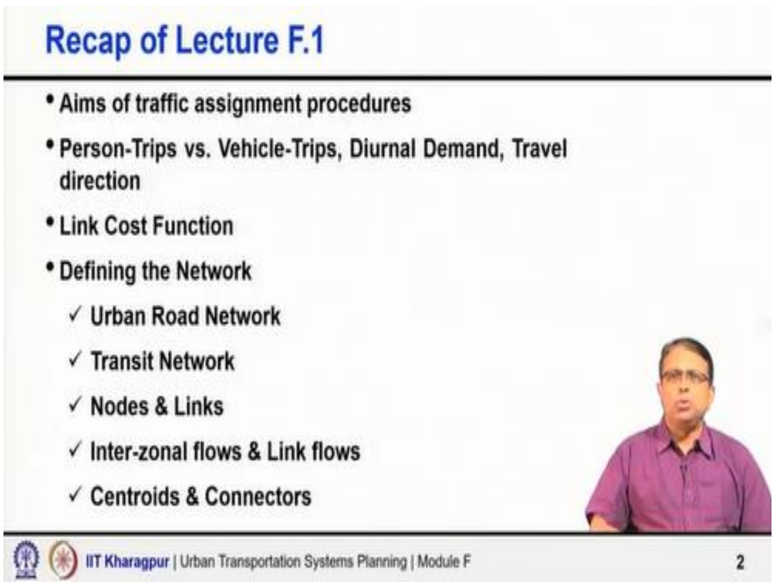
Urban Transportation Systems Planning
Prof. Bhargab Maitra
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture - 42
Network Algorithms-1

Module F Lecture 2: In first lecture we discussed about various aims of traffic assignment procedures.

(Refer Slide Time: 00:24)

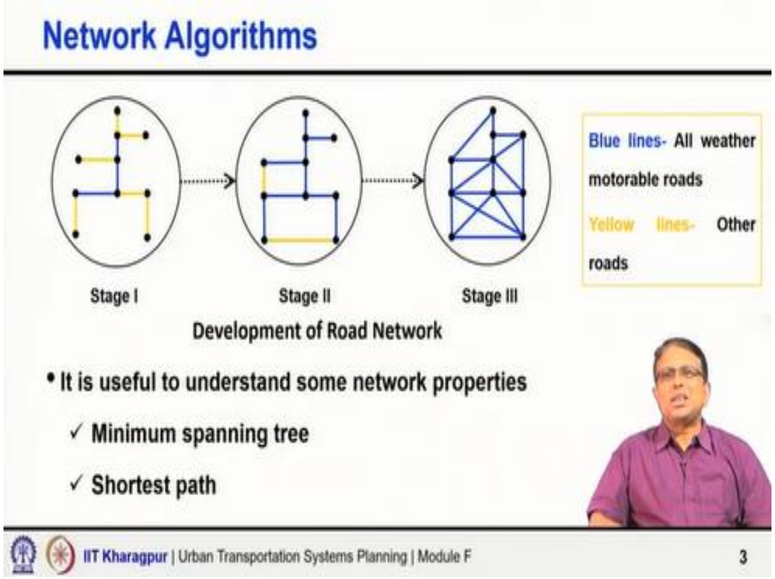


Why we do the traffic assignment? What about outcomes? Then the 3 aspects; what we should bear in mind like the percentage versus vehicle trips, diurnal demand or the time of the deviation of the demand and the direction of travel so, use of O-D matrix rather than production attraction matrix. These 3 aspects we mentioned very clearly that one should keep those things in mind while answering to any questions related to traffic assignment.

Then the Link Cost Function we said that how important it is and what is the link cost function, as for a given link how once the demand changes. How the travel time varies and that is what is represented by the link cost function. And then talked about how the network is defined with reference to say taking an urban road network what are the components linked or diagram representation then how the transit network is represented.

Then inter-zonal flow and the link flows how we can calculate a link may be part of multiple paths with respect to different O-D pairs. Then the centroid and how the centroids are connected using the centroid connectors or simply the connectors to the basic road network.

(Refer Slide Time: 01:57)



So, we shall continue our discussion today. Now before we, proceed further let me spend a few minutes just to give you a kind of understanding how the network developed. Think a small not even a town, but maybe a settlement mostly in the rural base where you have one or two major motorable roads passing through that area. And the remaining roads are not all weather roads or not the motorable roads probably.

Yes, people use them as road but they are just foot track or maybe I mean you can walk or you can go with the bicycle at best with the bike, but not by car or four wheelers. Also, maybe throughout the year this road may not be able to use this road. So, those are represented by yellow, so that look at the stage 1. So, the yellow line indicates are the real all all weather proper road which in engineering sense is the road and all weather facility.

Also, the yellows are other roads which are people call it road but they are not really in true sense engineering you know structure that way. That is the condition. So, people go from one house to another house or one village cluster to another village cluster using this kind of informal roads. And then you have one major road maybe a state highway passing through our interior passing through that area and that is the road which is a proper road. So, then at this stage, stage1 the first attempt is what? The first attempt should be developed the road network so that all these nodes if we consider all these nodes as village centroids or so. So, we want to then develop this network further to the best possible extent.

Primarily to satisfy the target of what? Target of ensuring connectivity to all these nodes. Connectivity was also there earlier but we were not using you know proper roads. So, now we should have all weather road connectivity to ensure that all the nodes are connected. So, that means my first target should be what? How to build minimum number of links or total road length? Minimum in the sense that we do not want duplication.

So, nowhere there will be any loop you cannot access one node using alternative path, but at least just you are able to ensure connectivity. So, any point to any point you can travel but then there is no alternative path for traveling to any node. That states to that is the immediate target. So, most cases the rural base we are targeting, that the whole PMGSY in a gross sense is aimed for that trying to ensure rural road connectivity using all weather rural road connectivity, that is the first stage.

So, yes, I can travel from any point to any point any village to any village any settlement to any settlement using all weather motorable road, that is the stage 2. But there is no alternative road.
So, at least the minimum that; if I can maintain my connectivity is maintained. So, all what we need to do is to ensure that? That network is maintained throughout the year. But then more and more people start living there, so this network may not be sufficient some cases people have to do a lot of detouring because there are no alternative roads.

So, then you start making more and more roads, that where which earlier you had to detour and take a much longer distance travelled much longer distance to reach. Now these two points are connected directly. Because you realize so many people want to travel from this point to that point, so it direct connectivity is necessary. Like that then you keep on adding more and more links because of the demand is growing and your current road does not have capacity or current road have route system is in terms of capacity.

But you know the people have to travel a longer distance it requires a lot of more time to travel. So, you want to make the connectivity or the travel faster with minimum time people should be able to travel, so you start developing the network. So, once you develop the network, then it is comes to a network like urban network alternative paths. Now not just connectivity but you can still travel using multiple path alternative path.

And intermittent states could be some small towns little bigger than that such number of paths are becoming, where alternatives are available becoming more and more because more and more roads are getting built. And finally, if you reach to Metro like Calcutta, Bombay, Delhi, virtually any point to any point you want to travel you have alternative path. Any two point most cases you will find, maybe 90, 90 % more than 95, 99 % case you want to travel from point A to point B you can travel using alternative path.

May not be through and through alternative path but some are events may be little bit common but still significantly different alternatives are available, that is stage 3. And any city any town any urban areas could be somewhere between stage 2 and stage 3 better than stage 2. The highest level is stage 3 where practically nearly 100% O-D pair you have alternative and then in between all other cities and towns will be there.

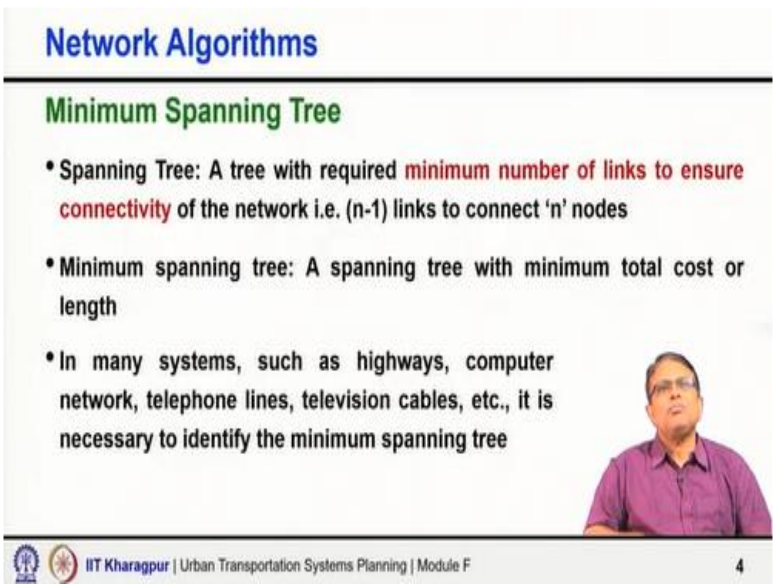
Somewhere maybe for some for od pairs you have alternative, some od pairs you do not have and then the proportion of O-D pairs where you have alternative links may increase as the city grow and you know cities are growing and as more and more infrastructure is being built. So, finally it comes to shape as I say near or around stage 3, where you have alternative paths. Most urban cases you want to travel you have alternative paths.

So, is useful at this stage to look at some of the network properties because I mentioned about stage 2 that where you have just you were ensuring connectivity with minimum road length. That I construct so many minimum what minimum road length I should construct. So, that my all nodes are connected just ensuring connectivity with minimum road length. That is the minimum spanning tree.

And then finally as you are moving towards stage 3, then you want to assign traffic stage 2 traffic has got no alternative. If somebody; has to travel along this path only, because there is no loop no alternative link. But as alternative starts coming and more and more O-D pairs were alternatives then the shortest path becomes important. Because every one of us each one of us individually would like to think which is the shortest path and we want to travel along the shortest path.

So, these two things I will discuss because I mentioned about this stage 2. So, I have to discuss about minimum spanning tree and then go to shortest path. So, with this algorithm once we know this then we will go to the next stage actual traffic assignment principle.

(Refer Slide Time: 11:16)



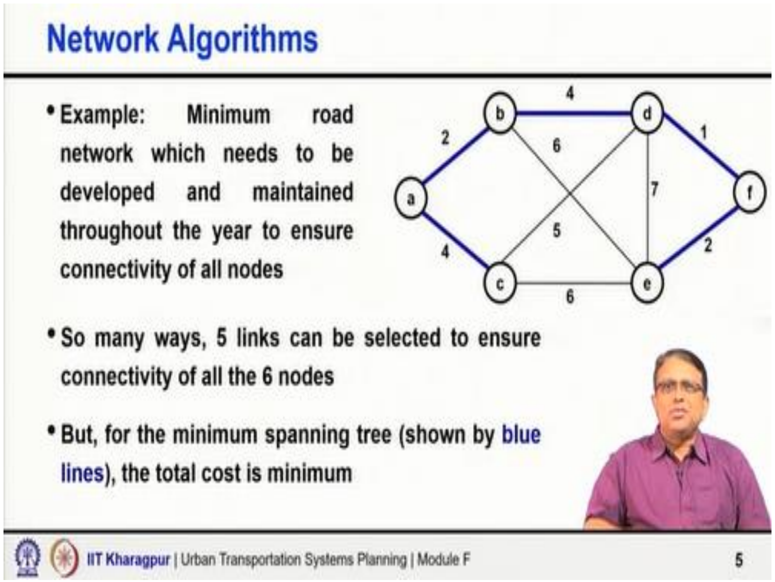
So, what is the Minimum Spanning Tree? As I said that to ensure connectivity of n nodes n number of nodes if I have to connect minimum number of links, how many? n - 1. So, I need n - 1 link minimum and only n - 1 link, because I do not want any loop to happen. Alternatives are not available you want to travel from one point to another point all point to all point you can travel but no alternative.

So, exactly then I will require minimum also n number one link to ensure connectivity and exactly n - 1 link only because I do not want to loop to happen. Now in a network there are so many ways you can select n - 1 link to ensure connectivity of n nodes. Now in each case may be the total length of all the link total length will be different. I want to pick up that n - 1 link that set of n - 1 link where the total length is minimum.

What is the implication? Or why I want to do that? That implication is that means if suppose the development authority or the government has to ensure connectivity, they would like to ensure that with what minimum road length. Because I can generally consider cost is proportional to the length of the road. If you have to make 200 kilometer road and 220 kilometer road making 220 kilometer road will be costlier in general.

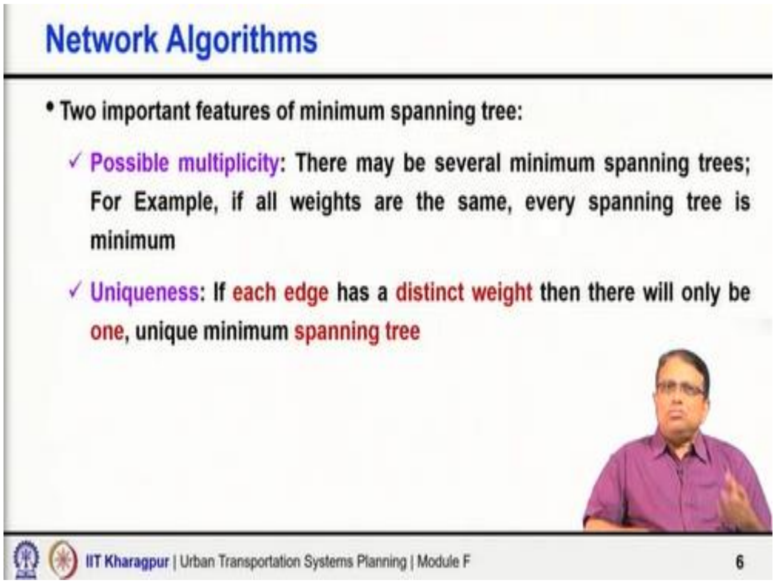
So, I want I have minimum length or total length of the road to ensure connectivity of this. So, I am optimizing in terms of my total length of the network. That I have to construct that I have to maintain operate that I have to maintain properly to ensure this connectivity. So, that is what is the minimum spanning tree, n - 1 link to ensure connectivity of n nodes with total minimum length of the links, n - 1 selected links.

(Refer Slide Time: 13:51)



That is what it is I have shown here. So, you can ensure connectivity if there are say 6 nodes there are so many ways, I can select 5 links and each case my all 6 nodes would be connected using exactly five number of links. But among those alternatives there will be one where the total length is minimum that is what is the spanning minimum spanning tree.

(Refer Slide Time: 14:25)



So, there are two important features of minimum spanning tree. As I said that development of something like a minimum spanning tree is the first objective. When a rural development based or rural area is getting developed slowly more and more population trying to get developed to more formal; going to some kind of urbanization is happening. Because all these developments happen slowly, you know the city of Calcutta walls once again also probably started yes it started with villages a set of villages.

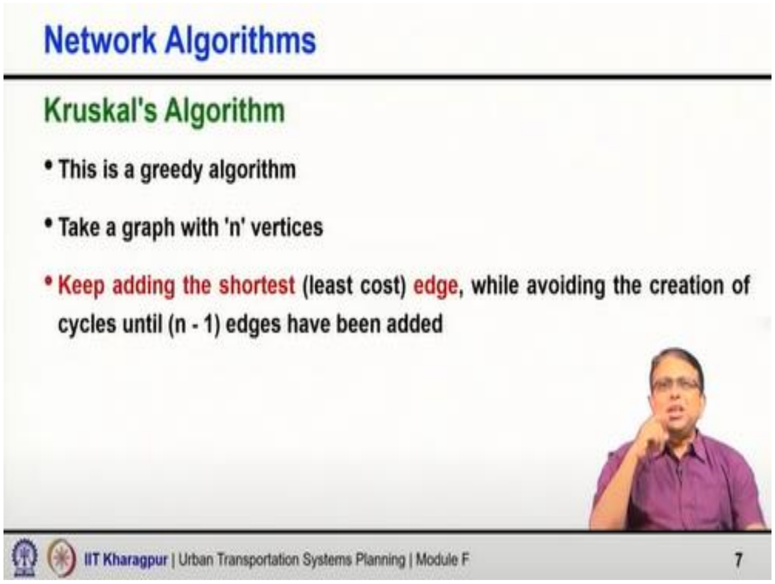
So, every place the big towns a big metro city they all started once upon a time with the cluster of villages. So, this the way that it gets developed is again very interesting. So, two important features, first step is again getting the minimum spanning tree, that is the basic minimum requirement. If people have to leave, they should be able to travel from any point to any point using all weather motorable roads.

That is the first objective and that we can achieve using minimum spanning tree. Now there are two important features of minimum spanning tree. One is the possible multiplicity, just consider that every link theoretical speaking every link the cost is same. So, there are there multiple solutions because every spanning tree is a minimum spanning tree. And I can have so many alternative minimum spanning trees. So, possible multiplicity is there.

So, similarly the sum of the links may be thing they are also you can you know get alternatives solutions. But if each edge has a distinct weight here in this case is let us a length we are talking. So, let us say length of each link is different then there can be only one spanning tree. So, no two-length link is having the same length, every link is length is different or the weight is different.

In that case the solution what you will get from minimum spanning tree is unique, no to minimum spanning tree you can get. So, it is only unique minimum spanning tree. The minimum spanning tree is to maybe unique, also, may have or you know multiplicity possible multiplicities there, depending on how the edges are there whether you have a proper link costs are same or different that will govern.

(Refer Slide Time: 17:31)



So, let us talk about two algorithms we shall discuss about two algorithms here. How to get the minimum spanning tree from a set of links? Now this set of links is can think that there are maybe you know connectivity is there. There are not always connectivity or they are may not be there motorable roads. You want to select that then which are those links which that I should upgrade to a more formal road to ensure connectivity with the minimum spanning tree.

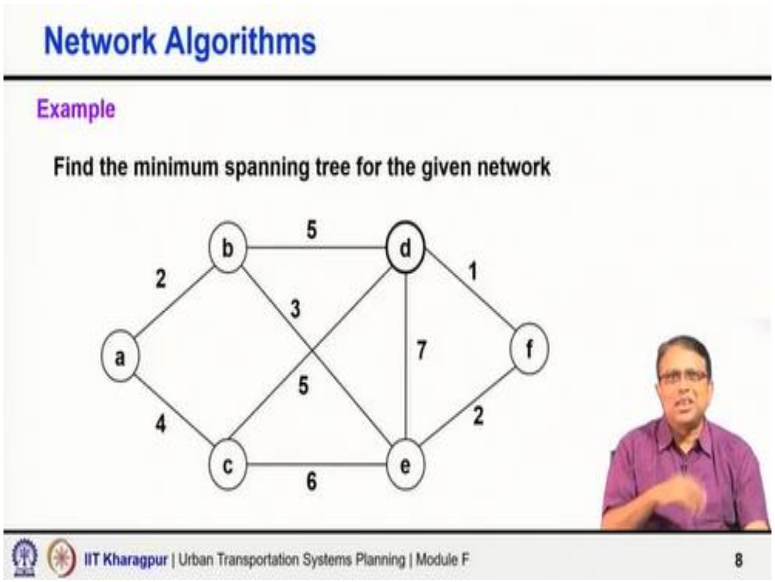
So, you can think all are potential candidates so that way I have got multiple links multiple connectivity exists, each case I know the length and I want to decide that which set of link n - 1 link I shall pick up to ensure connectivity of inputs. The Kruskal's Algorithms is one of possibility. The other possibilities the different algorithm prim's algorithm that I shall discuss after that this. First coming to Kruskal's algorithm, this is a greedy algorithm.

Because you will understand you have to remember all the links and their corresponding length. So, that what we need to do? We take a build a list in a very simple way in ascending order that means lowest length link on the top. Then the next mediate higher one, then next immediate higher one like that I prepare a list. Then first I take the shortest link try to see is all my connections are done, no then I go I pick up the next.

Immediate higher link higher cost link or higher link length. If I find I am forming a loop then loop it is not permissible not allowed I put it back. I know that I this I cannot include because then I am going to end up with loop. I cannot accept loop then I reject that take the next immediate higher link. Like that without creating loop if there is a loop reject that link. New link you pick up next higher cost link and you find that it is creating a loop.

So, you do not add to reject that. Like that I will pick up n - 1 edges on link my job is done. I picked up n - 1 link to ensure n connectivity of input how it works.

(Refer Slide Time: 20:18)

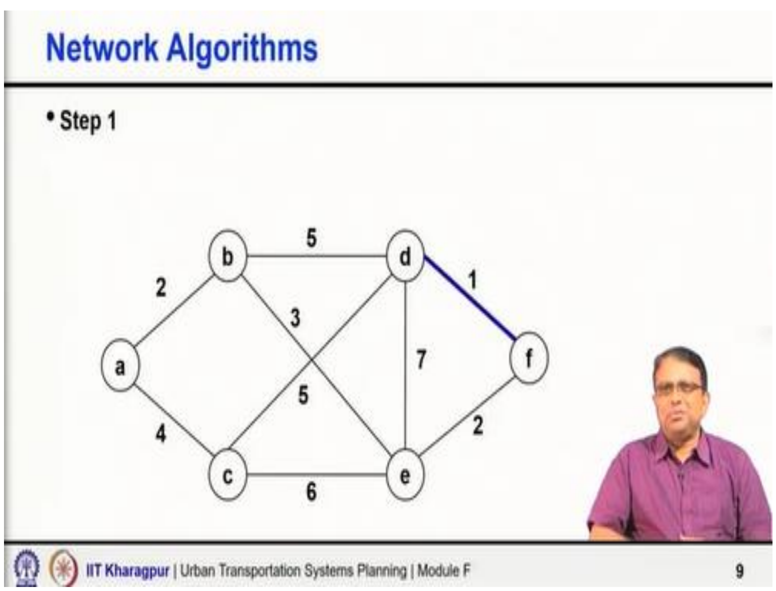


Let us take an example. Let us see that this is the base network that is given to us. These are all the possibilities maybe you can consider these are the existing roads, existing connections that people are using, by for walking and for use of by cycle and their maybe not blat of surface not bituminous road not concrete road. But they are just you know maybe some you know as I said that not really a proper engineering structure. And you want to convert them to all weather motorable roads.

So, how to select this? I know I have got a to b, b to d, d to e, 1, 2, 3, 4, 5, 6, 7, 8, 9 I have got 9 links here. Three threes, 3 or 7 to 9 yes, I have got 9 links. And how many nodes are there? I have got 6 nodes. So, how many links I want to pick up 5 links. So, from 9 I have to pick 5 links to ensure connectivity of 6 nodes that is what is my objective. As for this algorithm what I will do? I will put them in order.

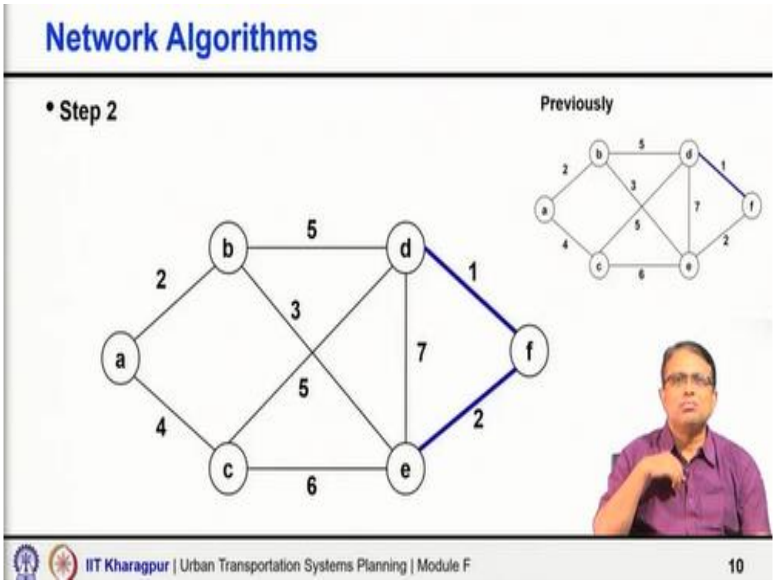
So, let us say e to f first is d to f, then you know e to f, a to b and e, in any order then b to e, with 3 then a to c, with cost 4. Then b to d with 5, then c to e 6, and e to d or d to e is 7. These are bidirectional link. So, no arrow is given that means two way links. And of course, when we are thinking of basing the basic connectivity, we are not we cannot think of one way or two way. All roads are two way roads.

(Refer Slide Time: 22:39)



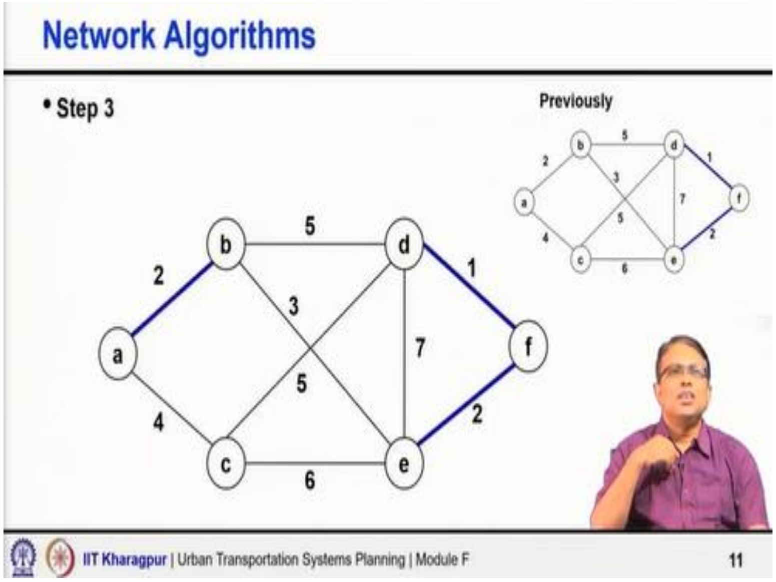
So, what we do? First, we take the d f that is the least cost that is on the top I pick up I have only one link but I know I have to select 5. So, I still need to select 4 more without creating loop. Next is e f or a b.

(Refer Slide Time: 23:03)



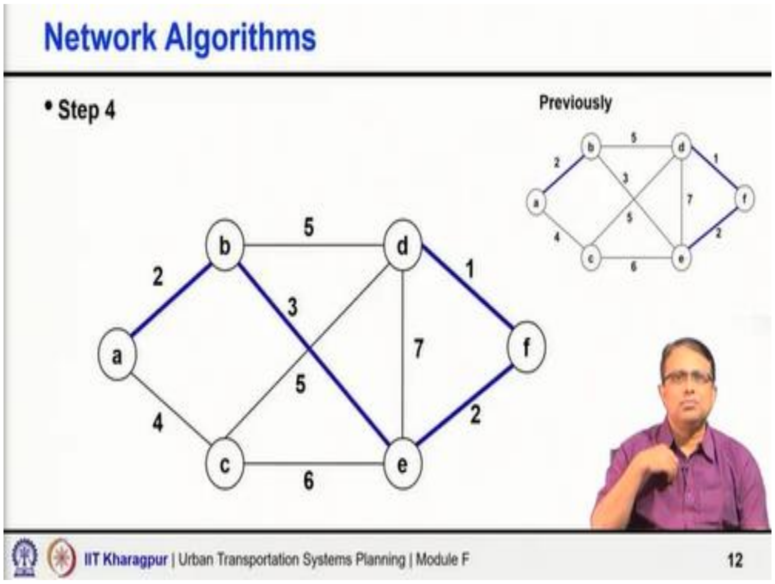
Anyone let us check e f, have I found the loop? No, I am going to only taken 2 links and no loop, so that is fine.

(Refer Slide Time: 23:15)



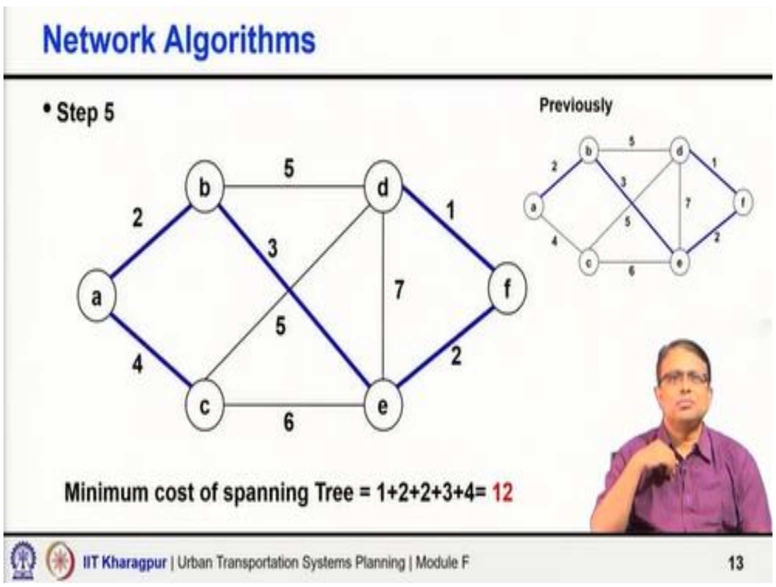
Third a to b again this also did not form loop. So, I can take it no problem then the next one is b to e.

(Refer Slide Time: 23:30)



Yes, again, no loop. Now next one in that order, what would be the next link, a to c cost is 4.

(Refer Slide Time: 23:55)



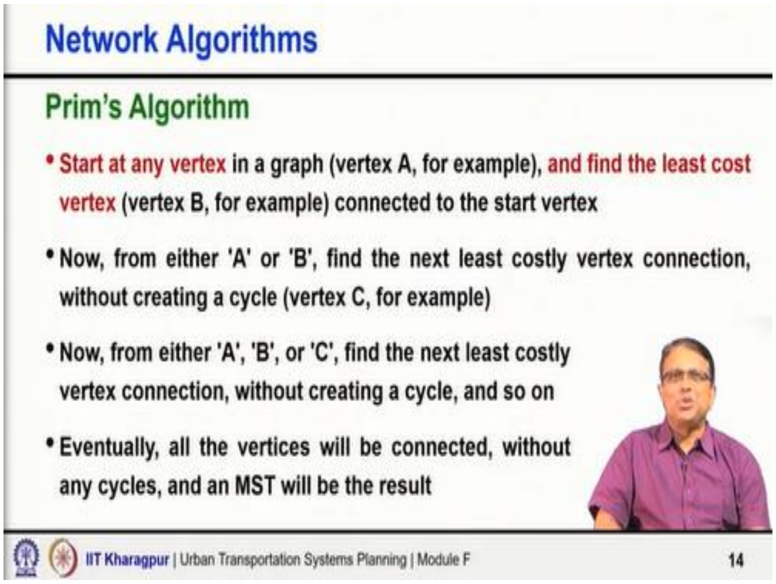
I got my link. So, here sequence is such that somehow the loop issued did not come, because the way I selected 1 after another the somehow the sequence was such that none of the link selected had to be rejected because of the loop that loop issue should not come. So, straight way I could select. May be in another case just the way the numbers are there you could find that. Say for example, this any other link may be change the cost there could be an issue of loop.

So, it could have been like four could have been this maybe e and d is 4, let us consider. And let us say a and c is 7. Then our next links as per the cost could have been d to e with 4. If I assume that d to e is 4, but I would not have taken that, why? Because then already I have connected d to f, e to f I do not want to connect d to e, because then it will form a loop. Then even if that d is 4, I would have still rejected that.

And then taken that next one whatever is the next higher if it is not forming the loop, I would have taken it. So, here the minimum cost of spanning tree is 12 and you can see that you will not get any other combination of 5 links, which are which will ensure connectivity of all these 6 nodes a, b, c, d, e, f with a total cost lesser than this 12. That is the way this particular algorithm works.

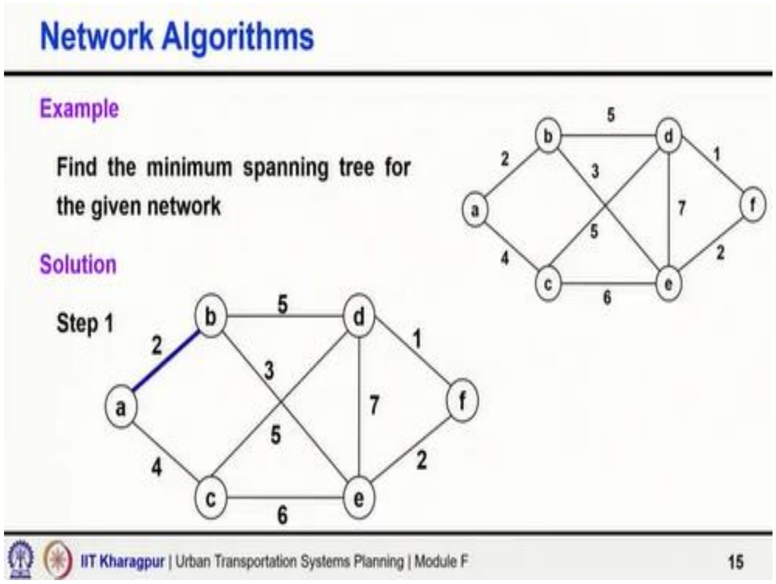
So, if I go back again, now you see this is a greedy algorithm. Take a graph with n vertices keep adding the shortest list cost edge while avoiding the creation of cycles or loop until n - 1 is just have been added without creating load starting from the list cost link I need to pick up n - 1 link without creating cycle or loop. That is what is the Kruskal's algorithms.

(Refer Slide Time: 26:52)



Next let me show you one more algorithm which is the Prim's algorithm. May be here also let me tell the example first and then come back here.

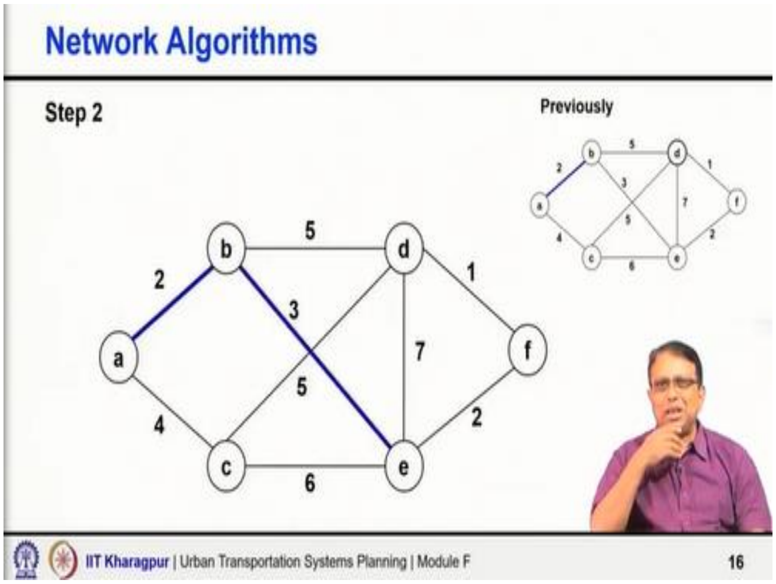
(Refer Slide Time: 27:03)



Let us take this example, same example what we start? We start with any node in this case, do not start with link. First start with any starting node, let us say I am starting with node a, you can start with any node no problem, but here in this case example, we are starting with node a. So, node a is connected to b, and a is connected to c, a to b the cost is how much 2, a to c the cost is how much 4.

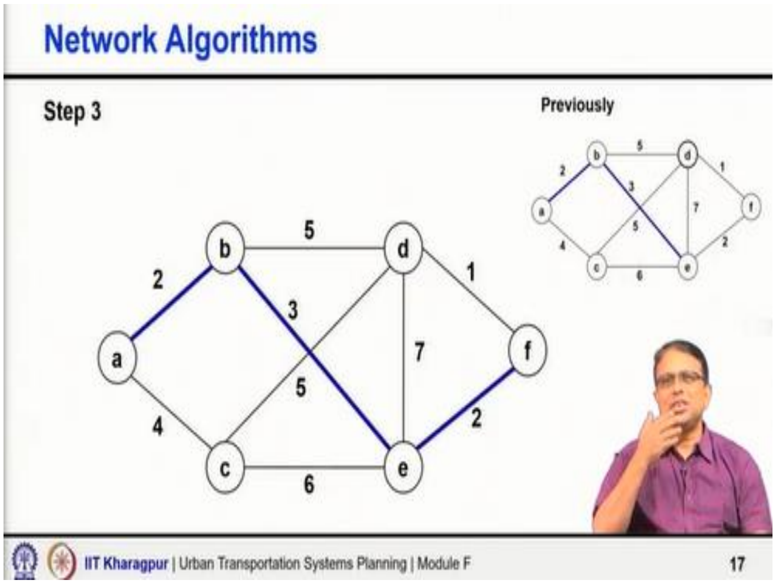
I have not explored any other node because I do not know still, I have started from a I do not know that remaining part of the network, so a b and a c are the two alternatives a b 2, a c 4, which one is minimum? a b, so I connect it to b. Once I have connected it to b then my whatever I have explored is c still there now b is newly connected. So, b d and b e are new connections new nodes d and e which I explored further. Now what are the cost a c 4, b e 3, b d 5. Which one is the minimum? b e with the cost of 3.

(Refer Slide Time: 28:46)



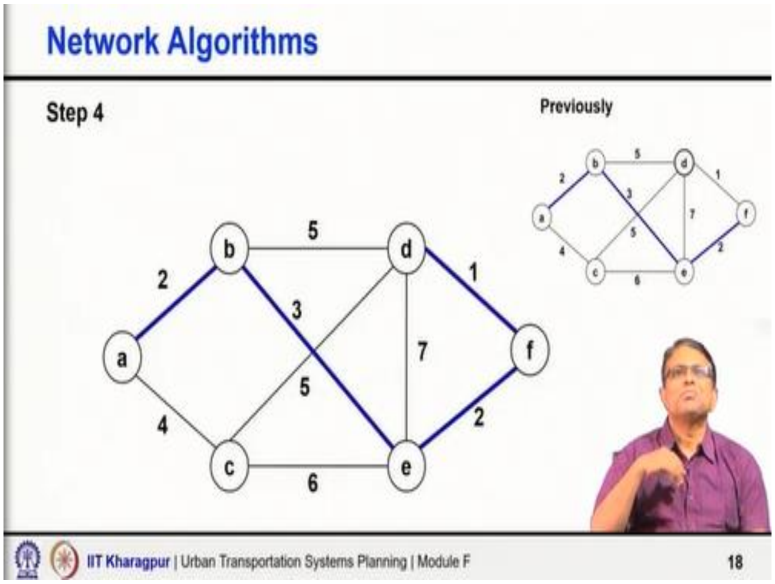
Now I connect that and again I am checking also I cannot form a loop that check or a cycle, that is continuously you have to check. Then what are my other connections now? a to c is still there with 4, b to d is still there with 5. But now e is also connected so e to f, is 2 e to d, is also 7. And e to c is 6, so I have a to c 4, c to e 6, e to d 7, e to f 2, b to d 5. Which is the least one? e to f.

(Refer Slide Time: 29:43)



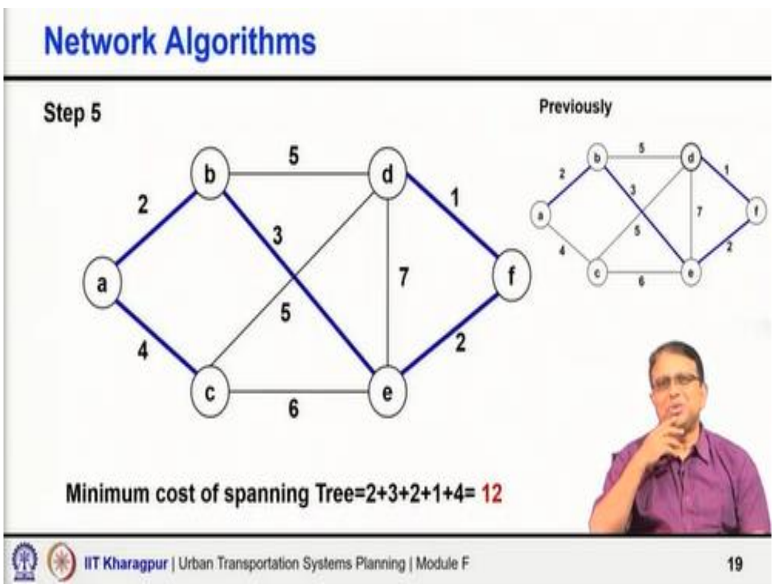
So, next e 2 f, then I have now I have explored f also. So, the new h also I find, f to d that is also connecting d, so that is another possibility. So, obviously you can say, here the next choice obviously is going to be f to d.

(Refer Slide Time: 30:09)



Then whatever are the remaining the next is going to be a to c with 4.

(Refer Slide Time: 30:24)



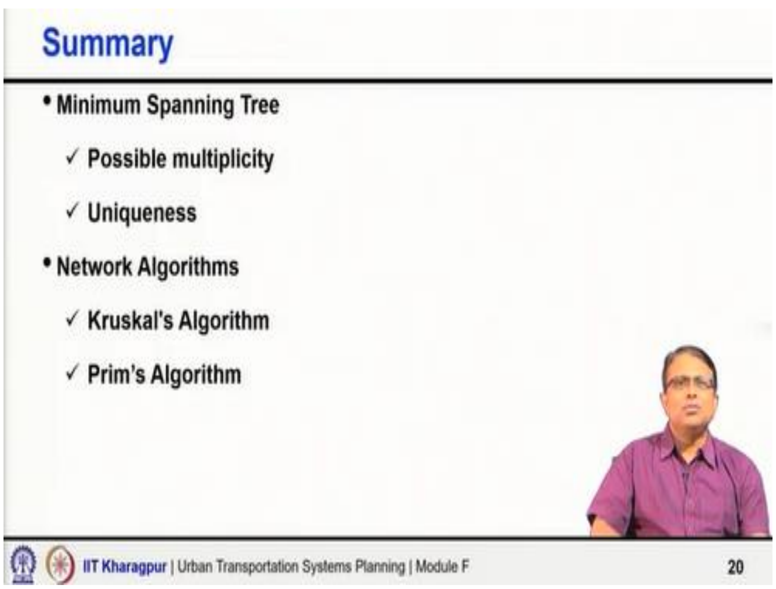
Because the other things b to d 5, c to e 6, e to d 7, all are higher than this 4. So, that is why I have again selected starting from all node I am exploring the connectivity and each segment whatever new links are there connecting to nodes which are still not connected, I am not reached to those nodes. Among those connections the least one cost connection I am taking. And like that going stage wise without creating loop or cycle nodes. Cycle it cannot create because n number of nodes are getting connected with exactly n - 1 link without creating loop.

You can start I started with a, you could start with b, you could start with c, any node. It does not matter which is your starting node, you will reach to the same answer. So, incidentally both algorithms give the same answer but two different approaches. Now, you can quickly see. How I describe this algorithm we said start at any vertex on the graph and find the least cost vertex what is connected to the start vertex.

Now from either A or B if a is my starting point and then least cost vertex is what I have discovered in that first stage is b. Then now either from A or B find the next least costly vertex connection without creating a cycle. Now again in the same way from either A, B or C find the next cost least costly vertex connection without creating a cycle and so on. Like that actually I will pick up n - 1 link without creating loop. So, that gives these are the prim's algorithm.

So, you see how we can get the solution and how the sequences of selection links are different in to algorithms. But the answer is same.

(Refer Slide Time: 32:52)



So, that is what it is. So, finally tell to summary. What we discussed? We discussed here about how the network gets developed. Finally, only a few motorized road or all weather road, the remaining are not very formal road in engineering sense and then the stitch to that time or attempt to way to at least ensure all weather road connectivity very habitation. And we do not want to create loop alternative path but basic minimum route network that can ensure connectivity.

And if I can maintain that road network throughout the year then people can travel from any point any point, that stage 2. How to get that? That I said there are spanning tree that is the minimum spanning tree. Possible multiplicity and uniqueness are that those both are explained clearly that if I have distinct you know link length then uniqueness will be there. If there are multiple links having the same cost, then it is possible that they have alternative solutions with same total cost.

Then two algorithms will discuss Kruskal's algorithm and Prim's algorithm which you can use to get the minimum spanning tree, that stage 2. Then the stage 3 is what? Now we start building even more roads, because more organization is happening more people are staying coming, they want to travel. Now you think like no a long detouring I do not want I want people to travel faster using minimum time, so if I connect this point to that point lot of people are traveling. Then so many people do not have to do so much detouring.

They can directly get connected somewhere the existing capacity is not adequate roads are congested you start building. And finally, you come to a full fledged network development like as you have seen big cities like Calcutta, Delhi, or Mumbai or Chennai. And where practically any point to any point you want to travel, nearly for all O-D pairs, you have alternative paths. So, there now the problem will be to find out the shortest path, so we shall continue our discussion in the next lecture about how to find out the shortest path. Thank you so much.