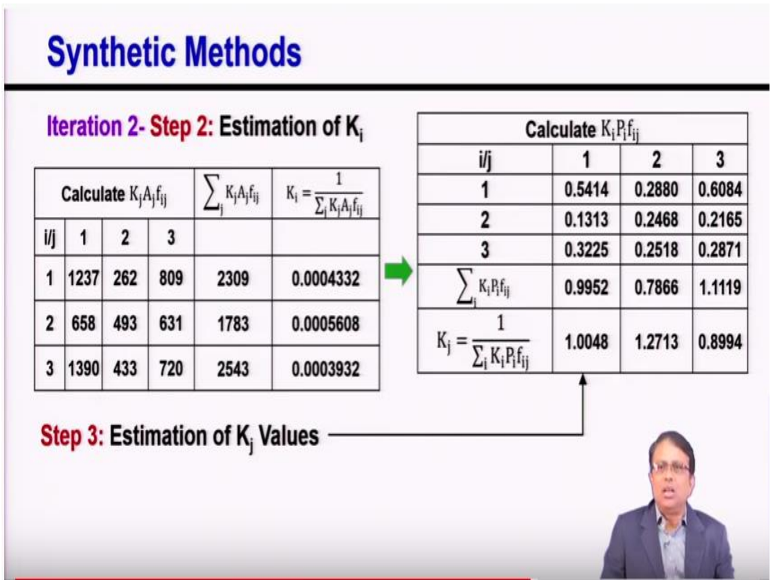

That is what is shown here. So, here we are again going for iteration 2 and going back to step 2. So, with that last set of K j whatever we got. Now, we are actually calculating K i and then the right side with those K i values. I have shown a table where we are calculating the K j values.

So, step 2 and 3 are repeated.

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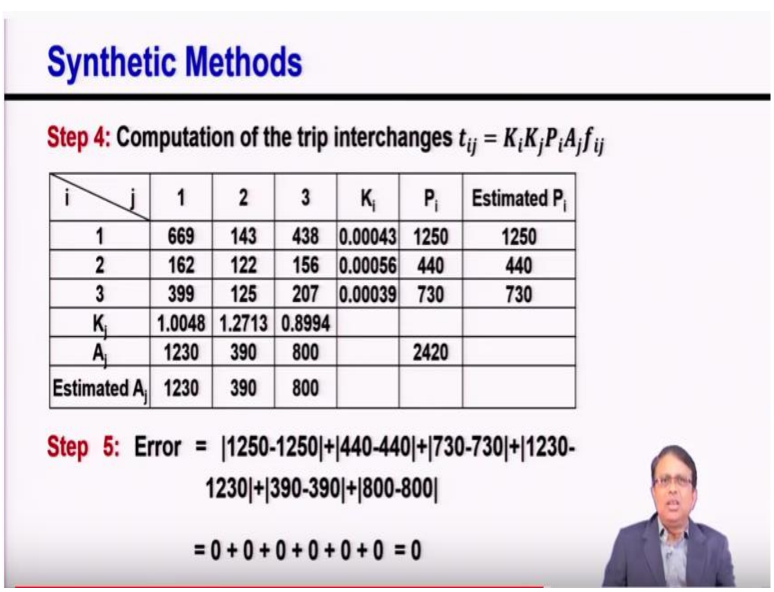

Then we are again doing the matrix and now it is matching. So, it is a small example, so it may match with just two iteration, but you take any example it may require more number of iteration and still there may be some error which are you know marginal and may be accepted for any such kind planning exercise. So, this is the overall procedure how you apply it.

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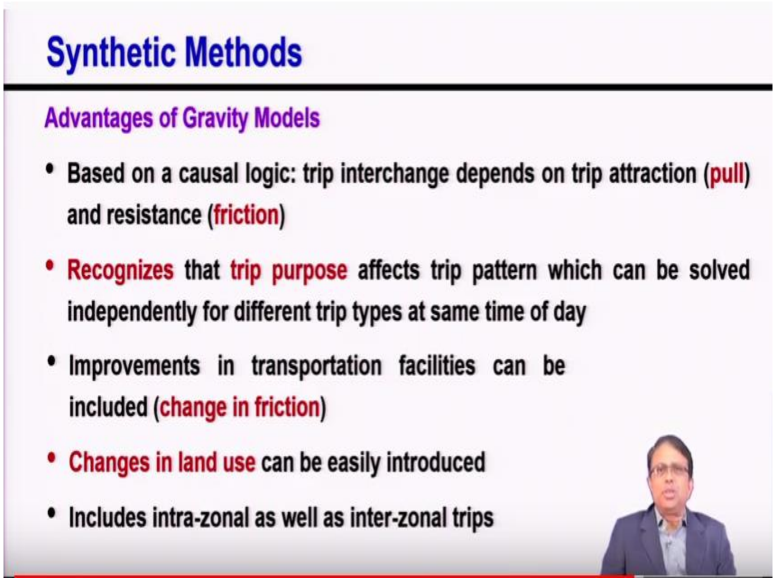

Now, before we close this discussion, you have discussed about gravity model. What is gravity models, Singly constraint, Doubly constraint, BPR calibration technique, how to apply simply constant model, how to do the balancing, how to apply the doubly constant model we have explained you here clearly. And with an example also we have shown you how you do it. Now before we close let us talk about some of the considerations.

What are the real potentials or advantages of gravity model. And in some cases we need to be little bit careful. So, I do not want to set disadvantage, but maybe some of the considerations where we have to be careful. We should understand them bear these things in mind. Let us go to the advantages. So, it used a causal relationship. So, the first thing is, you know, it uses a causal relationship.

That means why some places more trips are going as compared to other places or why to some places less number of trips are getting terminated as compared to other places. So, that logic is actually captured in terms of trip attraction and resistance. See we can call it as full how much trip I will be attracted to a zone j it will depends on what is the level pull that means what is that attraction? More that function more would be the pool.

Similarly what is the resistance? More the resistance less trips will finally get attracted to that zone. So, that causal logic is inbuilt. Second it recognizes that the trip purpose affects the pattern which can be solved independently by different reptiles at same time of trip. So, The effect of trip cost or travel time is not same on all trip purposes. That can also be considered advantageously. What we all need to do?

We need to calibrate separate f ij value for separated purposes, may be work trip work and business trip, one curve or one, you know set of calibration and some other trips a different calibration. So, this gives an opportunity for that. The same 15 minute, the way the increase or decrease in travel time or so the way it will influence the work trip the exactly the same way it may not influence the education trip or exactly in the same way it will not impact the recreational trips.

So, the impact of travel time or travel cost on friction factor may be different depending on the trip purpose and one can calibrate it separately. Improvement in the transportation facilities can be improved; that is a greatest advantage of using this kind of gravity model. As a matter of fact any kind of synthetic model the transportation system is improved, the travel time is improved, the travel cost is improved one can check with those now a revised distribution will come.

Because of inclusion of a f ij which is eventually again a function of cost or function of time or function of some characteristics of the transport network. And it includes intra-zonal as well as inter-zonal trips. All diagonal elements are also estimated and also the other elements which are inter-zonal elements, those are also estimated.

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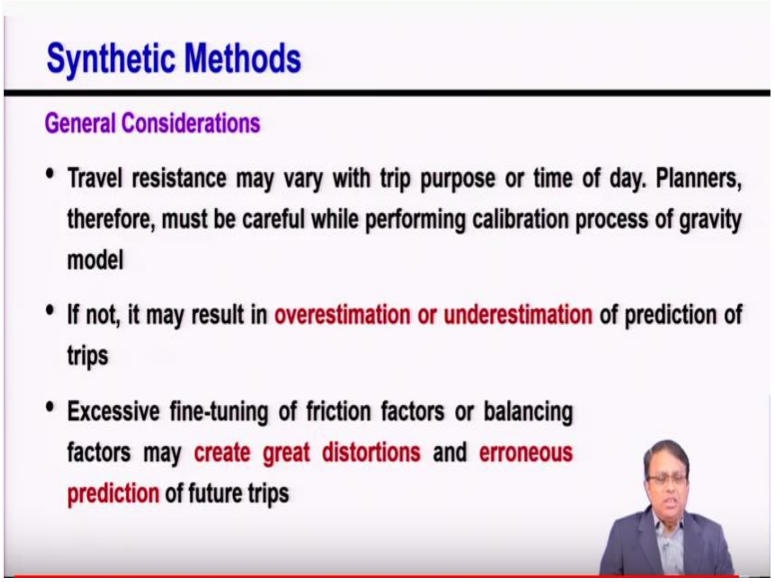

But there are also some kinds of general considerations. I do not like to say them as disadvantages because it all depends whether you are applying it properly if you apply it properly, then it may not be anything negative. But one has to understand and keep in mind. For example, the travel resistance made vary with trip purpose or even time of the day. The trip purpose we probably mentioned here that it gives an opportunity.

So, probability trip purpose it does not sense much logic for me to include it here, but even the time of the day variation also. It may not be so easy, but if you wish you can also develop several friction factors. So, I would not like to say it as a clear limitation but what has to keep in mind if I want that different times things should be different purpose things will be different or home based or none of this trip things should will be different.

If I want to consider their effect, I have to calibrate my friction factor. Yes, the limitation is you have to you have to go and disadvantage also you can say. In that way you have to go and calibrate it as well. The way you want to apply you have to then calibrate the friction factor and if you do not can calibrate it separately then it may result overestimation or underestimation. So, the way you want to apply your thing if you want to consider different times, peak and off peak differently.

Or trip purpose differently like shopping education work separately or maybe home-based and non-home trips separately. If you want to apply and if you want to distinguish between you know, or among different trips then you have to accordingly develop the friction factor value curves. Because you believe that the deterrence probably will be different, yes there may be different but in then you need to consider. So, if you do not consider and then try to you know account for that in your model, you will not be do that.

Then another thing is that remember this is true for the gravity models to for any other models we are doing. We should not do too much tuning of the model. The problem is these days with computer you can match everything using you know, probably with you know with high level of accuracy. Number two number, sell to sell I can match it. But please do not do that; that is the advice.

Because if you are doing excessive fine tuning; you might be actually doing lot of distortion. And when you want to predict you never know because of too much actually might have you might have done so much of distortion that you will incur much more error in prediction. So, remember that it is a planning, for planning purpose we are doing it. So, always there will be some error, some error in model estimation, some error in the data collection, anything. I mean the whole modeling work, you know cannot go without the error.

All what we need to ensure that you know, our error is not too much. So, that is why gross checking, gross level of matching that we want. But you know as I said it means zone to zone adjustment factor or socioeconomic adjustment factor beyond trip length frequency distribution, why the individual cells are not matching because of the composition. So, if you really use it too much to tune each and every cell you might be actually incurring some error because in future there may be some minor changes in the socioeconomic distribution.

And if there is an error and you have tuned in so much then the error also will be so it is better sometimes of course gross matching error within acceptable limit these are all very, you know important things I cannot work with the model where you are, you know, so much of very high 30% error, 25% error, probably it is not acceptable, but 5% error is fine, let us not tune them till the cells too much.

So, remember that tuning is good but too much of tuning maybe something you know wrong because you are actually maybe distorting it too much and you may encore actually higher error, higher level of error in your predictions.

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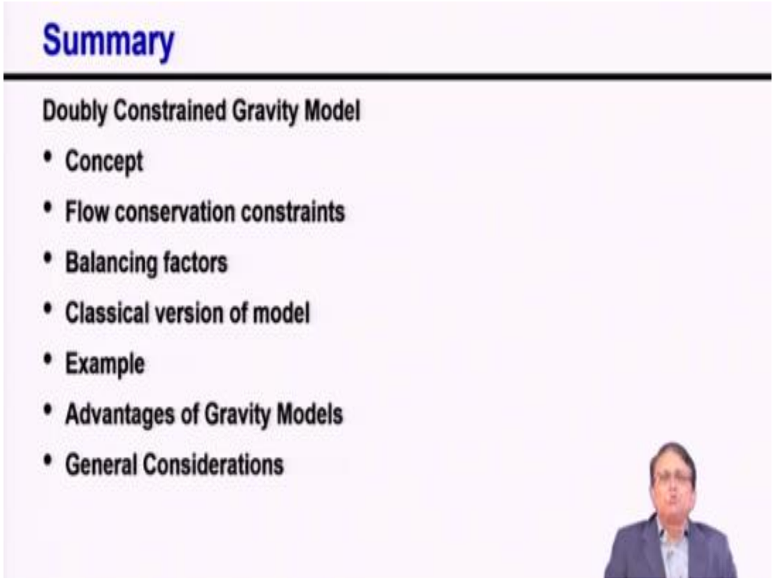

So, with this so what we discuss in this class, our lecture we basically discussed the f ij calibration related thing, what we are doing and take the role for zone to zone adjustment factor. Why we say zone to zone or socio economic adjustment factor, because this is something beyond the effect of deterrence due to time or deterrence due to cost something else. Which is likely to be due to socio economic composition then we talked about doubly constrained models.

We explained it clearly how we calibrate it and with an example and then we also discussed about various advantages of density model and some of the generally considerations where you need to be little bit careful when you are applying the gravity model. So, with this I close this lecture. Thank you so much.