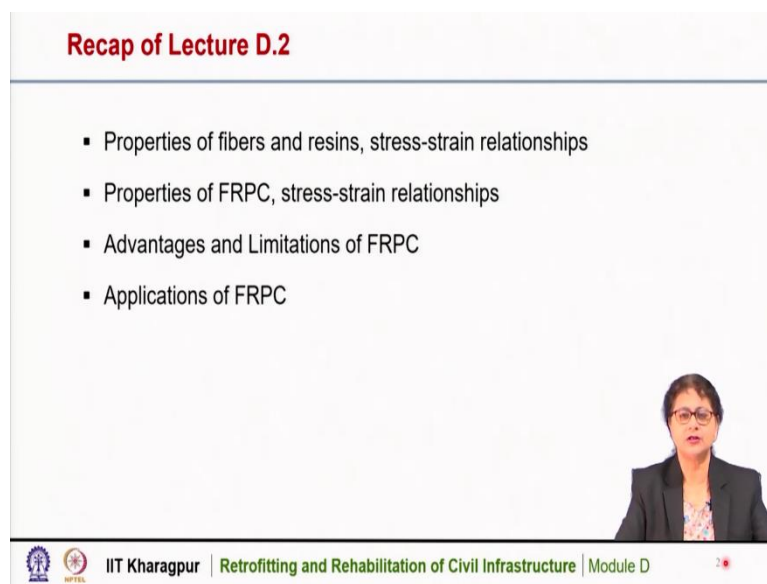


Retrofitting and Rehabilitation of Civil Infrastructure
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Indian Institute of Technology Kharagpur
Lecture 20
Micromechanics of Composites

Hello friends, welcome to the NPTEL Online Certification Course Retrofitting and Rehabilitation of Civil Infrastructure. Today, we will discuss Module D. The topic for Module D is Fiber Reinforced Polymer Composites and its Characteristics.

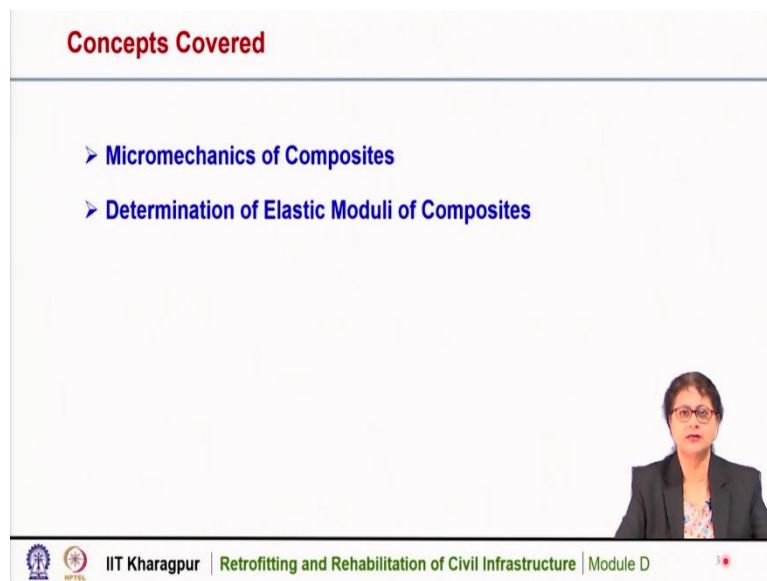
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The slide is titled "Recap of Lecture D.2" in red text at the top. Below the title, there is a list of four bullet points: "Properties of fibers and resins, stress-strain relationships", "Properties of FRPC, stress-strain relationships", "Advantages and Limitations of FRPC", and "Applications of FRPC". In the bottom right corner of the slide, there is a small video inset showing a woman with glasses and a dark jacket. At the bottom of the slide, there is a footer with logos for IIT Kharagpur and NPTEL, followed by the text "IIT Kharagpur | Retrofitting and Rehabilitation of Civil Infrastructure | Module D".

In the previous lecture, we have discussed the properties of different types of fibers and resins, and their stress-strain relationships. The properties of fiber reinforced polymer composites, and their stress-strain relationships. The advantages and limitations of fiber composites and its various application in different industries.

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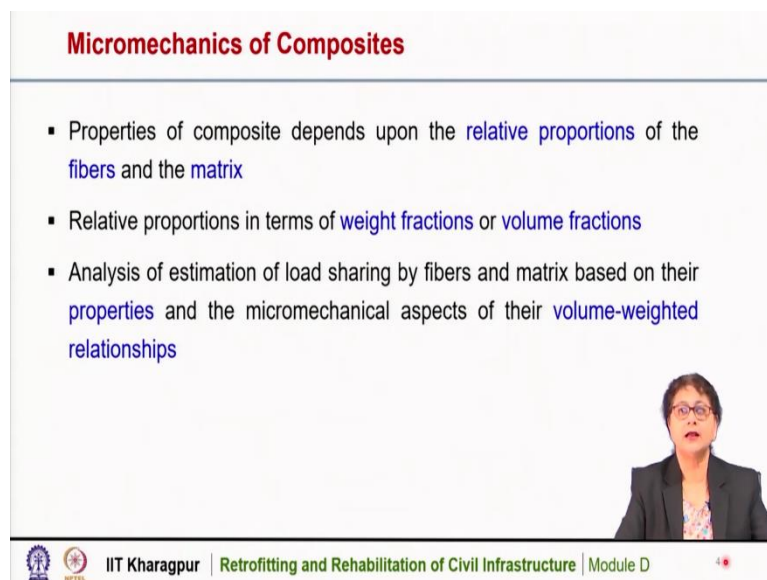
Concepts Covered

- **Micromechanics of Composites**
- **Determination of Elastic Moduli of Composites**

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In today's lecture, we will discuss the micromechanics of composites. In micromechanics of composite, we will discuss how we can determine the elastic modulus of composites by knowing the properties of the different phases that is the fiber and the matrix.

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Micromechanics of Composites

- Properties of composite depends upon the **relative proportions** of the **fibers** and the **matrix**
- Relative proportions in terms of **weight fractions** or **volume fractions**
- Analysis of estimation of load sharing by fibers and matrix based on their **properties** and the micromechanical aspects of their **volume-weighted relationships**

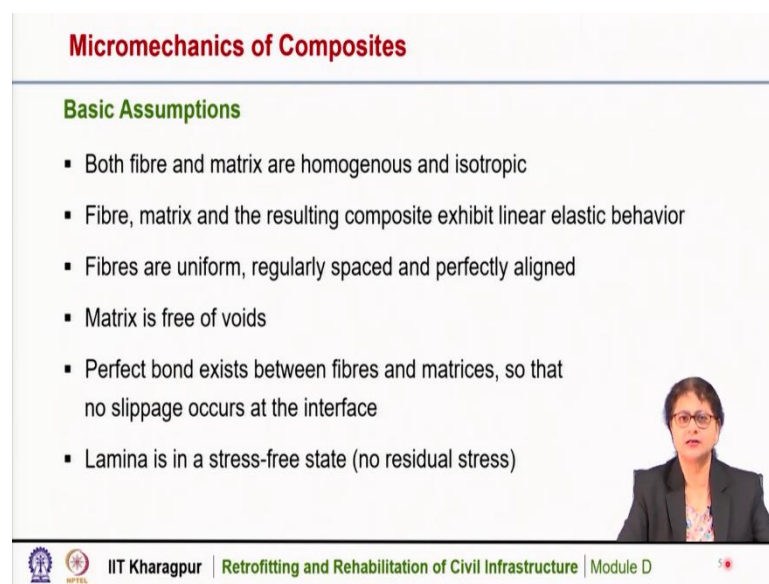
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In a composite, the properties depend significantly upon the properties of fibers and matrix, and their relative proportions. The relative proportions maybe in terms of weight fractions or volume fractions in a composite. The properties depend significantly on the amount of fiber and matrix in a composite. Micromechanics of composite is the analysis of the estimation of

load sharing by fibers and matrix based on their properties, and the micromechanical aspects of their volume-weighted relationships.

So, in micromechanics of composite we determine the elastic properties of the composite at the fiber level, so it is the analysis at the fiber level. And by knowing the properties of the individual fibers and the matrix phases, and their relative proportions in terms of their weight and volume the properties of the composite are determined.

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The slide is titled "Micromechanics of Composites" in red text. Below the title, the section "Basic Assumptions" is highlighted in green. It contains a bulleted list of six assumptions. In the bottom right corner of the slide, there is a small video inset showing a woman with glasses and a dark jacket. At the bottom of the slide, there are logos for IIT Kharagpur and NPTEL, along with the text "Retrofitting and Rehabilitation of Civil Infrastructure | Module D".

Micromechanics of Composites

Basic Assumptions

- Both fibre and matrix are homogenous and isotropic
- Fibre, matrix and the resulting composite exhibit linear elastic behavior
- Fibres are uniform, regularly spaced and perfectly aligned
- Matrix is free of voids
- Perfect bond exists between fibres and matrices, so that no slippage occurs at the interface
- Lamina is in a stress-free state (no residual stress)

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In determining the properties of composite there are several basic assumptions. The assumptions are that both fiber and matrix are homogeneous and isotropic. The composite may not be homogeneous because of the presence of fibers, but individual fibers are homogeneous and isotropic and also the matrix part.

The fiber matrix and the resulting composite exhibit linear elastic behavior, we have seen in the earlier lecture that the stress-strain relationship of fiber and matrix are generally linearly elastic and the resulting composite also. Fibers are uniform, they are regularly spaced within the composite and perfectly aligned within them composite.


The matrix is free of voids, the fibers are placed within the matrix and the matrix is free of voids, and there is perfect bond exists between the fiber and the matrix, so that no slippage occurs at the interface. So, we assume that the fibers are perfectly aligned, the matrix is free of voids and there is no delamination no slippage between the fiber and the matrix, and the lamina is in a stress free state.

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Micromechanics of Composites

Volume and Weight Fractions

V_c = Volume of Composite	V_m = Volume of Matrix
W_c = Weight of Composite	W_m = Weight of Matrix
ρ_c = Density of Composite	ρ_m = Density of Matrix
V_f = Volume of Fiber	
W_f = Weight of Fiber	
ρ_f = Density of Fiber	



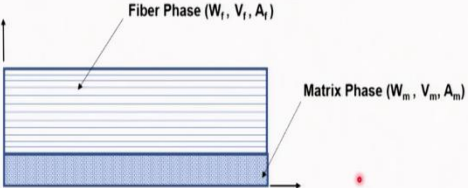
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To determine the volume and weight fractions there are several terminologies that are used. Capital V with suffix c is the volume of composite. For composite we use the suffix c for fiber, it is the f that is used and for matrix we use m as suffix, and the volume weight and density are denoted as V_w and ρ respectively. So, the weight of composite is denoted as W_c the volume of fiber is denoted as V_f and the density of matrix is denoted as ρ_m and similarly the other fractions.

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Micromechanics of Composites


- Fiber volume fraction
- Matrix volume fraction
- Fiber weight fraction
- Matrix weight fraction



Fiber Phase (W_f, V_f, A_f)

Matrix Phase (W_m, V_m, A_m)

Idealization of Unidirectional Composite



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Now, we need to determine the fiber volume fraction, the fiber weight fraction, the matrix volume fraction and matrix weight fraction. So, this is an idealization of a unidirectional

composite. We can see here this is the composite where there are two phases one is the fiber phase and the other is the matrix phase, though the fiber is suspended into the matrix more or less uniformly.

But, however, here to distinguish the two phases, it is shown schematically as this part is fiber phase and this part is the matrix phase. So, this is a unidirectional composite where the fibers are aligned along this axis. The properties are denoted as the weight of a matrix is W_m , the volume of matrix is denoted as V_m and the area of the matrix is denoted as A_m . The similarly for the fiber phase it is W_f , V_f and A_f .

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Micromechanics of Composites

Volume of composite, $V_c = V_f + V_m$ (1)

Volume fraction of fiber and Volume fraction of matrix


$$v_f = \frac{V_f}{V_c} \quad \text{and} \quad v_m = \frac{V_m}{V_c} \quad (2)$$

Now, $v_f + v_m = \frac{V_f}{V_c} + \frac{V_m}{V_c} = \frac{V_f + V_m}{V_c} = \frac{V_c}{V_c} = 1$ (3)

Weight fraction of fiber and Weight fraction of matrix

$$w_f = \frac{W_f}{W_c} \quad \text{and} \quad w_m = \frac{W_m}{W_c} \quad (4)$$

So, $w_f + w_m = \frac{W_f}{W_c} + \frac{W_m}{W_c} = \frac{W_f + W_m}{W_c} = \frac{W_c}{W_c} = 1$ (5)



Now, the volume of composite that is $V_c = V_f + V_m$ that is the volume of fiber plus the volume of matrix. And we have assumed that there is no void, so the volume of composite is the summation of the volume of fiber and the volume of matrix. Now, the volume fraction of fiber and the volume fraction of matrix is written as this. Small v_f is the volume of fiber by the volume of the composite.

So, it is the volume fraction of fiber that is equal to the volume of fiber by the volume of the composite. Similarly, the volume fraction of matrix is the volume of matrix divided by the volume of the total composite. Now, if we add the two volume fractions of fiber and matrix that is $v_f + v_m = 1$.

Similarly, we can write the weight fraction of the two phases. So, the weight fraction of fiber is small w_f . So, it is the weight of fiber divided by the weight of composite.

Similarly, the weight fraction of matrix is the weight of matrix by the weight of the composite. So, if we add the two weight fractions that is $w_f + w_m = 1$. So, the volume fraction of fiber plus volume fraction of matrix is equal to 1, and the weight fraction of fiber and the weight fraction of the matrix is also equal to 1.

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Micromechanics of Composites

Now, $W_c = W_f + W_m$ (6)


$$\rho_c \times V_c = \rho_f \times V_f + \rho_m \times V_m$$
 (7)

Dividing both sides by V_c , we get, $\rho_c = \rho_f \times v_f + \rho_m \times v_m$ (8)

Similarly, replacing volumes in terms of weights and densities in eqn. (1), we get

$$\frac{W_c}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}$$
 (9)

Dividing both sides by W_c , eqn. (9) can also be written as

$$\frac{1}{\rho_c} = \frac{w_f}{\rho_f} + \frac{w_m}{\rho_m}$$
 (10)


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Micromechanics of Composites

Volume of composite, $V_c = V_f + V_m$ (1)

Volume fraction of fiber and Volume fraction of matrix


$$v_f = \frac{V_f}{V_c} \quad \text{and} \quad v_m = \frac{V_m}{V_c}$$
 (2)

Now, $v_f + v_m = \frac{V_f}{V_c} + \frac{V_m}{V_c} = \frac{V_f + V_m}{V_c} = \frac{V_c}{V_c} = 1$ (3)

Weight fraction of fiber and Weight fraction of matrix

$$w_f = \frac{W_f}{W_c} \quad \text{and} \quad w_m = \frac{W_m}{W_c}$$
 (4)

So, $w_f + w_m = \frac{W_f}{W_c} + \frac{W_m}{W_c} = \frac{W_f + W_m}{W_c} = \frac{W_c}{W_c} = 1$ (5)



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Now, the total weight of the composite is the total weight of the fiber plus the total weight of the matrix. So, we can write the weight in terms of density and volume.

So, $\rho_c \times V_c = \rho_f \times V_f + \rho_m \times V_m$. So, dividing both sides by the volume of composite we get ρ_c that is the density of the composite. So, $\rho_c = \rho_f \times v_f + \rho_m \times v_m$.

So, by knowing the volume fraction of fiber and the volume fraction of matrix and also the densities of the two, we can get the density of the composite with this equation. Similarly, by replacing the volumes in terms of weights and density in equation 1. This is the equation 1, in this equation we are replacing the volume with weights and density we get

$$W_c/\rho_c = W_f/\rho_f + W_m/\rho_m \text{ and dividing both sides by the weight of composite we get } 1/\rho_c = w_f/\rho_f + w_m/\rho_m.$$

So, the density of the composite can also be written in terms of their weight fractions and the densities of fiber and matrix. Here in this equation the density of the composite can be written in terms of their volume fractions and here it is written in terms of their weight fractions, and also with the densities of fiber and matrix.

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Micromechanics of Composites

Now, $\frac{v_f}{w_f} = \frac{V_f/V_c}{W_f/W_c} = \frac{V_f}{W_f} \times \frac{W_c}{V_c} = \frac{\rho_c}{\rho_f}$ (11)

So, $v_f = w_f \times \frac{\rho_c}{\rho_f}$ (12)

Similarly, $v_m = w_m \times \frac{\rho_c}{\rho_m}$ (13)

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Now, we can also write

$$v_f/w_f = (V_f/V_c) / (W_f/W_c) = (V_f/W_f) \times (W_c/V_c) = \rho_c/\rho_f$$

W_f , we can also write V_f / W_f , that is volume fraction by weight fraction, which is equal to the total volume of fiber by the total volume of composite divided by the total weight of the fiber by the total weight of the composite and that gives us ρ_c/ρ_f . So, we can write the volume fraction of fiber is equal to $W_f \times (\rho_c/\rho_f)$.

Similarly, V_m that is the volume fraction of the matrix is equal to the weight fraction of the matrix into ρ_c/ρ_m . So, these are the two relationships that are used in the subsequent analysis


that is the weight fraction and the volume fractions are related. So, we can get the relationships between the volume fraction and the weight fraction of the fibers and also the metrics.

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Some Important terms

E_{cl} = Young's modulus of Composite in the direction of fiber
 σ_{cl} = Composite strength in the fiber direction
 ϵ_{cl} = Strain in composite in the fiber direction
 A_{cl} = Cross sectional area of the composite perpendicular to the direction of fiber

E_{ct} = Young's modulus of Composite in the transverse direction
 σ_{ct} = Composite strength in transverse direction
 ϵ_{ct} = Strain in composite in transverse direction
 A_{ct} = Cross sectional area of the composite parallel to the direction of the fiber




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Some Important terms

E_f = Young's modulus of fiber in the direction of applied load
 σ_f = Fiber strength in the direction of applied load
 ϵ_f = Strain in fiber in the direction of applied load
 A_f = Cross sectional area of the fiber perpendicular to the direction of applied load

E_m = Young's modulus of matrix in the direction of applied load
 σ_m = Matrix strength in the direction of applied load
 ϵ_m = Strain in matrix in the direction of applied load
 A_m = Cross sectional area of the matrix perpendicular to the direction of applied load



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Now, here are some important notations that we need to know for further analysis. E_{cl} is the elastic modulus or Young's modulus of the composite in the direction of the fiber. C denotes the composite and l denotes the longitudinal direction. So, it is the modulus of the composite in the direction of the fiber. Similarly, σ_{cl} is the strength of the composite in the direction of the fiber, ϵ_{cl} is the strain in the composite in the direction of the fiber, A_{cl} is the cross-sectional area of the composite perpendicular to the direction of the fiber.

Similarly, the transverse direction E_{ct} is the young's modulus of the composite in the transverse direction σ_{ct} is the composite strength in the transverse direction, ϵ_{ct} is the strain in the composite in the transverse direction and A_{ct} is the cross-sectional area of the composite parallel to the direction of the fiber.

There are some more terms. Similar notations are used for fibers, as well as matrix. For example, E_f is the modulus of the fiber in the direction of the applied load, and σ_f is the fiber strength in the direction of the applied load. Similarly, E_m is the modulus of the matrix in the direction of the applied load etc.

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Longitudinal Strength and Stiffness of Composite

Unidirectional Composite member
longitudinally stressed

Assumptions

- The rod extends uniformly with no delamination between the matrix and fiber
- Transverse sections that were plain before bending remains plain after bending. This means, strain in the matrix and the strain in the fiber are the same

$$\epsilon_{cl} = \epsilon_f = \epsilon_m = \frac{\Delta L}{L} \quad (14)$$

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Now, we will discuss how to determine the elastic modulus of the composite along the direction of the fiber. So, it is the longitudinal strength and stiffness of the composite, and it is along the direction of the fiber. So, here is an idealized schematic diagram. This is a unidirectional composite member. We can see here the two phases are shown differently. This is the fiber phase and this is the matrix phase.

The fibers are aligned along this direction, and this is the matrix. The composite is of length l . Though the fibres are suspended into the matrix however, for the analysis purpose we are showing it in this way the two different phases of the composite. The composite is subjected to a load you can see here a load that is the tensile force P_c . And due to this tensile force, the composite undergoes deformation and this is the deformed section of the composite.

So, it elongates and this is the deformed section. The assumptions for the analysis is that the rod extends uniformly with no delamination between the matrix and the fiber, so here we assume that though it is subjected to loading the composite extends uniformly and there is no delamination, no slippage between the fiber and the matrix. The transverse sections that were plane before bending remains plain after bending.

This indicates that the strain in the matrix and the strain in the fiber are the same. So, the ϵ_{cl} that is the strain in the composite is equal to the strain in the fiber, and also that is the same strain is applied under matrix parts. So, $\epsilon_{cl} = \epsilon_f = \epsilon_m = \Delta L/L$. This part the deformation is the ΔL , and the strain is equal to $\Delta L/L$. That is the original length of the composite.

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Longitudinal Strength and Stiffness of Composite

Now $E_f = \frac{\sigma_f}{\epsilon_f}$ $\sigma_f = E_f \times \epsilon_f$ (15)


$E_m = \frac{\sigma_m}{\epsilon_m}$ $\sigma_m = E_m \times \epsilon_m$ (16)


Load taken by the Composite, $P_c = P_f + P_m$ (17)

Or, $\sigma_{cl} \times A_{cl} = \sigma_f \times A_f + \sigma_m \times A_m$ (18)

$= E_f \times \epsilon_f \times A_f + E_m \times \epsilon_m \times A_m$

$= (E_f \times A_f + E_m \times A_m) \epsilon_{cl}$ (19)




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Now, the elastic modulus can be written as stress by strain, so, for the fiber $E_f = \sigma_f/\epsilon_f$. So, $\sigma_f = E_f \times \epsilon_f$, for the matrix $E_m = \sigma_m/\epsilon_m$ or $\sigma_m = E_m \times \epsilon_m$.

Now, the load is taken by the entire composite and that load is shared by the fiber part and the matrix part. So, the total load P_c is equal to the load shared by the fiber which is P_f plus the load shared by the matrix which is P_m . So, $P_c = P_f + P_m$.

Now, P_c can be written as $\sigma_{cl} \times A_{cl}$ and this is equal to $(\sigma_f \times A_f + \sigma_m \times A_m)$, which is also written as in terms of the strain that is $(E_f \times \epsilon_f \times A_f + E_m \times \epsilon_m \times A_m)$. We are replacing the stress by the strain and the elastic modulus.

Since the strain is the same in the fiber as well as in the matrix so we can write that $(E_f \times A_f + E_m \times A_m) \epsilon_{cl}$.

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Longitudinal Strength and Stiffness of Composite

$$\sigma_{cl} \times A_{cl} = (E_f \times A_f + E_m \times A_m) \epsilon_{cl} \quad (19)$$

Or, $\frac{\sigma_{cl}}{\epsilon_{cl}} = E_{cl} = (E_f \times \frac{A_f}{A_{cl}} + E_m \times \frac{A_m}{A_{cl}}) \quad (20)$


Or, $E_{cl} = \left[E_f \times \frac{A_f \times L}{A_{cl} \times L} \right] + \left[E_m \times \frac{A_m \times L}{A_{cl} \times L} \right] \quad (21)$

Or, $E_{cl} = \left[E_f \times \frac{V_f}{V_{cl}} \right] + \left[E_m \times \frac{V_m}{V_{cl}} \right] \quad (22)$

Or, $E_{cl} = E_f \times v_f + E_m \times v_m \quad (23)$

And, eqn. (20) can be written as

$$\sigma_{cl} = \sigma_f \times \frac{A_f}{A_{cl}} + \sigma_m \times \frac{A_m}{A_{cl}} = \sigma_f \times v_f + \sigma_m \times v_m \quad (24)$$



Now, by changing this epsilon cl here, we are putting this ϵ_{cl} in the left side and A_{cl} in the right side, we get $\sigma_{cl}/\epsilon_{cl} = E_{cl} = (E_f \times A_f/A_{cl} + E_m \times A_m/A_{cl})$.

This can also be written as, this equation, equation 21 by multiplying the length of the composite in the denominator as well as in the numerator in both these components are fiber and matrix. So, this $A_f \times L$ is nothing but the volume of the fiber. $A_m \times L$ is the volume of the matrix and $A_{cl} \times L$ is the volume of the composite.

So, this is the volume of fiber and this is the volume of composite. So, this gives the volume fraction of the fiber. So, this is the volume fraction of the fiber which is V_f . Similarly, this is the volume fraction of the matrix, which is written as V_m .

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Longitudinal Strength and Stiffness of Composite

$E_{cl} = E_f \times v_f + E_m \times v_m$

(25)

$\sigma_{cl} = \sigma_f \times v_f + \sigma_m \times v_m$

(26)

Equations (25) and (26) are known as the **Rule of Mixtures**

The fraction of the load carried by fibers in a unidirectional lamina is


$\frac{P_f}{P_c} = \frac{\sigma_f \times v_f}{\sigma_f \times v_f + \sigma_m(1 - v_f)} = \frac{E_f v_f}{E_f v_f + E_m(1 - v_f)}$


(27)

Equation (25) can also be written as,

$E_{cl} = (E_f - E_m)v_f + E_m$

(28)




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So, $E_{cl} = E_f \times v_f + E_m \times v_m$. This equation can also be written as $\sigma_{cl} = (\sigma_f \times A_f / A_{cl} + \sigma_m \times A_m / A_{cl}) = (\sigma_f \times v_f + \sigma_m \times v_m)$.

That is, (the elastic modulus of fiber \times the volume fraction of fiber) + (the elastic modulus of the matrix \times the volume fraction of the matrix). Similarly, we have derived σ_{cl} that is the stress in the composite is equal to the stress in the fiber ($\sigma_f \times$ the volume fraction) + (the stress in the matrix \times the volume fraction).

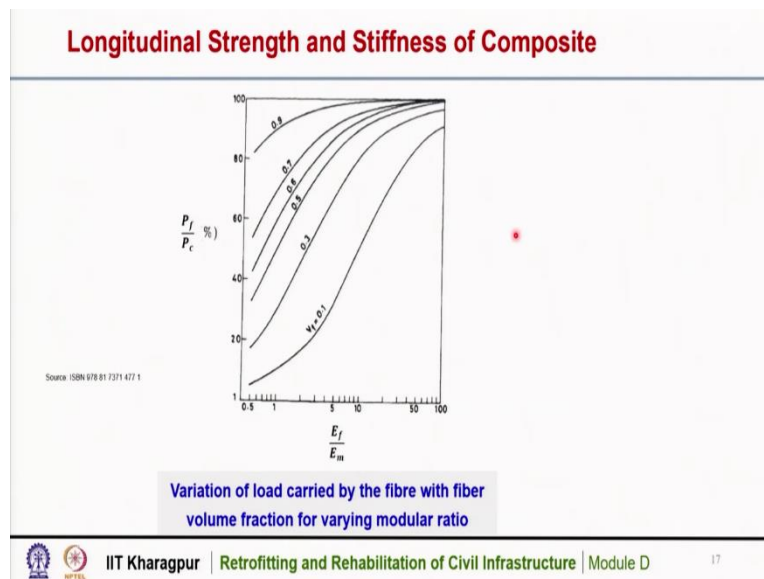
This is known as rule of mixture. So, if we know the volume fraction of the individual components, that is V_f or V_m and the properties like say E_f or E_m or here it is σ_f or σ_m , we can find out the properties of the composite and this is the rule of mixture. So, to determine the elastic modulus of composite the rule of mixture can be applied.

And by knowing the volume fraction of the fiber and the volume fraction of the matrix and their elastic modulus, we can find out the elastic modulus of the composite in the direction of the fiber. Similarly, the stress in the composite can also be obtained from the rule of mixture. Now, the fraction of the load carried by fibers in the unidirectional lamina can be determined as P_f/P_c . P_f is the load taken by the fiber and P_c is the load taken by the entire composite.

So, this way we can write it this is equal to $P_f/P_c = (\sigma_f \times v_f) / [(\sigma_f \times v_f) + \{\sigma_m(1 - v_f)\}] = (E_f \times v_f) / [(E_f \times v_f) + \{E_m(1 - v_f)\}]$. So, by knowing the volume fraction of the fiber and the matrix and their elastic modulus, we can find out the fraction of the load carried by the fiber.

This equation can also be written as $E_{cl} = (E_f - E_m) v_f + E_m$

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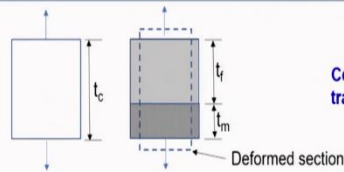
This is a graph that shows the variation of the load carried by the fiber with different fiber volume fraction for varying modular ratio. So, this graph shows that when the E_f / E_m is very same, different same 1 to 100 or so, and these are the different fiber volume fraction. So, what is the load shared by the fiber?

So, we can see that when the E_f / E_m is more than 10. As you can see, this is plotted in a logarithmic scale so when it is plotted, and when the elastic modulus of fiber is significantly high as compared to the elastic modulus of matrix and with a significant high fiber volume fraction the fiber carries most of the load of the composite.

So, we can see here when this is 10, when E_f / E_m is approximately 10 or so, and the fiber volume fraction is a point 6 or point 7, then, about 80 percent of the load is shared by the fiber itself in the composite. So, it is the property of the fiber that dominates predominantly in the composite. So, the load is shared by the fiber and the matrix. And depending on the fiber volume fraction and their elastic modulus, the load carried by the fiber can be estimated. And significant amount of load is carried by the fiber.

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Transverse Modulus of Composite



Composite under transverse stress

Assumption

- No separation between fiber and matrix
- Each layer has the same area on which the load acts. This means each layer experiences the same load and the same stress

$$\sigma_{ct} = \sigma_f = \sigma_m \quad (29)$$

$$\text{Total transverse displacement is, } \Delta t_c = \Delta t_f + \Delta t_m \quad (30)$$

$$\text{Also, } BLt_f = V_f ; \quad BLt_m = V_m ; \quad BLt_c = V_c \quad (31)$$



Now, we will determine the transverse modulus of composite. To determine the transverse modulus composite here also we will have some assumptions. The first assumption is that there is no separation between the fiber and the matrix, and the fiber composite is subjected to a transverse loading. As we can see here, this is also an idealized schematic diagram of the composite, the two phases the fiber phase and the matrix phase are denoted here and this composite is under transverse loading.

So, this is the transverse load applied on the composite. The thickness of the composite is t_c , whereas, it is idealized as two different phases and the thickness of the fiber phase is t_f and the thickness of the matrix phase is t_m . Because of this transverse loading there is deformation in the composite in the transverse direction and this is the deformed shape of the composite under the loading.

Now, in this transverse loading case each layer has the same area on which the load acts, so this means that each layer experiences the same load and the same stress. So, here in case of transverse loading the composite, the fiber and the matrix experiences the same stress. So, $\sigma_{ct} = \sigma_f = \sigma_m$. σ_{ct} is the stress in the composite in the transverse direction and that stress is the same in the fiber and also in the matrix.

Now, the total transverse displacement is Δt_c and that is equal to the transverse displacement of the individual faces that is the fiber and the matrix. So, $\Delta t_c = \Delta t_f + \Delta t_m$ where Δt_f is the displacement of the fiber phase and Δt_m is the displacement of the matrix phase.

Now, the volume of fiber that is V_f can be written as BLt_f , where t_f is the thickness of the fiber part, L is the length of the composite and B is the width. Similarly, the volume of matrix can be written as V_m is equal to BLt_m and the volume of composite V_c can be written as BLt_c .

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Transverse Modulus of Composite


Total transverse displacement is, $\Delta t_c = \Delta t_f + \Delta t_m$ (32)

Now $\epsilon_{ct} = \frac{\Delta t_c}{t_c} = \frac{\Delta t_f}{t_c} + \frac{\Delta t_m}{t_c}$ (33)

Or, $\epsilon_{ct} = \frac{\Delta t_f \times t_f}{t_f \times t_c} + \frac{\Delta t_m \times t_m}{t_m \times t_c}$ (34)

Or, $\epsilon_{ct} = \frac{\epsilon_f \times (t_f \cdot L \cdot B)}{(t_c \cdot L \cdot B)} + \frac{\epsilon_m \times (t_m \cdot L \cdot B)}{(t_c \cdot L \cdot B)}$ (35)

So, $\epsilon_{ct} = \epsilon_f \times v_f + \epsilon_m \times v_m$ (36)



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Transverse Modulus of Composite


Replacing strain terms by their corresponding modulus and stresses, yields

$$\frac{\sigma_{ct}}{E_{ct}} = \frac{\sigma_f}{E_f} v_f + \frac{\sigma_m}{E_m} v_m \quad (37)$$

From equation (37), the relation becomes

$$\frac{1}{E_{ct}} = \frac{v_f}{E_f} + \frac{v_m}{E_m} \quad (38)$$

Or, $\frac{1}{E_{ct}} = \frac{v_f}{E_f} + \frac{1 - v_f}{E_m}$ (39)



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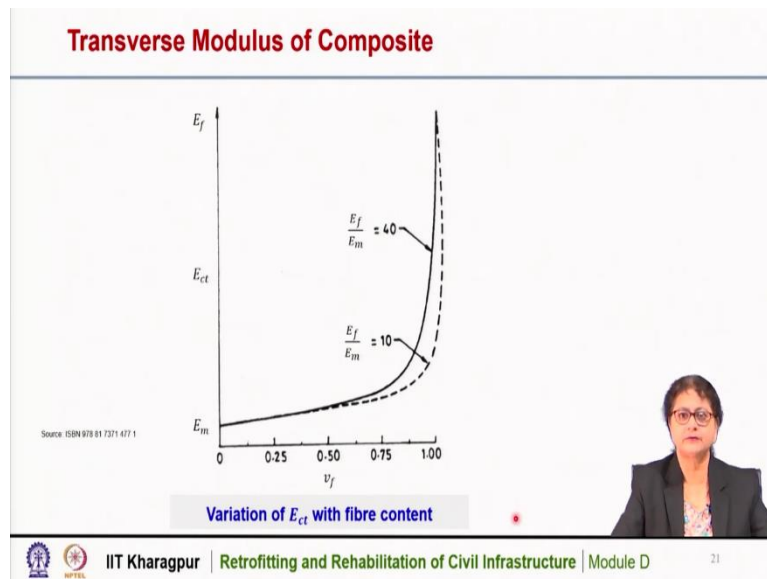
Now, the total transverse displacement is written as Δt_c is equal to $\Delta t_c = \Delta t_f + \Delta t_m$ as we have written earlier. Now, the strain can be written as this ϵ_{ct} is the strain in the composite in the transverse direction that is equal to $\Delta t_c/t_c$. So, this is the strain in the composite in the transverse direction, so $\Delta t_c = \Delta t_f + \Delta t_m$.

Now, $\epsilon_{ct} = \Delta t_c/t_c = \Delta t_f/t_c + \Delta t_m/t_c = (\Delta t_f \times t_f)/(t_f \times t_c) + (\Delta t_m \times t_m)/(t_m \times t_c) = [\epsilon_f \times (t_f \cdot L \cdot B)]/(t_c \cdot L \cdot B) + [\epsilon_m \times (t_m \cdot L \cdot B)]/(t_c \cdot L \cdot B) = \epsilon_f \times v_f + \epsilon_m \times v_m$

So, from the equation 37, the relation becomes $1/E_{ct} = v_f/E_f + v_m/E_m$. So, this is the equation that can be used to determine the elastic modulus of the composite in the transverse direction. So, it is derived from the elastic modulus of the fiber phase and the matrix phase and by knowing their volume fractions.

So, this equation 38 can also be written as $1/E_{ct} = v_f/E_f + (1-v_f)/E_m$. So, by knowing the volume fraction of fiber and the matrix and their elastic modulus we can determine and derive the elastic modulus of the composite in the transverse direction.

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This graph shows the variation of the elastic modulus of the composite in the transverse direction with fiber content. The fiber content are plotted in the x axis and this is the E_{ct} for different E_f / E_m values, so we can see here that at lower values of the volume fraction of fiber, there is not significant increase in the transverse modulus of the composite.

With higher V_f values, higher is the E_{ct} value of the composite. So, the volume fraction of fiber influences significantly the transverse modulus of the composite as well.

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In-Plane Shear Modulus


The composite element is subjected to uniform shearing and complementary shearing stresses along the boundaries

Composite under in-plane shear stress

Shearing stresses in the fibre and matrix are equal. Thus,

$$\tau_c = \tau_f = \tau_m \quad (40)$$



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Now we will derive the in-plane shear modulus of the composite. The composite element is subjected to uniform shearing and complementary shearing stresses along the boundaries. So, here this is the composite, similarly, the two phases are shown separately or idealization so this is the fiber part and this is the matrix part, and it is subjected to shear force. You can see here shear stresses are shown.

Now with this shear stresses there is deformation. This is the overall deformation, that overall deformation of the composite is denoted as Δ_c and the deformation of the fiber phase is denoted as Δ_f whereas the formation of the matrix phase is denoted as Δ_m , so this is the shear deformation. The shearing stress in the fiber and the matrix are equal.

So, in this case since it is in-plane shear stress, the shear stresses in the fiber and the matrix phase are equal, thus $\tau_c = \tau_f = \tau_m$ which is equal to the stress in the composite. So, τ denotes for the shearing stress.


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In-Plane Shear Modulus

Total shear deformation of composite,

$$\Delta_c = \Delta_f + \Delta_m \quad (41)$$
$$\Delta_c = \gamma_c t_c \quad (42)$$
$$\Delta_f = \gamma_f t_f \quad (43)$$
$$\Delta_m = \gamma_m t_m \quad (44)$$

Therefore,

$$\gamma_c t_c = \gamma_f t_f + \gamma_m t_m \quad (45)$$


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The total shear deformation of the composite can be written as Δ_c and that has shown also in the figure that is $\Delta_c = \Delta_f + \Delta_m$. That is the deformation of the fiber part and the deformation of the matrix part. The deformation of the composite can also be written as the shearing strain into the t_c , that is the you can see here this is the total shear deformation of the composite is Δ_c and it is equal to $\Delta_m + \Delta_f$.

So, $\Delta_c = \gamma_c t_c$, $\Delta_f = \gamma_f t_f$, $\Delta_m = \gamma_m t_m$

Therefore, $\gamma_c t_c = \gamma_f t_f + \gamma_m t_m$

Now, dividing both sides by t_c we get $\gamma_c = \gamma_f t_f / t_c + \gamma_m t_m / t_c$. And again, we are multiplying the numerator and the denominator with B into L that is width and the length we are getting, this equation 47.

Now $t_f \times B \times L$ is nothing but the volume of the fiber and this is the volume of the composite. Similarly, this is the volume of the matrix. So, this ratio gives the volume fraction. So, this is nothing but the volume fraction of the fiber and this is the volume fraction of the composite. So, the shear strain of the composite can be written as that is $\gamma_c = \gamma_f v_f + \gamma_m v_m$.

So, this is the shear strain in the composite by knowing the shear strain of the fiber and the matrix and their volume fractions we can determine the shear strain in the composite. So, this

can also be written in terms of the shear modulus and the shear stress that is τ_c / G_c similarly for the fiber part and the for the matrix part.

And since the shearing stresses are same, we can eliminate it so we get equation 50 which is $1/G_c$ that is the shear modulus of the composite $1/G_c = v_f/G_f + v_m/G_m$. So, we can determine the shear modulus of the composite by knowing the shear modulus of the fiber part and the matrix part and their volume fractions,

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In-Plane Shear Modulus

$$\gamma_c t_c = \gamma_f t_f + \gamma_m t_m$$

Or, $\gamma_c = \gamma_f \frac{t_f}{t_c} + \gamma_m \frac{t_m}{t_c}$ (46)


Or, $\gamma_c = \gamma_f \frac{t_f BL}{t_c BL} + \gamma_m \frac{t_m BL}{t_c BL}$ (47)



Or, Shear strain of the composite

$$\gamma_c = \gamma_f v_f + \gamma_m v_m$$
 (48)

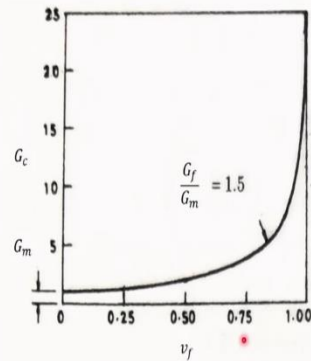
Or, $\frac{\tau_c}{G_c} = \frac{\tau_f}{G_f} v_f + \frac{\tau_m}{G_m} v_m$ (49)

So, $\frac{1}{G_c} = \frac{v_f}{G_f} + \frac{v_m}{G_m}$ (50)








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In-Plane Shear Modulus



Variation of G_c with fibre content





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This graph shows the variation of G_c with fiber content here in the x axis V_f is plotted and this is the G_c . Now, with increase in fiber volume fraction the G_c also increases. So, initially

the value is quite low nearly the G_m value but with increase in the fiber content the shear modulus of the composite also increases significantly.

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Poisson's Ratio

For in-plane loading of a composite,

- ν_{cl} = major Poisson's ratio, which relates longitudinal strain and transverse strain
- ν_{ct} = minor Poisson's ratio, which relates transverse strain and longitudinal strain

Model for prediction of Poisson's ratio

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Now, we will discuss the Poisson's Ratio of the composite. For in-plane loading of a composite we have two Poisson's ratio for the composite. One, is the major Poisson's ratio which is denoted as ν_{cl} , which relates the longitudinal strain and the transverse strain. The other one is the minor Poisson's Ratio, which is denoted as ν_{ct} , which relates the transverse strain and the longitudinal strain.

So, here is also a composite, which is idealized as the two-phased system. This is the fiber phase and this is the matrix phase, and they are separately shown here for idealization, and it is subjected to some loading. And because of that loading there is deformation, and this is the deformed shape of the composite. Now, due to this deformation the fiber phase undergoes a deformation, as you can see, this is denoted as Δ_f and the matrix part also undergoes a deformation that is Δ_m .

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Poisson's Ratio

So from the definition, major Poisson's ratio

$$\nu_{cl} = -\frac{\varepsilon_t}{\varepsilon_l} \quad (51)$$

Transverse strain in fiber and in matrix are

$$\varepsilon_{tf} = -\nu_f \varepsilon_l \quad \text{and} \quad \varepsilon_{tm} = -\nu_m \varepsilon_l \quad (52)$$


Also,


$$\varepsilon_{tf} = \frac{\delta_f}{t_f} ; \quad \varepsilon_{tm} = \frac{\delta_m}{t_m} ; \quad \varepsilon_t = \frac{\delta_c}{t_c} \quad (53)$$

Now,

$$\delta_c = \delta_f + \delta_m \quad (54)$$

Or,

$$\varepsilon_t \times t_c = \varepsilon_{tf} \times t_f + \varepsilon_{tm} \times t_m \quad (55)$$



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So, from the definition the major Poisson's Ratio can be written as the lateral strain by the longitudinal strain. So, here for the composite the major Poisson's Ratio that is the $\nu_{cl} = -\varepsilon_t/\varepsilon_l$.

The transverse strain in the fiber and in the matrix can be written as $\varepsilon_{tf} = -\nu_f \varepsilon_l$. Similarly, $\varepsilon_{tm} = -\nu_m \varepsilon_l$. So, these are the, this is the Poisson's Ratio of the matrix and this is the Poisson's Ratio of the fiber part. Also, we can write that the strain in the fiber that is $\varepsilon_{tf} = \delta_f/t_f$ that is the deformation in the fiber by the total thickness.

Similarly, $\varepsilon_{tm} = \delta_m/t_m$ and $\varepsilon_t = \delta_c/t_c$, which is for the composite. Now, the total deformation of the composite δ_c is the summation of the deformation of the fiber part and the matrix part. So, $\delta_c = \delta_f + \delta_m$. Now, δ_c can be written in terms of the strain values, so δ_c can be written as $\varepsilon_t \times t_c$ which is equal to $\varepsilon_{tf} \times t_f + \varepsilon_{tm} \times t_m$.

Now, we can substitute with these equations with Poisson's Ratio what we get $-t_c \nu_{cl} \varepsilon_l = -t_f \nu_f \varepsilon_l - t_m \nu_m \varepsilon_l$. So, we get $\nu_{cl} = \nu_f t_f / t_c + \nu_m t_m / t_c$.

Now, this t_f/t_c is nothing but the volume fraction. We can multiply B into L in numerator and denominator and this gives the volume fraction of the fiber. And this gives the volume fraction of the matrix part. So, we can write ν_{cl} , which is the major Poisson's Ratio is equal to $\nu_f \nu_f + \nu_m \nu_m$.

So, it follows again the rule of mixture. So, by knowing the volume fraction of the fiber and the matrix and their individual Poisson's Ratio, we can find out the Poisson's Ratio of the composite. So, this is also as per the rule of mixture.

Now, the minor portions ratio can be obtained similarly, as and we can get this relationship $\nu_{cl}/E_{cl} = \nu_{ct}/E_{ct}$. By knowing the major Poisson's Ratio and the elastic modulus of the composite in the longitudinal direction and the transverse direction we can find out the minor Poisson's Ratio of the composite using equation 59.

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Poisson's Ratio

Substituting,


$$-t_c \nu_{cl} \varepsilon_l = -t_f \nu_f \varepsilon_l - t_m \nu_m \varepsilon_l \quad (56)$$



Or,
$$\nu_{cl} = \nu_f \frac{t_f}{t_c} + \nu_m \frac{t_m}{t_c} \quad (57)$$

Or,
$$\nu_{cl} = \nu_f \nu_f + \nu_m \nu_m \quad (58)$$

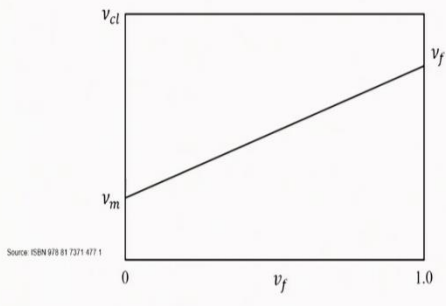
The major Poisson's ratio follows the **Rule of Mixtures**

Minor Poisson's ratio (ν_{ct})


$$\frac{\nu_{cl}}{E_{cl}} = \frac{\nu_{ct}}{E_{ct}} \quad (59)$$






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Poisson's Ratio



Variation of ν_{cl} with fibre content

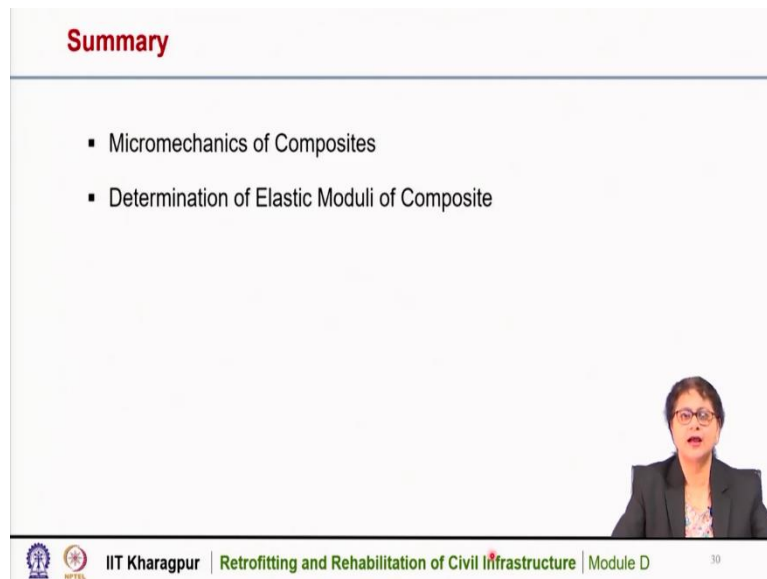




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This is the variation of the Poisson's Ratio the major Poisson's Ratio of the composite with fiber content. Here this is the fiber content is plotted in this axis and this is the Poisson's

Ratio of the composite. Now, with increase in the volume fraction of the composite, the Poisson's Ratio the major Poisson's Ratio also increases significantly. So, this variation is linear, as we increase the volume fraction of the composite. The Poisson's Ratio of the composite in the direction of the fiber also increases.

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Summary

- Micromechanics of Composites
- Determination of Elastic Moduli of Composite

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So, this shows that the influence of fiber on the composite the properties of composite can be derived from the volume fraction of the fiber part and the matrix part, and also by knowing their individual properties. By knowing their elastic modulus values, by knowing their poisson's ratio, we can determine the elastic modulus of the composite in the direction of the fiber or in the transverse direction of the fiber.

The fiber volume fraction plays a significant role in determining the properties of the composite. If the fiber volume fraction is high, that means, the amount of fiber is high in a composite, the property of the composite also is better. So, the superior property of the composite is mainly due to the amount of fiber present in it, and also on, it depends on the elastic modulus of the fibers and the matrix.

So, in today's lecture, we have discussed the micromechanics of composites, and how to determine the elastic modulus of composite. We have discussed the derivation of the elastic modulus of the composite along the direction of the fiber and perpendicular to the direction of the fiber how to determine the Poisson's Ratio and shear modulus of the composite by knowing the individual properties of the fiber and the matrix, and their volume fractions. So, that is all for today's lecture. Thank you.