Urban Transportation Systems Planning
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Lecture - 30
Competing Opportunities Model &
Linear Programming Approach

Welcome to module D lecture 10. This is the last lecture in this week and also the last lecture of this module. Here we shall discuss about two other modeling approaches, computing opportunities model and linear programming approach.

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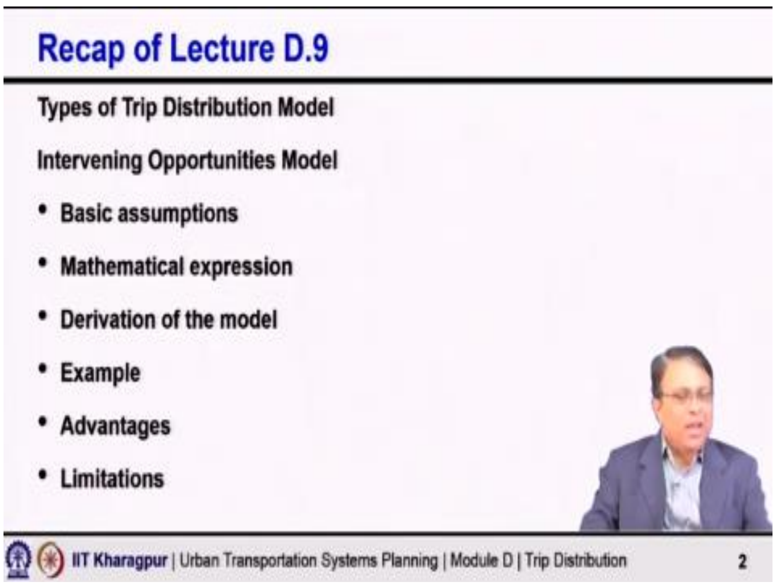

In the last lecture we discussed about intervening opportunity models we explained you clearly what are the basic assumptions, the mathematical formulation and then of course not the complete derivation. But some part of it rather and then took an example to show how it can be applied and what really is very very important for this model is the order of closeness. For every origin zone what is the order of closeness of the destination zone.

Because that order of closeness is very very important and that is where we need to be careful when you are applying this model. We of course discussed about the advantages and some of the limitations as well.

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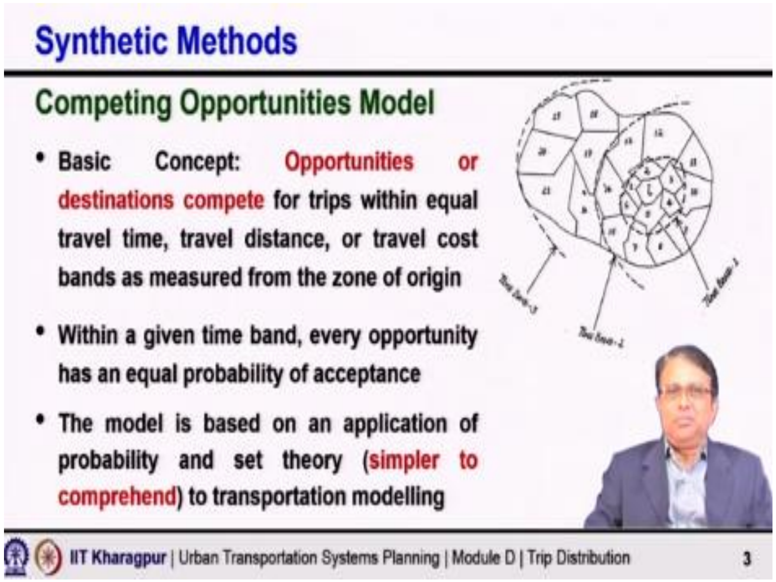

Now with this let us take another kind of model what is called competing opportunities. So, here again we are talking about opportunities, you know the concept but we are saying competing opportunities, so what is competing? The basic concept here is that opportunities or destination compete for trips with equal travel time, travel distance or travel cost bands as measured from the zone of origin.

I should read it again, let us come to computing opportunities model. You are already familiar with the term opportunity or you have already studied also opportunity model. The new thing here is competing opportunity. So, what is competing? Who are competing? The basic concept here is that opportunities or destinations compete for trips with equal travel time or travel distance or travel cost bands as measured from the zone of origin.

So, you can think that we are talking about a ring or about a band based on the travel cost and travel distance. So, the opportunities or destinations with equal travel time or equal opportunities they compete. So, with a given time band every opportunity has equal probability of acceptance. So, the model is based on an application of probability and set theory of course simpler to comprehend in transportation planning.

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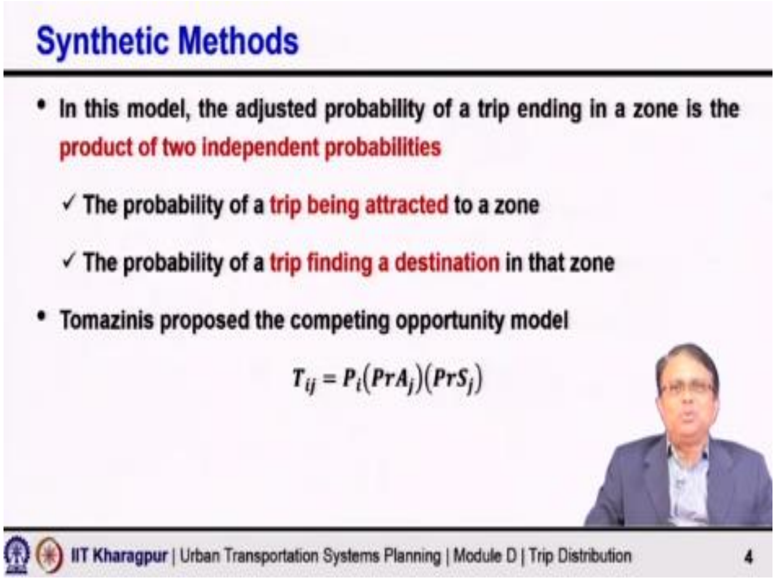

In this model the adjusted probability of a trip ending in a zone or in a ring or a particular destination zone is the product of two independent probabilities, what are those? First the probability of a trip being attracted to a zone. What is the probability that a trip will be attracted to a particular zone j? We can call this PrA j, what is the probability that a trip will get attracted to a zone j?

The second is the probability of a trip finding a destination in that zone. Two things are different. What is the probability that a trip being attracted to a zone, second the probability of a trip finding a destination in that zone. So, the following formulation is used. T ij equal to P i that is produced in zone i the simple part now how many will go to zone j is basically the product of PrA j probability of a trip;

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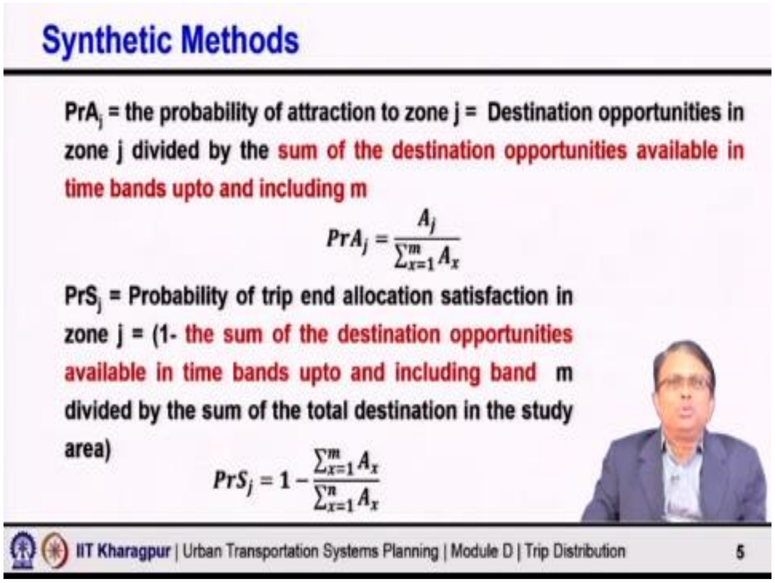

Getting attracted to zone j multiplied by PrS j that is probability of trip end allocation satisfaction in that zone. What I said in this earlier slide the probability of a trip finding a destination in that zone. Because all destinations are competing. So, what is then probability of a j that the probability of trip attraction to zone j? We can say destination opportunities in zone j divided by the sum of destination opportunities available in time band up to and including m.

Here we suddenly say m because we are considering there are m number of zones up to zone j. So, what we are talking again something like what I said earlier v and dv just think exactly in the similar manner. That one what is the probability that it will get you know located it will get attracted to a particular j. It will be what is the attraction in zone j divided by attraction of all the zones which are there up to zone j inclusive of zone j all total what is that opportunity.

So, if there are you know m such opportunities, m such zones, we can then say that probability of A j equal to A j what is the attraction in zone j divided by A x is equal to 1 to m. m zone in order of preference in order of closeness. In order of closeness all the zones that means what we are saying if I have to again you know bring back my earlier example of this 5th station. So, we are saying what is the opportunity available in 5th station, divided by what is the total opportunities available up to 5th station.

That means opportunities in station 1, station 2, station 3, station 4 and station 5. That is what is the probability of A j. Now what is then probably PrS j or probability of trip and allocation satisfaction in zone j. It is 1 minus sum of destination opportunities available in the band up to and including band m divided by some of the total destination in the study area. Let me go back to the same example I remember today so I am you know referring to that example only.

That suburban station or the route does not end at station 5 maybe it goes up to station 10. So, that in that urban area whatever you are considering people can go up to station 10 if they want to stay. We are talking only about 5, so there is something beyond that also is there its available station 6, 7, 8, 9, 10 is also available up to 10. So, that number is n suppose, that number is n. So, we have n number of station up to x and there are total n number in n number of stations which are there.

That means there when we talked about probability of A j we considered only up to station 5. 5th station, what is the opportunity divided by total up to 5th station what is the opportunity and here what we are saying what is the opportunity up to 5th station divided by what is the total opportunity up to 10 station. 1 minus that is basically probability of trip end allocation satisfaction in zone j.

That is what I say one minus what I explained sum of destination opportunities available in time band up to and including band m. That means total up to station catchment 5 divided by sum of the total destination in the study area. So, divided by up to not 5 but up to total 10, 10 station catchments up to that the study area is there. So, total opportunity is available in the study area. That is why you wrote PrS j equal to 1 - sum over A x where x equal to 1 to m divided by sum over A x again but this x equal to 1 to n.

So, the difference between m and n hopefully is clear; m is remember that 5th station catchment and n is the 10th station catchment up to ten station catchment. So, when you are saying sum over up to m we are talking up to 5th station and when we are considering n sum over n that means up to 10th, whatever is my limit for the study area or the zone.

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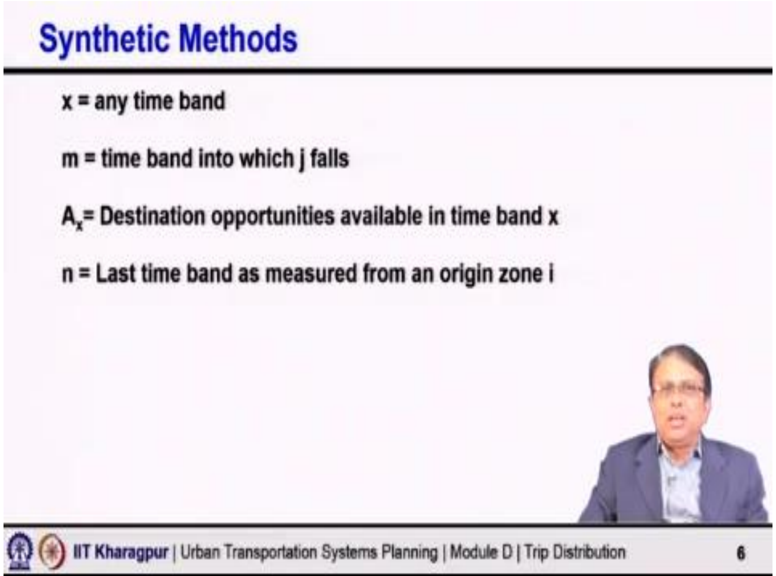

So, now with this we can calculate and as I said x we have used which is indicates any time band. m is the time band which j falls up to 5th station, the station 5 falls within that and A x is the destination opportunities available in time band x, n is the last time band as measured from an origin zone i. So, when I say 10th station last station in the suburban route.

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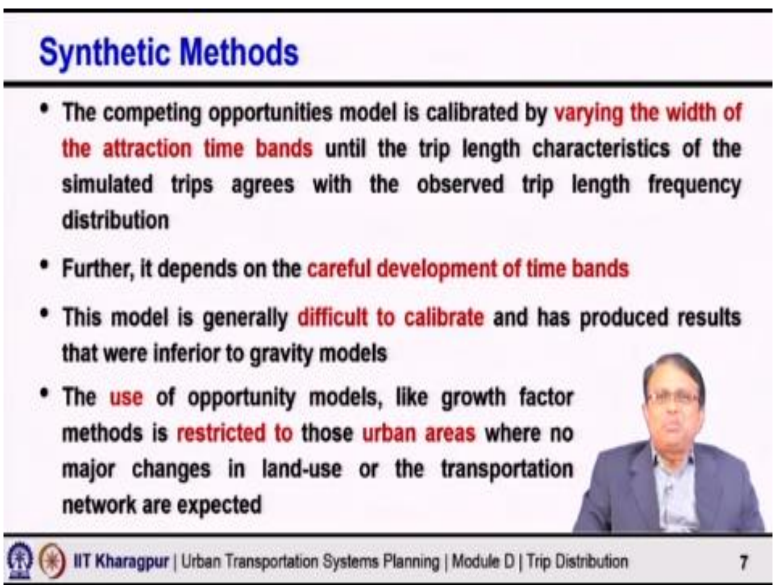

Now remember that the competing opportunity model is calibrated by varying the width of the attraction time band. How you are adjusting the time bands? That is where you know it is very important during calibration, you can make it okay every 2 minute, every 3 minute or it may be unequal 3 minute and then 5 minute then maybe 7 minute unequal band or equal band both are possible.

So, the computing opportunities model is calibrated by varying this width of attraction time bands until what the trip length characteristics of the simulated trip agrees with the observed trip length frequency distribution. So, model trip length frequency distribution and observed trip length frequency distribution we try to match by adjusting the width of attraction time bands. That is the basic philosophy.

I shall not take example problems to explain this. Second further it depends on careful development of the time band for obvious reason and this model is generally difficult to calibrate and has procedure has produced results. This model is generally difficult to calibrate and this model has produced results that were inferior to gravity model. So, even though we consider all this still the gravity model most cases give even the better results.

The use of opportunity models like growth factor methods is restricted in urban area. We know that there are you know the use of growth factor model is restricted only in certain case you use otherwise we prefer generally wherever it is possible and feasible to go for synthetic model. So, same thing we are saying the use of opportunity model like growth factor method is restricted to those urban areas where no major change in land use or transportation networks are expected.

So, that is where is the key. So, both cases this model as well as growth factor based models are having restricted applications in cases where no major change in the land use or transport network is expected.

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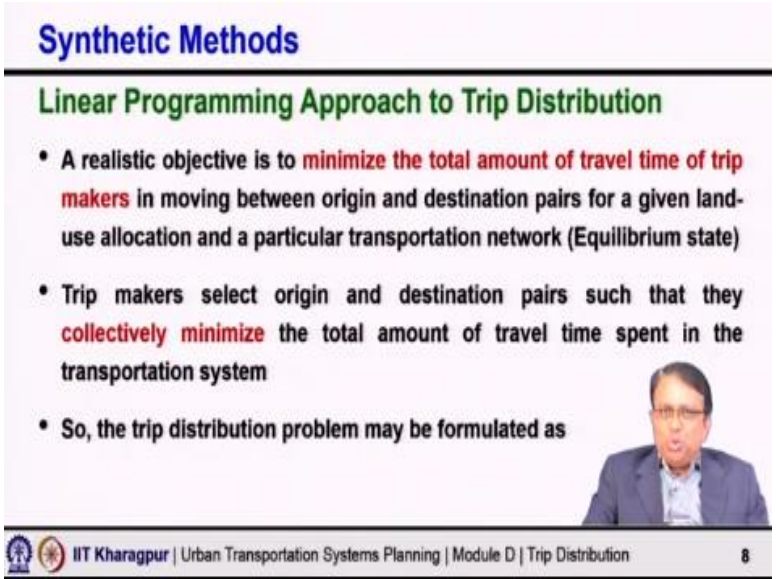

Then we also have another approach for modeling that is called linear programming approach. What we do in this case is based on certain basic assumption that the assumption is that to minimize the trip get distributed in such a manner. That the total amount of travel time of all trip makers in moving between origin and destination pairs for a given land use allocation and a particular transport network is minimized.

That means how the trips will get distributed trips will get distributed. So, that the total travel time of all the trips in that zone aggregate trip time or travel time we are saying is minimized. That is what is the kind of equilibrium state which is possible. This is like a dream if we could do that actually in urban area we do the travel distribution in a manner. So, that the aggregate travel time is minimized for all the travelers for all the pairs of zones.

That would have been really wonderful. Unfortunately the reality, in real life the distribution does not happen exactly following in this manner. So, with that assumption the trip makers select origin and destination pairs such that they collectively minimize the total amount of travel time spent in the transportation system. So, the way the origin destination pairs and the travel distributions are done.

So, that they collectively minimize the total amount of travel time spent in the transportation system. So, that is almost like a dream it would be really wonderful that if we have a trip distribution that will minimize the overall travel time in a given transportation network. So, if so if we assume that it is so maybe it is a dream, it does not affect in reality the real behavior is somewhat different.

But this is what we would like to have anything you know away from this one is basically a deviation from the ideal solution. So, this is kind of an ideal solution what we would like to have. Anyhow if this is true like people collectively try to minimize the total amount of travel time spent in the whole network for every people together for all the origin destination zones;

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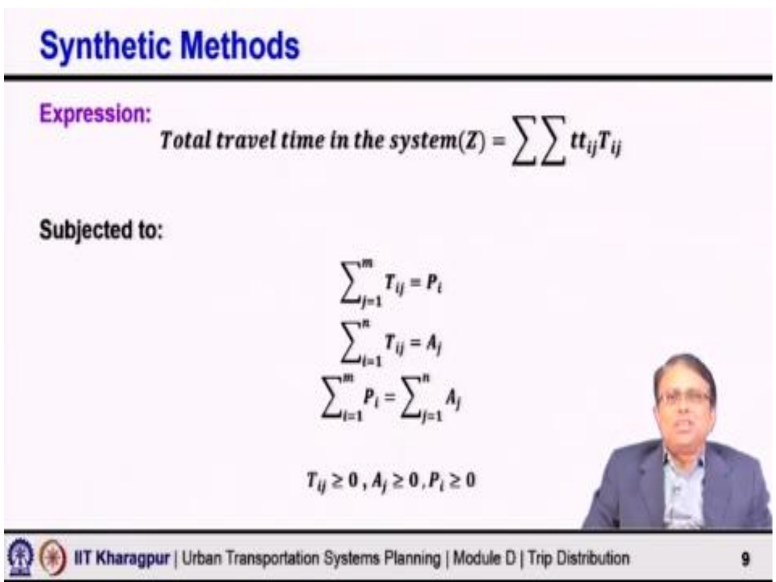

Then it can be formulated like this, that the total travel time in the system is tt ij that is travel time for travel between i and j into T ij. T ij is the number of trips that will happen between i and j when the travel time is tt ij and then we are trying to sum it over all i all j. I have shown here two summations but I have not indicated that sum over i sum over j. So, you can say that sum over i equal to 1 to you know m and 1 to n and j also equal to 1 to n. Because same number of origins destination will be there naturally.

Of course you can use I have used here two different variables but they indicate the same number. So, that is the total travel time is getting minimized. So, travel time and the number of travel between i and j that is the product that is the total travel time for people travelling between i and j like that. Consider all the i all the j you have all the production zones i and j may be different for all production zones and attraction zones may be number of production zone attraction zone may be different.

But generally even it would be equal number of zones but maybe some zones will have 0 production, zero attraction something like that and subject to satisfying the constant. What are those constant? T ij sum over j is P i, T ij sum over i is A j, sum over i P i equal to sum over j A j. Both will be equal you remember we talked about some constants, this constant also we discussed when I mentioned about different types of gravity model in my lecture 9.

There also I mentioned about this. So, all these constraints are to be satisfied. These are the boundary these are the constraints which one has to satisfy. Also the T ij value has to be greater than zero travel time cannot be negative A j all attractions has to be greater than equal to 0 non negative, all productions have to be greater than 0 non-negative. So, that gives you a formulation linear programming formulation.

Now I am not going to teach you linear programming here in this course. But I will only show you the formulation. So, with an assumption that if we really try to see that the total aggregate travel time is minimized and we try to find out what is that distribution. Trip distribution that can minimize this aggregate travel time for all the travelers considering all origin destination zones in the study area.

Then that is what we should try to optimize and we must satisfy all these constraints productions constraints, attraction constraints. And then total number of productions for all the zones must be equal to total number of attractions from all the zones and then the non negativity constraint like T ij can be only 0 or positive attractions to each zone could be 0 or positive production also could be 0 or positive.

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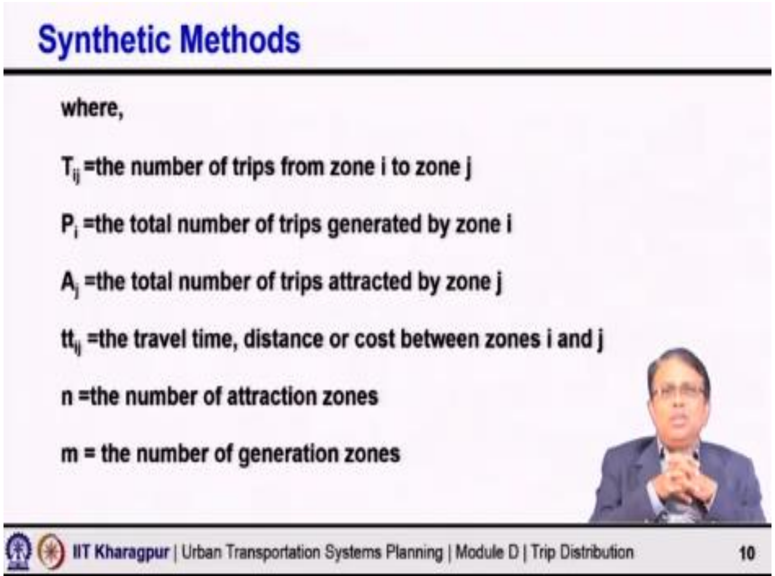

Now let us take an example and here of course I said that T ij is the number of trips from zone i to zone j, P i is the total number of trips generated by zone i, A j is the total number of trips attracted by zone j and tt ij is the travel time distance or cost between zone i and zone j. So, far I said they want to minimize the travel time but you can also minimize the distance traveled, you can also minimize the cost of travel.

There is a deeper meaning if we can achieve that is what is really the ideal thing what we would like to have. But the real behavior is never show because people act in a selfish manner. I would like to choose the way it is convenient for me. My own choice the collective things do not happen. So, this is almost will give you kind of ideal solution and you know how far you are from ideal.

So, you have to make all our attempts to take it towards the ideal solution. Then n is the number of attraction zones m is the number of production zones and as I said normally number of zones in the study area we use that. So, we do not use separate nrm but you can consider that not all zones will have productions not all zones will have attractions may be some will have zero value or so. So, that way this m and n is considered separately they can always be equal as well.

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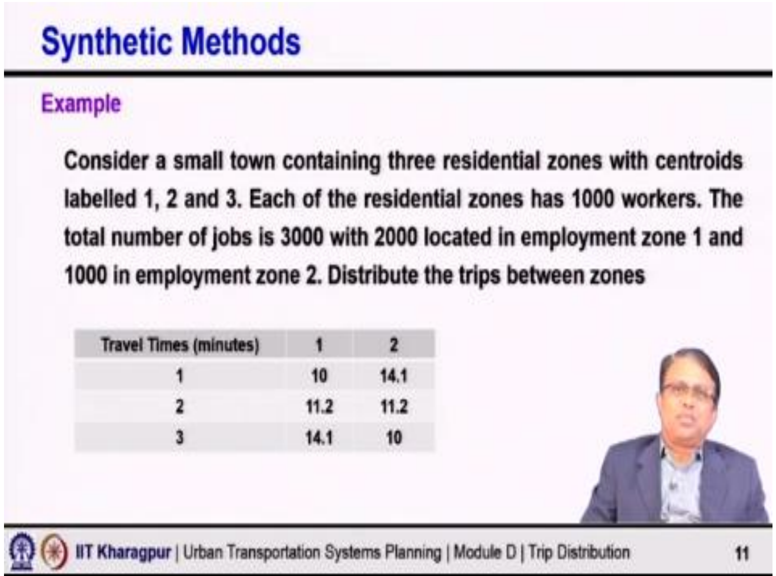

Now this example is taken considering a small town consisting of three residential zones named as 1, 2 and 3 and each zone has residential worker of above 1000. So, total 3000 workers are staying residential areas. The total number of jobs in the whole study area is 3000 and then 2000 located in employment zone 1 and 1000 located employment zone 2 and zone 3 does not have any employment.

Now distribute the trips between zones and we know the travel times, 1 to 1, 1 to 2, 2 to 1, 2 to 2, 3 to 1, 3 to 2. This travel time matrix is given. So, with that if we have to distribute the zone, distribute the trips following the linear programming approach.

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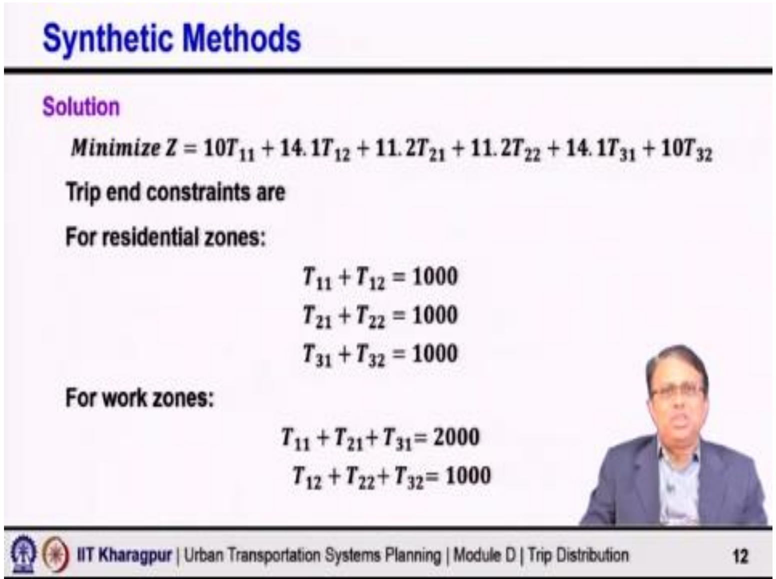

So, how would formulate we would like to minimize Z, minimize the aggregate travel time. How 10 into T 11, 14.1 into T 12 where this 10, 14 are coming? These are the travel times. 1 to 1 10, 1 to 2 14.1, so 10 into T 11 number of trips that will happen from 1 to 1 14. 1 into T 12, number of trips which will happen from 1 to 2 plus like that 11. 2 T 21, 11.2 T 31 by 2 2 and like this. So, we define the objective function and then we define the constraints.

We know that T 11+ T 12 total productions from zone 1 has to be 100, T 21 + T 22 again has to be 100, T 31 + T32 also has to be 100. Because each zone is producing 1000 trips. Similarly the attractions we have two zones only 1 and 2. So, T 11 + T 21 + T 31 has to be 2000, similarly T 12 + T 22 + T 32 has to be 1000. So, and of course all this T 11,T 12, T 21,T 22, T 31, T 32 all have to be 0 or any positive value they cannot be negative.

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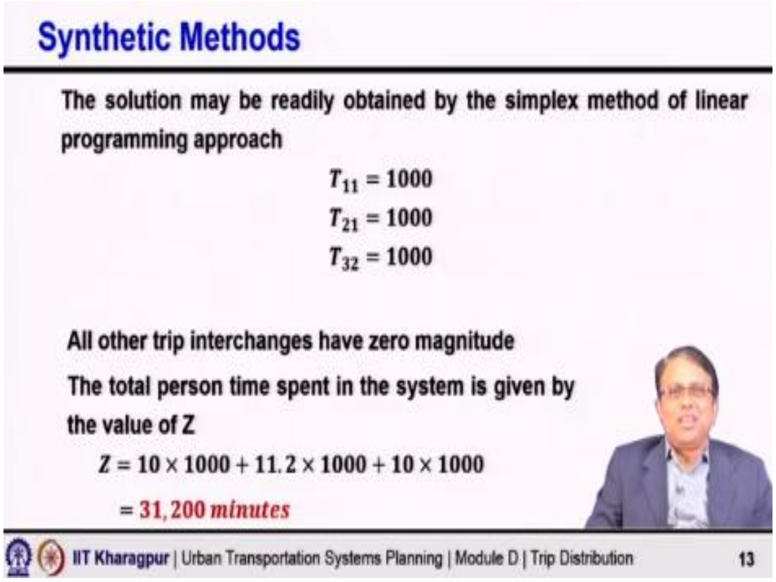

So, now I am not going to show you how you solve it you can use simplex method to solve it even you can use you know excel to solve it or any other platform to get a solution. But end of the day if you do it maybe you get solution like this. That in this case T 11 1000, T 21 1000, T 32 1000 and all other trip interchanges of zero magnitude. This is of course very simple example so no wonder.

So, in that case you find the total percent time spent also once you know this you know the trip matrix, the trip distribution values T 11, T 22, T 32 and you know already the corresponding travel time. So, you can find out then if this is the distribution then what will be my aggregate travel time for all the people who will be travelling and that is what is calculated here, 31200 minutes.

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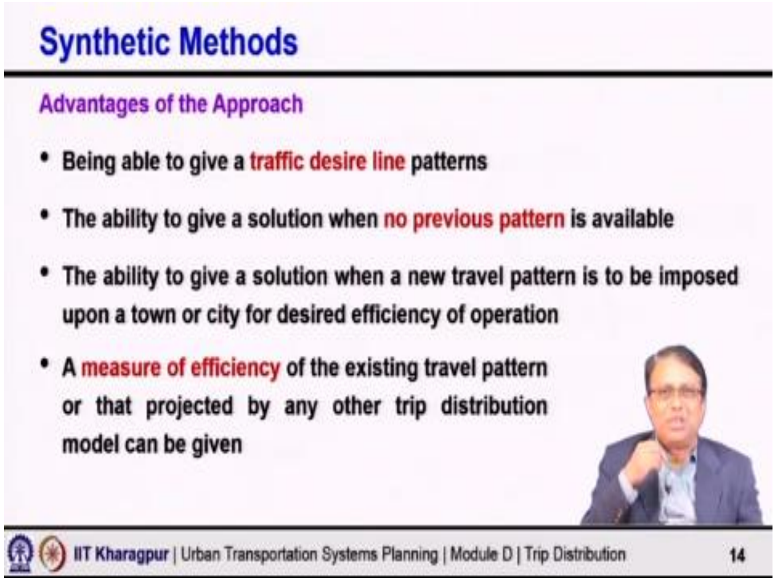

So, with this let us talk about the advantages being able to give a traffic desire line pattern. I could say that what is kind of ideal solution for me. The ability to give a solution when no previous pattern is available you do not need any previous pattern understanding. The ability to give a solution with a new travel time pattern is to be imposed upon a town or city for desired efficiency of operation.

So, may be the complete change you expect in the travel pattern. Because the whole metro network has been opened up and is operational now. So, you can use it and you can say that what would be the travel distribution trip distribution. A measure of efficiency of the existing travel pattern or that projected by any other trip distribution model can be given. That is again another positive thing because I say that this is actually some kind of a ideal solution or a kind of a dream solution what we would like to have if possible.

But it may not you may not get it because of so many other reasons. People do not think collectively and you know every individual make his or her own decision making based on own consideration. So, the real distribution is somewhat very different. But it has its own meaning actually if we know what is the solution we get.

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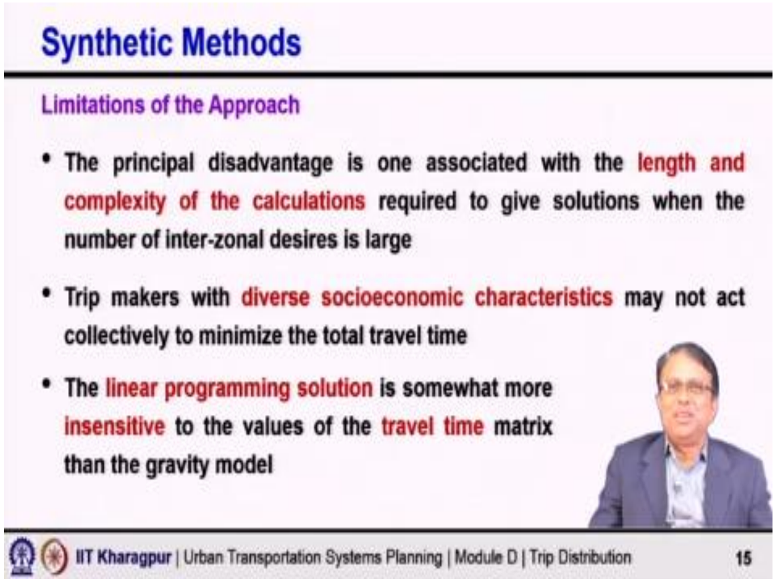

It has got certain limitations as well. The principal disadvantage is one associated with the length and complexity of calculations required. You know you we have considered here only three zones so it looks very simple. Now you just imagine that if you in a realistic network if you have you know large number of zones and so many cells are there. So, how many variables you will have like this T 11, T 12, T 13 like that how many variables you will have.

If you have 50, 60 or 100 zones in a transport network and then how many cells will be there. Trip makers with diver socioeconomic characteristics may not act collectively to minimize the total travel time what I indicated earlier to some extent the same thing we are saying the diverse economic characteristics you are you expect. And people may not really act collectively to minimize the total travel time.

Everybody is more likely to act selfishly I would select something which will minimize my own travel time. But collective thing is very different the linear programming solution is somewhat more insensitive to the values of travel time matrix as compared to the gravity model. So, again end of the day people have found out that probably gravity model is not that bad it is rather good.

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So, what we discussed here we discussed two types of other synthetic model one is the computing opportunity model and also the linear programming approach. And we said end of the day that you know you can apply them they are meaningful they have some you know deeper meaning or sense if you get solution. But in terms of overall performance still gravity model is not that bad rather the solution what you get is likely to be better. So, with this I close this module and this lecture as well. Thank you so much.