Surface Water Hydrology
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Lecture – 10
Analysis of Precipitation: IDF and PMP

In this lecture 10 discussing two very important concepts. One is the intensity duration and frequency curve and the other one is the concept of probable maximum precipitation.

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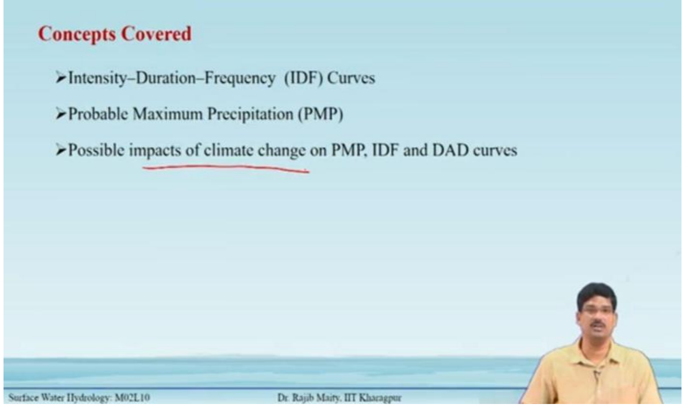

Concepts Covered

- Intensity–Duration–Frequency (IDF) Curves
- Probable Maximum Precipitation (PMP)
- Possible impacts of climate change on PMP, IDF and DAD curves

IDF is the Intensity Duration Frequency curve and Probable Maximum Precipitation is PMP. Apart from these two concepts that we covered, finally another very important in the current age is the impact of climate change on these curves..

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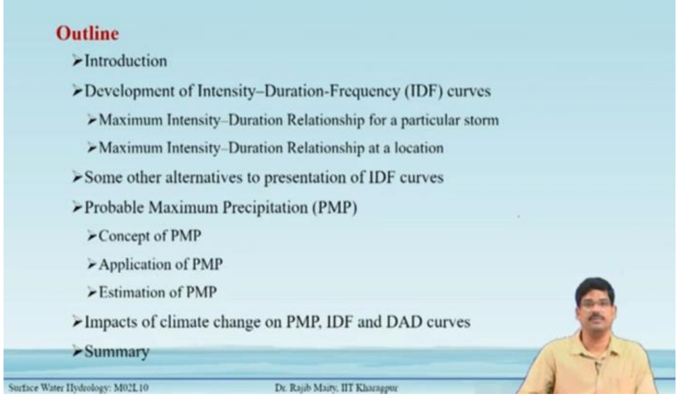

Outline

- Introduction
- Development of Intensity–Duration–Frequency (IDF) curves
  - Maximum Intensity–Duration Relationship for a particular storm
  - Maximum Intensity–Duration Relationship at a location
- Some other alternatives to presentation of IDF curves
- Probable Maximum Precipitation (PMP)
  - Concept of PMP
  - Application of PMP
  - Estimation of PMP
- Impacts of climate change on PMP, IDF and DAD curves
- Summary

The outline goes like this, first some general introduction, then the development of IDF curves, and within that the maximum intensity and duration relationship for a particular storm, and then the same for a particular location. Then, comes that some alternative presentation of the IDF curve. Next, we take up the probable maximum precipitation, though we should see its concept its application, it is the estimation of PMP for a particular event or particular region. And finally, eimpact of climate change on PMP, IDF, and DAD curves and the summary.

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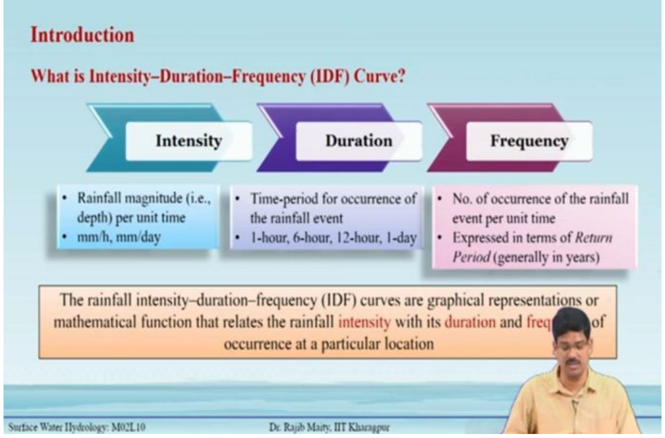

Introduction

What is Intensity–Duration–Frequency (IDF) Curve?

Intensity
- Rainfall magnitude (i.e., depth) per unit time
- mm/h, mm/day

Duration
- Time-period for occurrence of the rainfall event
- 1-hour, 6-hour, 12-hour, 1-day

Frequency
- No. of occurrence of the rainfall event per unit time
- Expressed in terms of *Return Period* (generally in years)

The rainfall intensity–duration–frequency (IDF) curves are graphical representations or mathematical function that relates the rainfall intensity with its duration and freq of occurrence at a particular location

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Introduction

What is this intensity duration and frequency curve?

There are three characteristics are there for a storm event, the first one is the intensity. The intensity is the rainfall magnitude that is a depth per unit time. So, the rainfall we measure with the depth per unit time. The unit time can be an hour or 3 hours or a day. So, this intensity is the one characteristic, the second characteristic is the duration. So, how long does the storm event or the rainfall event occur? 1 hour, 6 hours, 12 hours, or sometimes one day, those are the durations.And the third one is the frequency means, how frequent is that particular event is. So, a number of occurrences of that rainfall event per unit time. But here we express it in terms of the return period generally in the years.. In brief, the return period is the average on an average in a statistical sense over a long period, the recurrence interval of a particular event. So, coming to the IDF, The rainfall intensity–duration–frequency (IDF) curves are graphical representations or mathematical function that relates the rainfall intensity with its duration and frequency of occurrence at a particular location.

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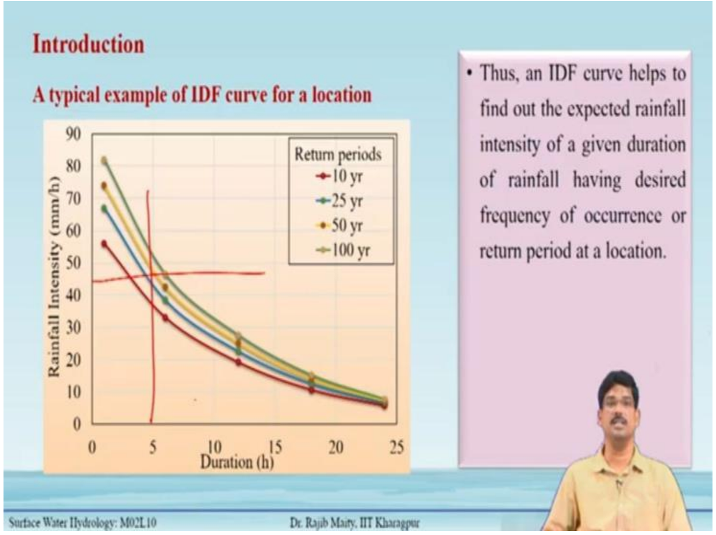

Introduction

A typical example of IDF curve for a location

- Thus, an IDF curve helps to find out the expected rainfall intensity of a given duration of rainfall having desired frequency of occurrence or return period at a location.

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A typical example of an IDF curve at a location show in fig.1. If there are three attributes are there in a particular plot, then we generally keep one variable constant for each line. So, in this case, the different colors are indicating different return periods, starting from 10 years, 25 years, 50 years, 100 years like that. And in the x-axis, the duration in an hour, and the y-axis rainfall intensity in millimeter per hour.

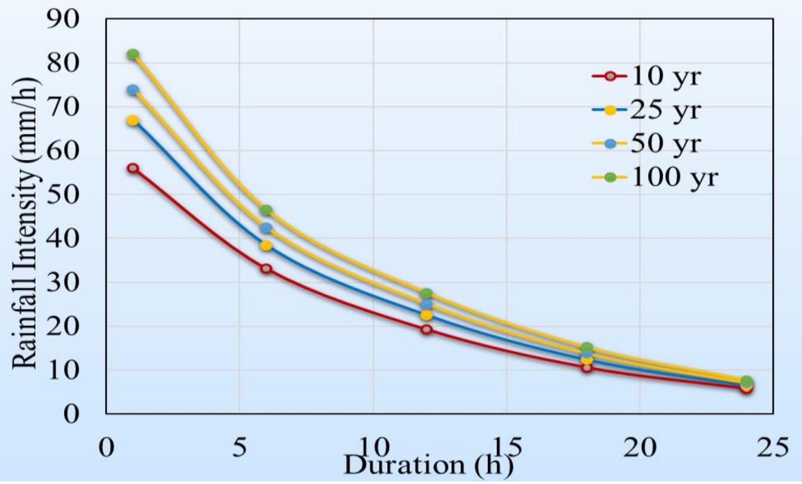

Fig.1 shows the typical example of IDF curve

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Thus an IDF curve help to find out the expected rainfall intensity for a given duration of rainfall having the desired frequency of the occurrence of the return period at a particular location. So, this information of intensity is then extensively used for hydrological analysis and engineering applications.

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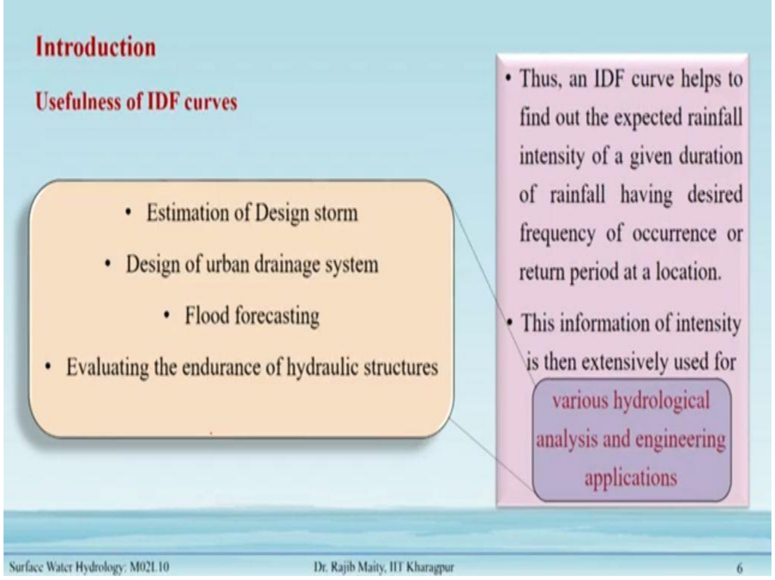

Introduction

Usefulness of IDF curves

- Estimation of Design storm
- Design of urban drainage system
- Flood forecasting
- Evaluating the endurance of hydraulic structures

- Thus, an IDF curve helps to find out the expected rainfall intensity of a given duration of rainfall having desired frequency of occurrence or return period at a location.
- This information of intensity is then extensively used for various hydrological analysis and engineering applications

Usefulness of IDF curve

- Estimation of Design storm
- Design of urban drainage system
- Flood forecasting
- Evaluating the endurance of hydraulic structures

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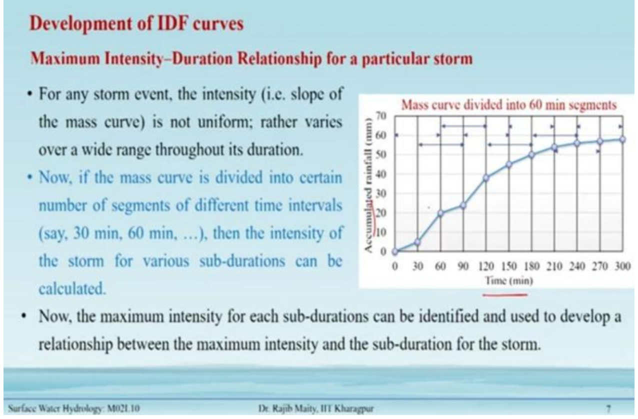

Development of IDF curves

Maximum Intensity–Duration Relationship for a particular storm

- For any storm event, the intensity (i.e. slope of the mass curve) is not uniform; rather varies over a wide range throughout its duration.
- Now, if the mass curve is divided into certain number of segments of different time intervals (say, 30 min, 60 min, …), then the intensity of the storm for various sub-durations can be calculated.
- Now, the maximum intensity for each sub-durations can be identified and used to develop a relationship between the maximum intensity and the sub-duration for the storm.

Development of IDF curves

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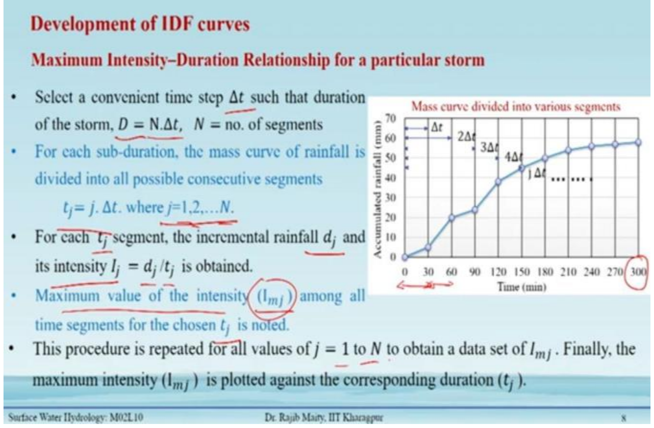

Development of IDF curves

Maximum Intensity–Duration Relationship for a particular storm

- Select a convenient time step Δt such that duration of the storm, $D = N.\Delta t$, N = no. of segments
- For each sub-duration, the mass curve of rainfall is divided into all possible consecutive segments

  $t_j = j.\Delta t$. where j=1,2,…N.
- For each t_j segment, the incremental rainfall d_j and its intensity $I_j = d_j / t_j$ is obtained.
- Maximum value of the intensity (I_{mj}) among all time segments for the chosen t_j is noted.
- This procedure is repeated for all values of j = 1 to N to obtain a data set of I_{mj}. Finally, the maximum intensity (I_{mj}) is plotted against the corresponding duration (t_j).

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Development of IDF curves

Maximum Intensity–Duration-frequency Relationship at a location

Using the same procedure of developing intensity-duration relationship for a particular storm, IDF curves can be developed at a particular location, provided the rainfall data is available for a long period of time (at least 20 years). Suppose, we have X years of rainfall data, then,

- At first, M numbers of heavy storms in a particular year are selected.
- Each of these storms are analysed for maximum intensity-duration relationship, as explained in earlier slide. This gives a set of maximum intensity, I_m as a function of duration for the considered year.
- The procedure is repeated for all X years of record to obtain the maximum intensity $I_m(t_j)_k$ for all segmented sub-durations, j = 1 to N and for all years k =1 to X.

Maximum Intensity–Duration-frequency Relationship at a location

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At first, M numbers of heavy storms in a particular year are selected.

Each of these storms is analyzed for a maximum intensity-duration relationship, as explained in the earlier slide. This gives a set of maximum intensity, I_m as a function of duration for the considered year.

The procedure is repeated for all X years of record to obtain the maximum intensity $I_m\ (t_j)_k$ for all segmented sub-durations, j= 1 to N and for all years k=1 to X.

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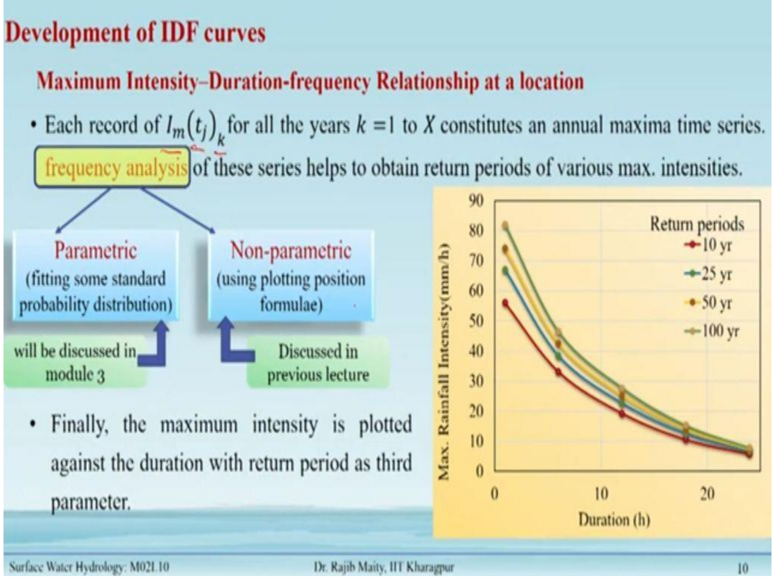

Development of IDF curves

Maximum Intensity–Duration-frequency Relationship at a location

- Each record of $I_m(t_j)_k$ for all the years $k = 1$ to X constitutes an annual maxima time series.
- frequency analysis of these series helps to obtain return periods of various max. intensities.

Parametric (fitting some standard probability distribution)

will be discussed in module 3

Non-parametric (using plotting position formulae)

Discussed in previous lecture

- Finally, the maximum intensity is plotted against the duration with return period as third parameter.

Now, for each of the records of $I_m(t_j)_k$ for all the years k=1 to X constitutes an annual maxima time series. The frequency analysis of this series will help us to obtain the return period of the various maximum intenses intensity. This frequency analysis now for those particular series are available can be done in two different ways. One is parametric, that we can fit some standard probability distributions.

And the second thing is the non-parametric that we use some plotting position formulae. Finally after this frequency analysis, this maximum intensity is plotted against the duration with the different return periods, as the third parameter to get get the IDF curve as it is shown in fig.1.

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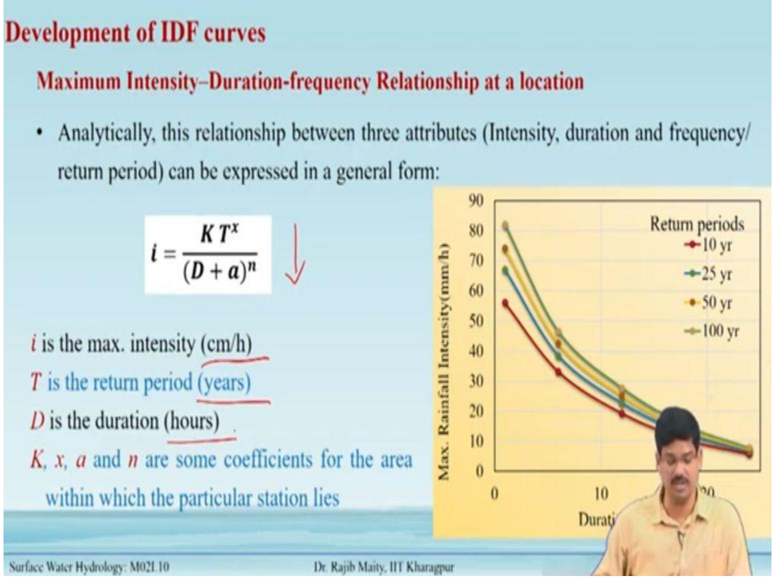

Development of IDF curves

Maximum Intensity–Duration-frequency Relationship at a location

- Analytically, this relationship between three attributes (Intensity, duration and frequency/ return period) can be expressed in a general form:

$$i = \frac{K\,T^x}{(D+a)^n}$$

i is the max. intensity (cm/h)

T is the return period (years)

D is the duration (hours)

K, x, a and n are some coefficients for the area within which the particular station lies

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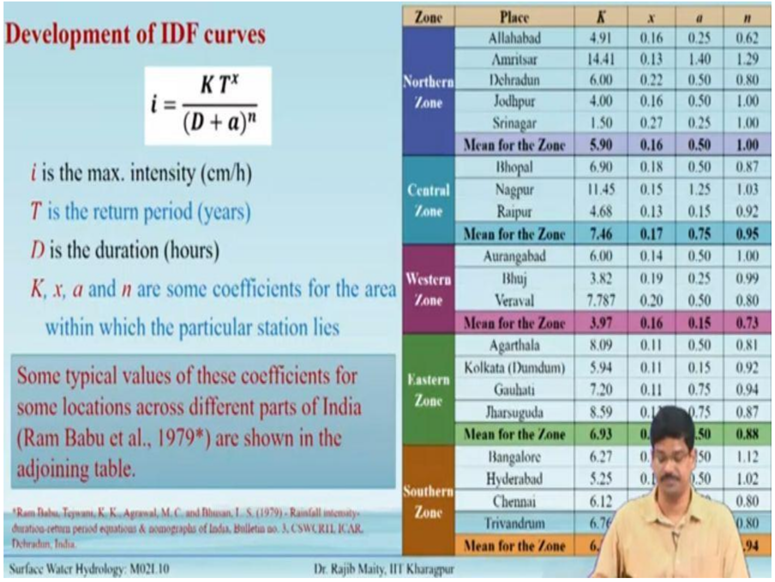

Development of IDF curves

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Some typical values of these coefficients for some locations across different parts of India (Ram Babu et al., 1979*) are shown in the adjoining table.

*Ram Babu, Tejwani, K. K., Agrawal, M. C. and Bhusan, L. S. (1979) - Rainfall intensity-duration-return period equations & nomographs of India, Bulletin no. 3, CSWCRTI, ICAR, Dehradun, India.

Zone	Place	K	x	a	n
Northern Zone	Allahabad	4.91	0.16	0.25	0.62
	Amritsar	14.41	0.13	1.40	1.29
	Dehradun	6.00	0.22	0.50	0.80
	Jodhpur	4.00	0.16	0.50	1.00
	Srinagar	1.50	0.27	0.25	1.00
	Mean for the Zone	**5.90**	**0.16**	**0.50**	**1.00**
Central Zone	Bhopal	6.90	0.18	0.50	0.87
	Nagpur	11.45	0.15	1.25	1.03
	Raipur	4.68	0.13	0.15	0.92
	Mean for the Zone	**7.46**	**0.17**	**0.75**	**0.95**
Western Zone	Aurangabad	6.00	0.14	0.50	1.00
	Bhuj	3.82	0.19	0.25	0.99
	Veraval	7.787	0.20	0.50	0.80
	Mean for the Zone	**3.97**	**0.16**	**0.15**	**0.73**
Eastern Zone	Agarthala	8.09	0.11	0.50	0.81
	Kolkata (Dumdum)	5.94	0.11	0.15	0.92
	Gauhati	7.20	0.11	0.75	0.94
	Jharsuguda	8.59	[illegible]	[illegible]	0.87
	Mean for the Zone	**6.93**	[illegible]	[illegible]	**0.88**
Southern Zone	Bangalore	6.27	[illegible]	[illegible]	1.12
	Hyderabad	5.25	[illegible]	[illegible]	1.02
	Chennai	6.12	[illegible]	[illegible]	0.80
	Trivandrum	[illegible]	[illegible]	[illegible]	[illegible]
	Mean for the Zone	[illegible]	[illegible]	[illegible]	[illegible]

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Some typical example for these different parts of India, from one region that one study that is shown in table 1. So, this K, x, a and n can vary for these different regions and all,

Table 1: shows the typical values of these coefficients for some locations across different parts of India (Ram Babu et al., 1979*)

Zone	Place	K	x	a	n
Northern Zone	Allahabad	4.91	0.16	0.25	0.62
	Amritsar	14.41	0.13	1.40	1.29
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	Chennai	6.12	0.16	0.50	0.80
	Trivandrum	6.76	0.15	0.50	0.80
	Mean for the Zone	**6.31**	**0.15**	**0.50**	**0.94**

*Ram Babu, Tejwani, K. K., Agrawal, M. C. and Bhusan, L. S. (1979) - Rainfall intensity-duration-return period equations & nomographs of India, Bulletin no. 3, CSWCRTI, ICAR, Dehradun, India.

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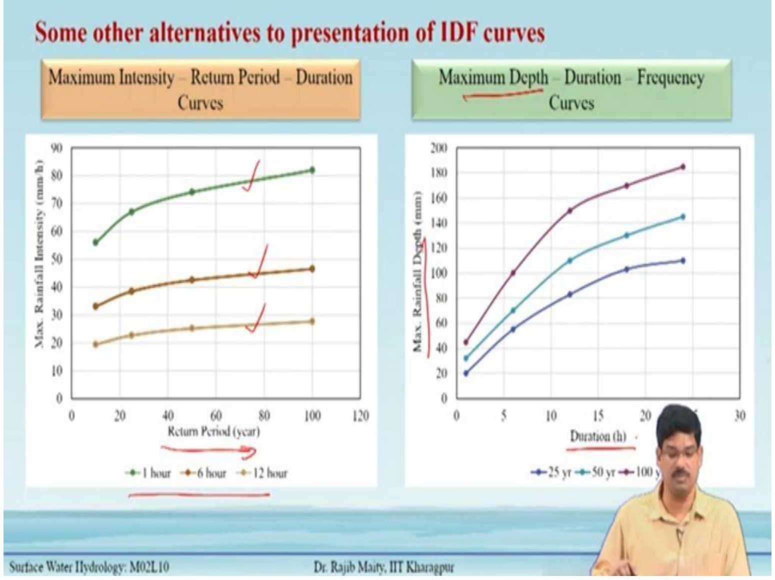

Some other alternatives to presentation of IDF curves

Maximum Intensity – Return Period – Duration Curves

Maximum Depth – Duration – Frequency Curves

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Some other alternatives to presentation of IDF curves

Maximum Intensity – Return Period – Duration Curves

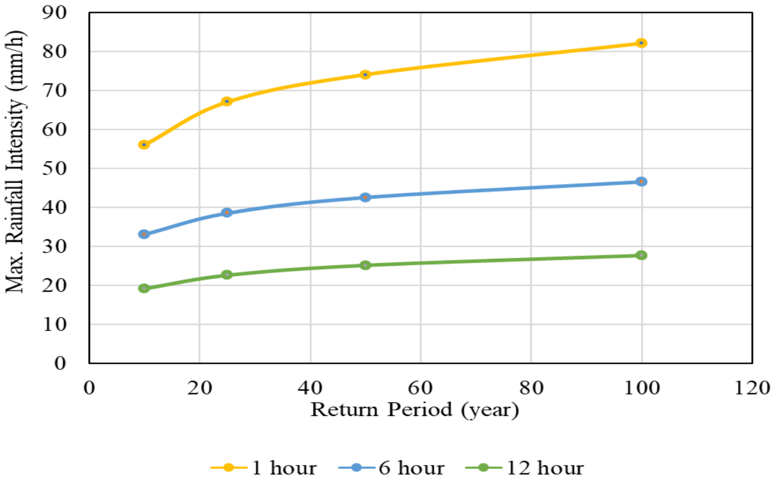

Fig.2 shows the Maximum Intensity – Return Period – Duration Curves

In fig.2 the return period showing in x-axis, and the durations are shown in different colors here. So, each different curve is this is for say 1-hour duration, this is 6 hours, and this is 12-hour duration curve. And the y-axis shows the maximum rainfall intensity

Maximum Depth – Duration – Frequency Curves

In the fig.3 the y-axis, is the maximum depth, the x-axis is duration. Again, here the frequencies are frequencies in terms of different return periods are the different lines that are shown in fig.3..

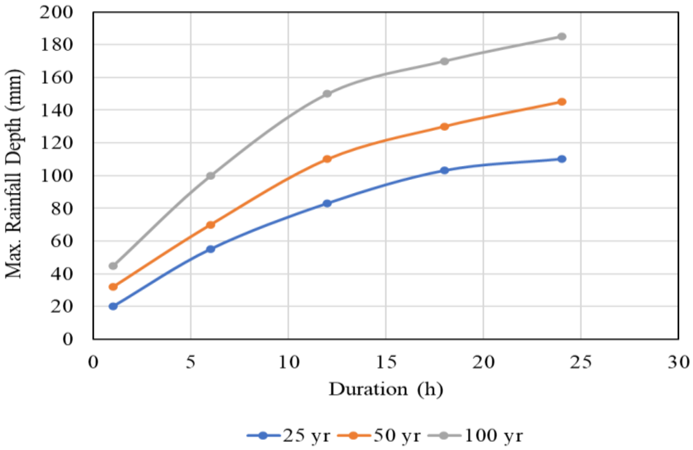

Fig.3 shows the Maximum Depth – Duration – Frequency Curves

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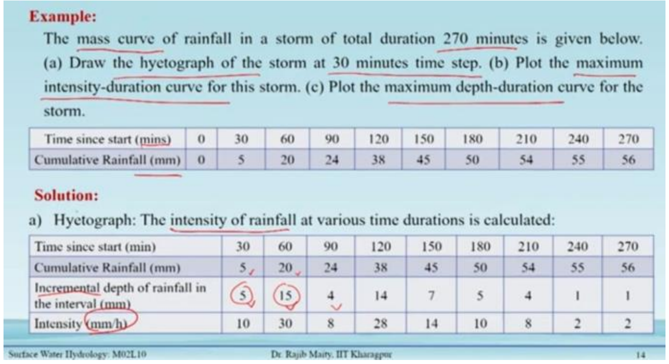


Example:

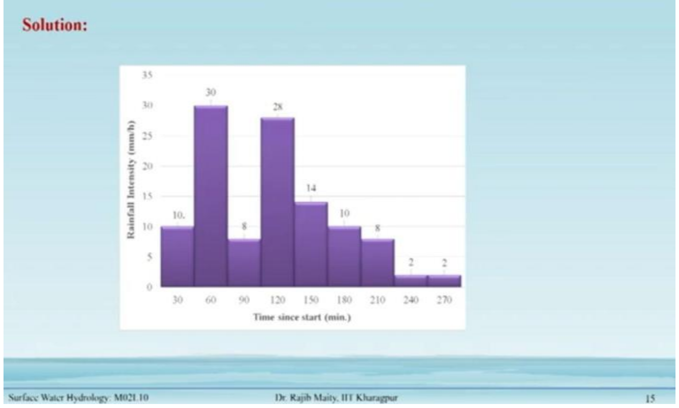
The mass curve of rainfall in a storm of total duration 270 minutes is given below. (a) Draw the hyetograph of the storm at 30 minutes time step. (b) Plot the maximum intensity-duration curve for this storm. (c) Plot the maximum depth-duration curve for the storm.

Time since start (mins)	0	30	60	90	120	150	180	210	240	270
Cumulative Rainfall (mm)	0	5	20	24	38	45	50	54	55	56

Solution:

a) Hyetograph: The intensity of rainfall at various time durations is calculated:

Time since start (min)	30	60	90	120	150	180	210	240	270
Cumulative Rainfall (mm)	5	20	24	38	45	50	54	55	56
Incremental depth of rainfall in the interval (mm)	5	15	4	14	7	5	4	1	1
Intensity (mm/h)	10	30	8	28	14	10	8	2	2



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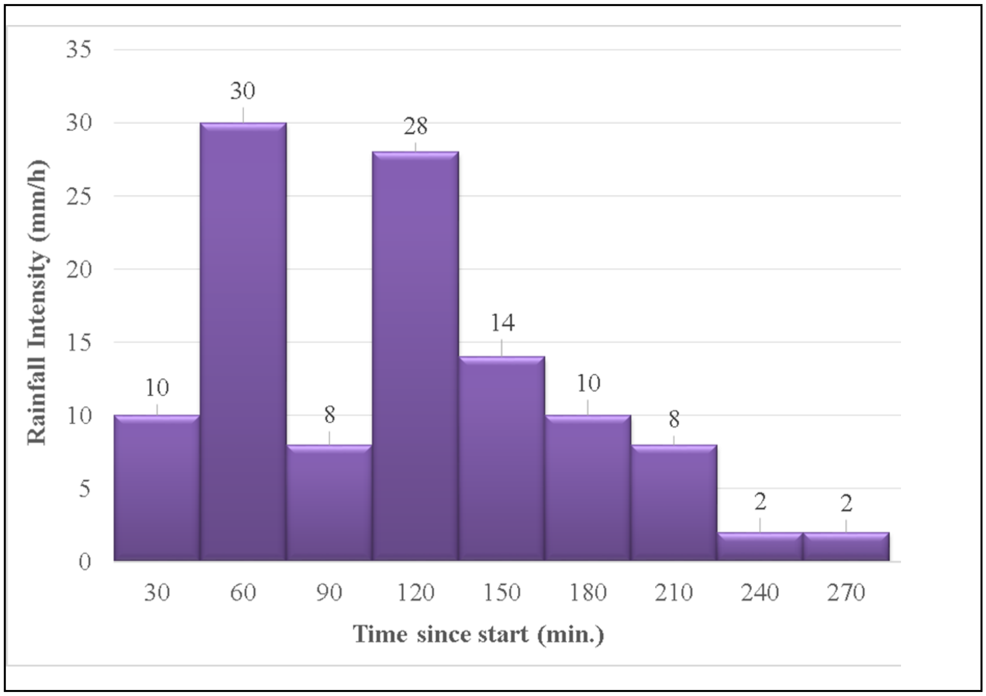

Fig.4 shows the Hyetograph of the Strom

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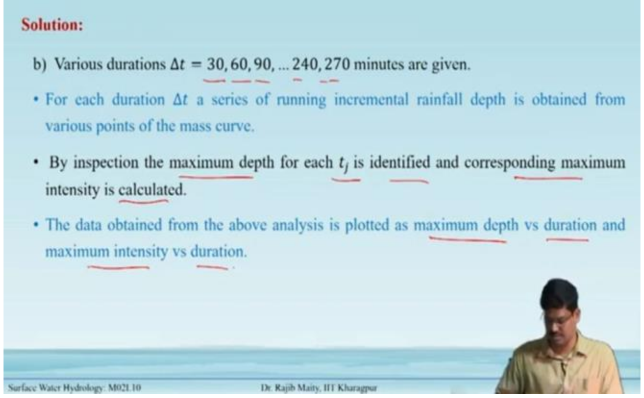

Solution:

b) Various durations $\Delta t = 30, 60, 90, \ldots 240, 270$ minutes are given.

- For each duration Δt a series of running incremental rainfall depth is obtained from various points of the mass curve.
- By inspection the maximum depth for each t_j is identified and corresponding maximum intensity is calculated.
- The data obtained from the above analysis is plotted as maximum depth vs duration and maximum intensity vs duration.

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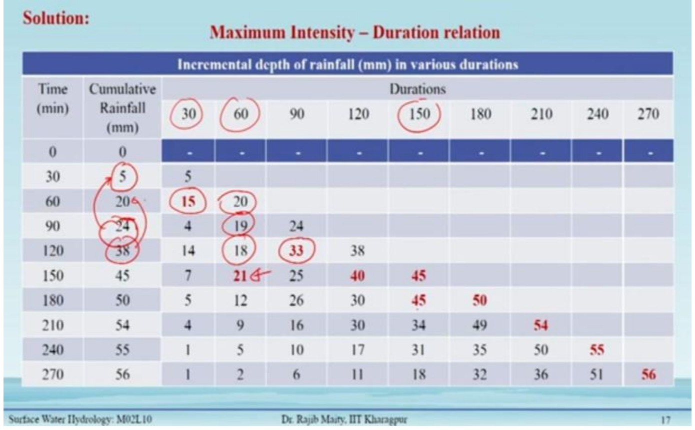

Solution:

Maximum Intensity – Duration relation

Incremental depth of rainfall (mm) in various durations

Time (min)	Cumulative Rainfall (mm)	Durations								
		30	60	90	120	150	180	210	240	270
0	0	-	-	-	-	-	-	-	-	-
30	5	5								
60	20	**15**	20							
90	24	4	19	24						
120	38	14	18	**33**	38					
150	45	7	**21**	25	**40**	**45**				
180	50	5	12	26	30	**45**	**50**			
210	54	4	9	16	30	34	49	**54**		
240	55	1	5	10	17	31	35	50	**55**	
270	56	1	2	6	11	18	32	36	51	**56**

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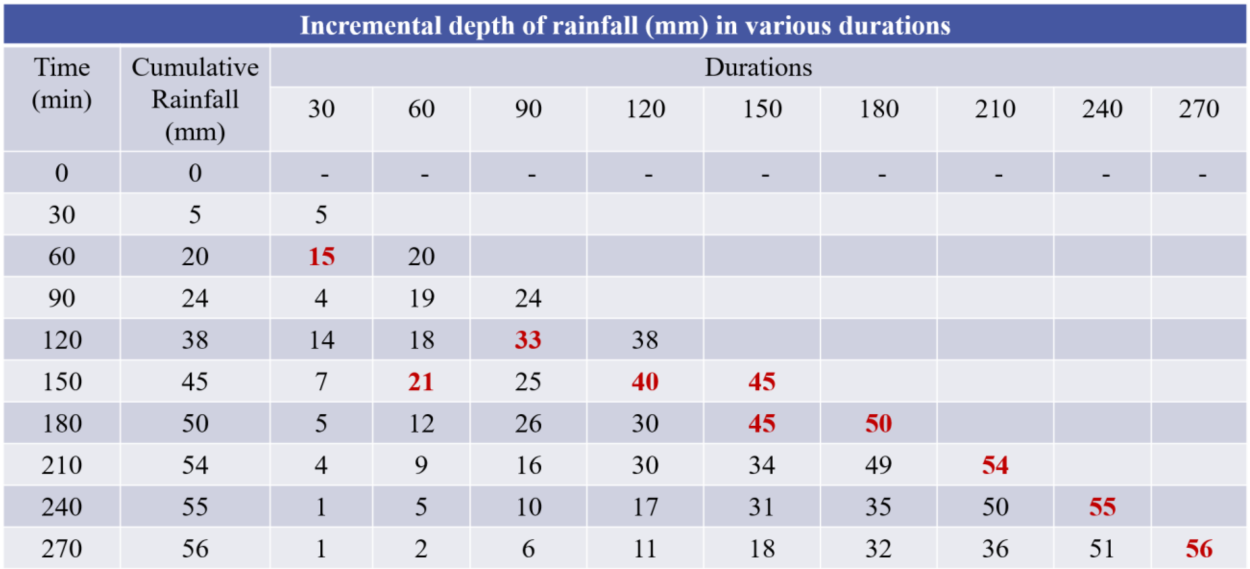
Solution:

Maximum Intensity – Duration relation



Incremental depth of rainfall (mm) in various durations										
Time (min)	Cumulative Rainfall (mm)	Durations								
		30	60	90	120	150	180	210	240	270
0	0	-	-	-	-	-	-	-	-	-
30	5	5								
60	20	**15**	20							
90	24	4	19	24						
120	38	14	18	**33**	38					
150	45	7	**21**	25	**40**	**45**				
180	50	5	12	26	30	**45**	**50**			
210	54	4	9	16	30	34	49	**54**		
240	55	1	5	10	17	31	35	50	**55**	
270	56	1	2	6	11	18	32	36	51	**56**

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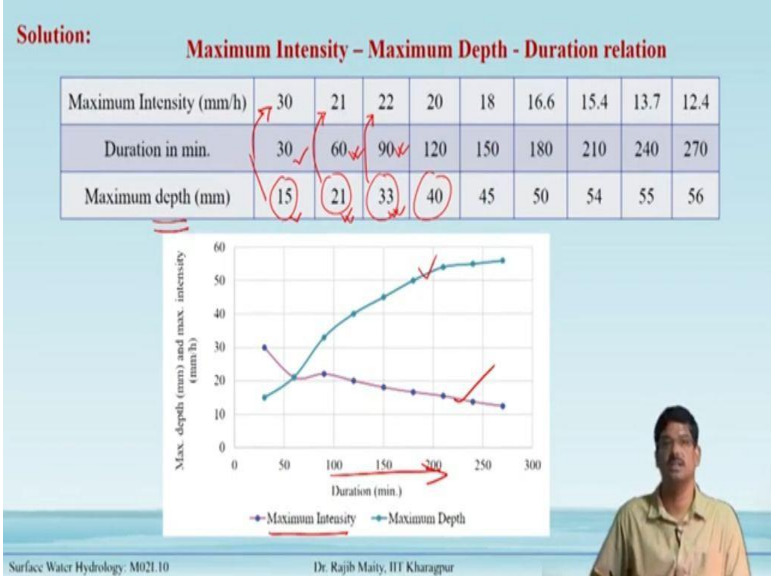

Solution:

Maximum Intensity – Maximum Depth - Duration relation

Maximum Intensity (mm/h)	30	21	22	20	18	16.6	15.4	13.7	12.4
Duration in min.	30	60	90	120	150	180	210	240	270
Maximum depth (mm)	15	21	33	40	45	50	54	55	56

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Maximum Intensity – Maximum Depth - Duration relation

Maximum Intensity (mm/h)	30	21	22	20	18	16.6	15.4	13.7	12.4
Duration in min.	30	60	90	120	150	180	210	240	270
Maximum depth (mm)	15	21	33	40	45	50	54	55	56

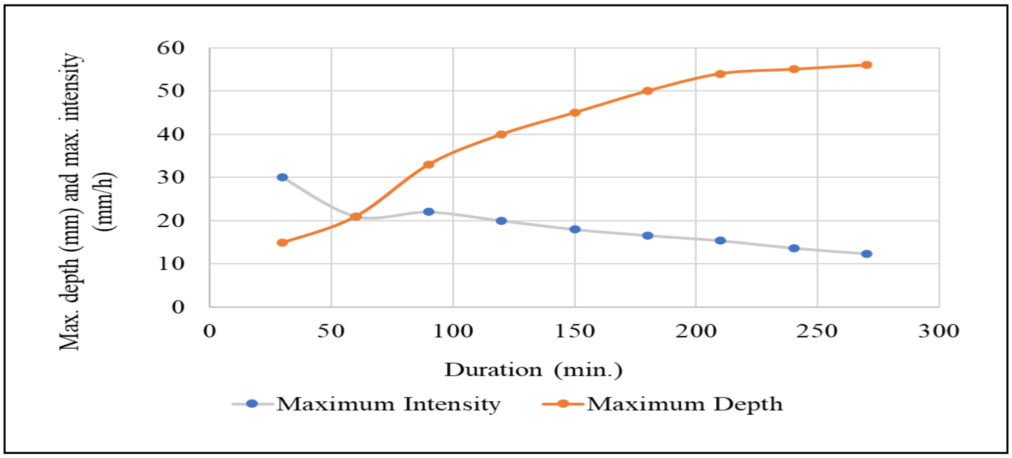

Fig.5: Maximum Intensity-Duration and maximum Depth-duration curves For the Storm

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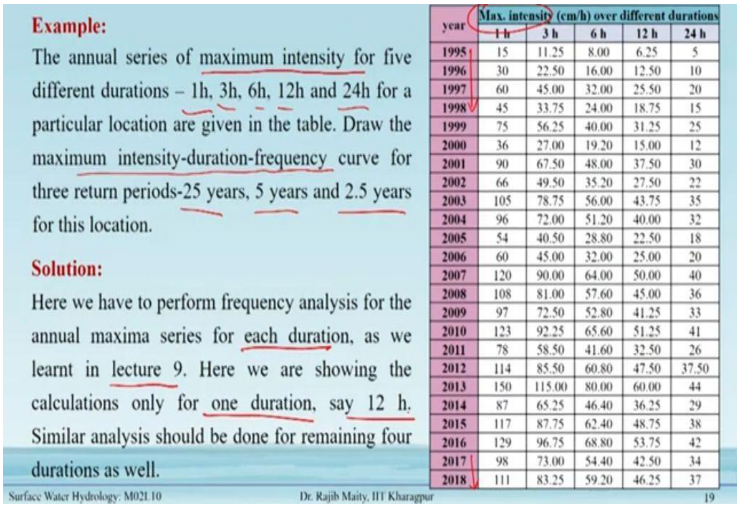

Example:

The annual series of maximum intensity for five different durations – 1h, 3h, 6h, 12h and 24h for a particular location are given in the table. Draw the maximum intensity-duration-frequency curve for three return periods-25 years, 5 years and 2.5 years for this location.

Solution:

Here we have to perform frequency analysis for the annual maxima series for each duration, as we learnt in lecture 9. Here we are showing the calculations only for one duration, say 12 h. Similar analysis should be done for remaining four durations as well.

year	Max. intensity (cm/h) over different durations				
	1 h	3 h	6 h	12 h	24 h
1995	15	11.25	8.00	6.25	5
1996	30	22.50	16.00	12.50	10
1997	60	45.00	32.00	25.50	20
1998	45	33.75	24.00	18.75	15
1999	75	56.25	40.00	31.25	25
2000	36	27.00	19.20	15.00	12
2001	90	67.50	48.00	37.50	30
2002	66	49.50	35.20	27.50	22
2003	105	78.75	56.00	43.75	35
2004	96	72.00	51.20	40.00	32
2005	54	40.50	28.80	22.50	18
2006	60	45.00	32.00	25.00	20
2007	120	90.00	64.00	50.00	40
2008	108	81.00	57.60	45.00	36
2009	97	72.50	52.80	41.25	33
2010	123	92.25	65.60	51.25	41
2011	78	58.50	41.60	32.50	26
2012	114	85.50	60.80	47.50	37.50
2013	150	115.00	80.00	60.00	44
2014	87	65.25	46.40	36.25	29
2015	117	87.75	62.40	48.75	38
2016	129	96.75	68.80	53.75	42
2017	98	73.00	54.40	42.50	34
2018	111	83.25	59.20	46.25	37

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Example:

The annual series of maximum intensity for five different durations – 1h, 3h, 6h, 12h and 24h for a particular location are given in the table. Draw the maximum intensity-duration-frequency curve for three return periods-25 years, 5 years and 2.5 years for this location.

Solution:

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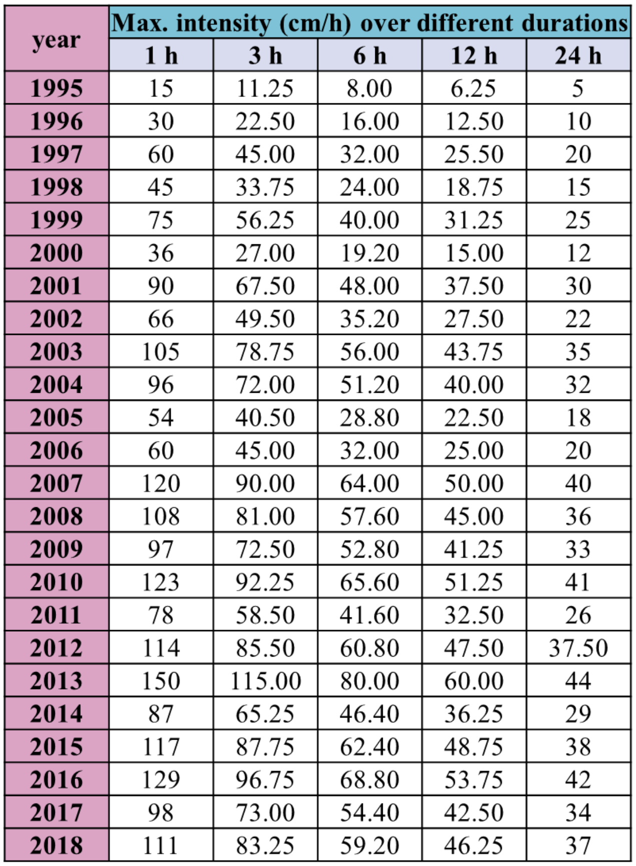

year	Max. intensity (cm/h) over different durations				
	1 h	3 h	6 h	12 h	24 h
1995	15	11.25	8.00	6.25	5
1996	30	22.50	16.00	12.50	10
1997	60	45.00	32.00	25.50	20
1998	45	33.75	24.00	18.75	15
1999	75	56.25	40.00	31.25	25
2000	36	27.00	19.20	15.00	12
2001	90	67.50	48.00	37.50	30
2002	66	49.50	35.20	27.50	22
2003	105	78.75	56.00	43.75	35
2004	96	72.00	51.20	40.00	32
2005	54	40.50	28.80	22.50	18
2006	60	45.00	32.00	25.00	20
2007	120	90.00	64.00	50.00	40
2008	108	81.00	57.60	45.00	36
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2011	78	58.50	41.60	32.50	26
2012	114	85.50	60.80	47.50	37.50
2013	150	115.00	80.00	60.00	44
2014	87	65.25	46.40	36.25	29
2015	117	87.75	62.40	48.75	38
2016	129	96.75	68.80	53.75	42
2017	98	73.00	54.40	42.50	34
2018	111	83.25	59.20	46.25	37

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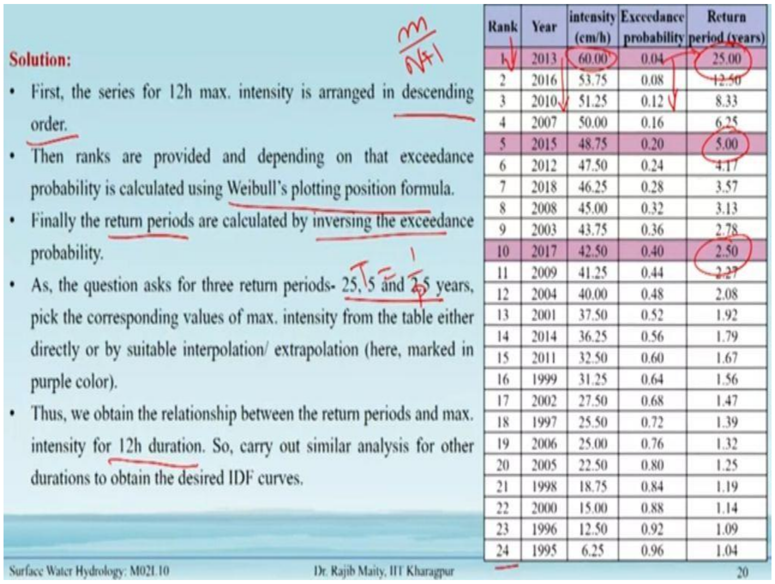


$\frac{m}{N+1}$

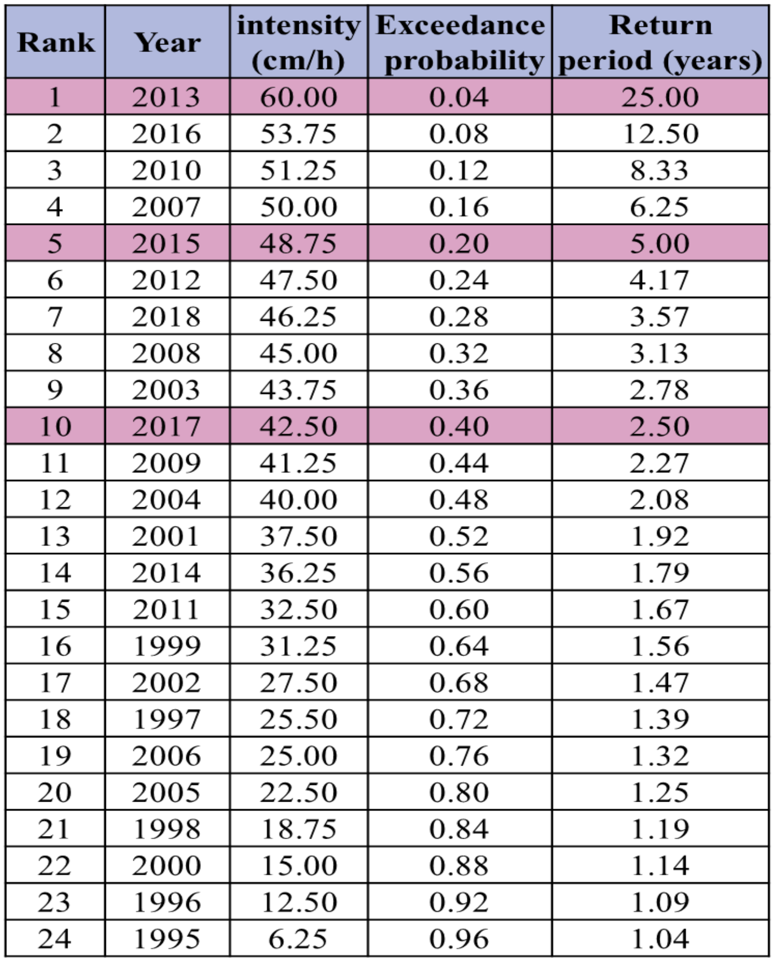
Solution:

- First, the series for 12h max. intensity is arranged in descending order.
- Then ranks are provided and depending on that exceedance probability is calculated using Weibull's plotting position formula.
- Finally the return periods are calculated by inversing the exceedance probability.
- As, the question asks for three return periods- 25, 5 and 2.5 years, pick the corresponding values of max. intensity from the table either directly or by suitable interpolation/ extrapolation (here, marked in purple color).
- Thus, we obtain the relationship between the return periods and max. intensity for 12h duration. So, carry out similar analysis for other durations to obtain the desired IDF curves.

$T = \frac{1}{p}$

Rank	Year	intensity (cm/h)	Exceedance probability	Return period (years)
1	2013	60.00	0.04	25.00
2	2016	53.75	0.08	12.50
3	2010	51.25	0.12	8.33
4	2007	50.00	0.16	6.25
5	2015	48.75	0.20	5.00
6	2012	47.50	0.24	4.17
7	2018	46.25	0.28	3.57
8	2008	45.00	0.32	3.13
9	2003	43.75	0.36	2.78
10	2017	42.50	0.40	2.50
11	2009	41.25	0.44	2.27
12	2004	40.00	0.48	2.08
13	2001	37.50	0.52	1.92
14	2014	36.25	0.56	1.79
15	2011	32.50	0.60	1.67
16	1999	31.25	0.64	1.56
17	2002	27.50	0.68	1.47
18	1997	25.50	0.72	1.39
19	2006	25.00	0.76	1.32
20	2005	22.50	0.80	1.25
21	1998	18.75	0.84	1.19
22	2000	15.00	0.88	1.14
23	1996	12.50	0.92	1.09
24	1995	6.25	0.96	1.04

- First, the series for 12h maximum intensity is arranged in descending order.
- Then ranks are provided and depending on that exceedance probability is calculated using Weibull's plotting position formula.
- Finally, the return periods are calculated by inversing the exceedance probability.
- As, the question asks for three return periods- 25, 5, and 2.5 years, pick the corresponding values of maximum intensity from the table either directly or by suitable interpolation/ extrapolation (here, marked in purple color).
- Thus, we obtain the relationship between the return periods and maximum intensity for 12h duration. So, carry out a similar analysis for other durations to obtain the desired IDF curves



Rank	Year	intensity (cm/h)	Exceedance probability	Return period (years)
1	2013	60.00	0.04	25.00
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9	2003	43.75	0.36	2.78
10	2017	42.50	0.40	2.50
11	2009	41.25	0.44	2.27
12	2004	40.00	0.48	2.08
13	2001	37.50	0.52	1.92
14	2014	36.25	0.56	1.79
15	2011	32.50	0.60	1.67
16	1999	31.25	0.64	1.56
17	2002	27.50	0.68	1.47
18	1997	25.50	0.72	1.39
19	2006	25.00	0.76	1.32
20	2005	22.50	0.80	1.25
21	1998	18.75	0.84	1.19
22	2000	15.00	0.88	1.14
23	1996	12.50	0.92	1.09
24	1995	6.25	0.96	1.04

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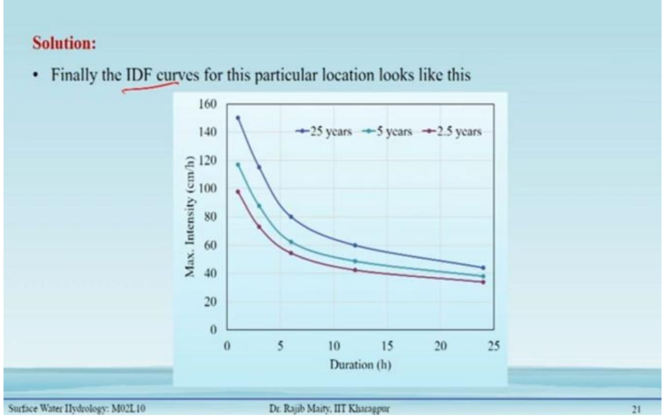
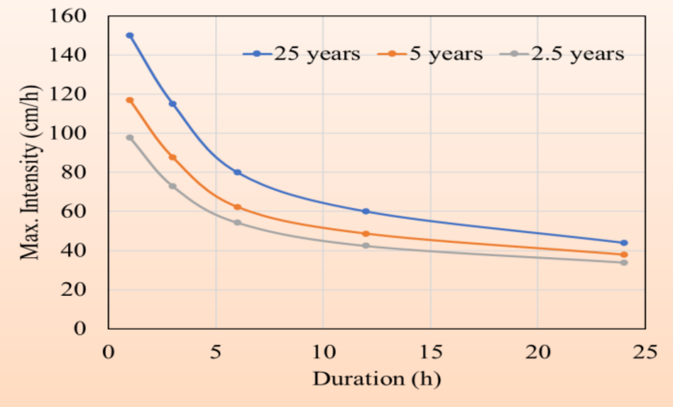


Solution:

- Finally the IDF curves for this particular location looks like this

Finally the IDF curves for this particular location looks like this



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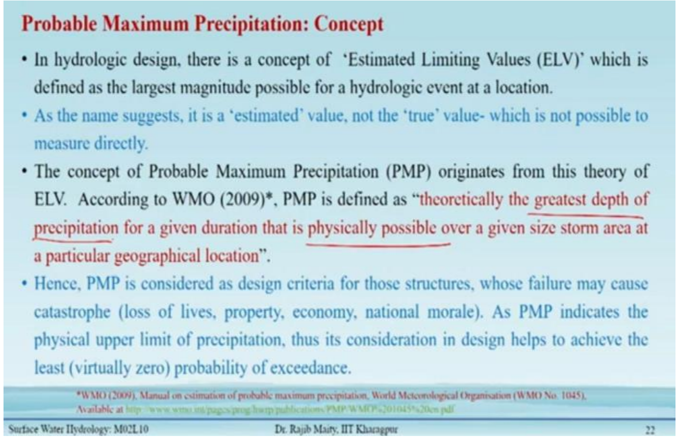

Probable Maximum Precipitation: Concept

- In hydrologic design, there is a concept of 'Estimated Limiting Values (ELV)' which is defined as the largest magnitude possible for a hydrologic event at a location.
- As the name suggests, it is a 'estimated' value, not the 'true' value- which is not possible to measure directly.
- The concept of Probable Maximum Precipitation (PMP) originates from this theory of ELV. According to WMO (2009)*, PMP is defined as "theoretically the greatest depth of precipitation for a given duration that is physically possible over a given size storm area at a particular geographical location".
- Hence, PMP is considered as design criteria for those structures, whose failure may cause catastrophe (loss of lives, property, economy, national morale). As PMP indicates the physical upper limit of precipitation, thus its consideration in design helps to achieve the least (virtually zero) probability of exceedance.

*WMO (2009), Manual on estimation of probable maximum precipitation, World Meteorological Organisation (WMO No. 1045), Available at http://www.wmo.int/pages/prog/hwrp/publications/PMP/WMO%201045%20en.pdf

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Probable Maximum Precipitation: Concept

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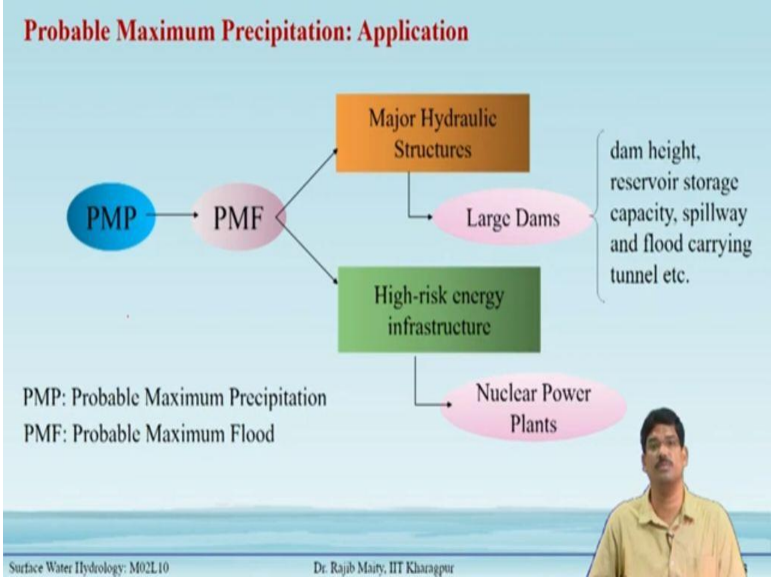

Probable Maximum Precipitation: Application

PMP

PMF

Major Hydraulic Structures

Large Dams

dam height, reservoir storage capacity, spillway and flood carrying tunnel etc.

High-risk energy infrastructure

Nuclear Power Plants

PMP: Probable Maximum Precipitation

PMF: Probable Maximum Flood

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Probable Maximum Precipitation: Application

So, its applications are multifold. The first one is the PMP, what we take first one is the PMF, which stands for the Probable Maximum Flood. And this Probable Maximum Flood is for the major hydraulic structure and the high-risk energy infrastructure. Depending on the major hydraulic structure, there are some large dams, particular the dam height, reservoir storage capacity, reservoir storage capacity, spillway, and flood-carrying tunnels and all. So, these are the different things where this PMP information is used through that PMF.

So, PMP information is utilized to estimate the Probable Maximum Flood and that is neutralized. This is one category, the other one is a high-risk energy structure. For example, the nuclear power plants, where you know that it cannot be flooded in any situation. So, those are the things are this information can be utilized.

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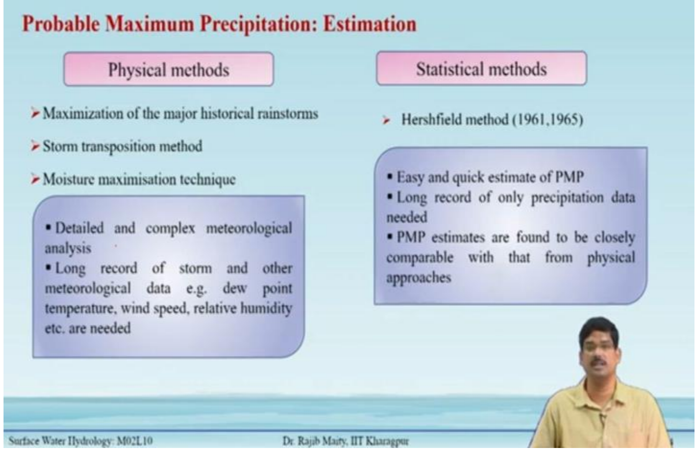

Probable Maximum Precipitation: Estimation

Physical methods

- Maximization of the major historical rainstorms
- Storm transposition method
- Moisture maximisation technique

- Detailed and complex meteorological analysis
- Long record of storm and other meteorological data e.g. dew point temperature, wind speed, relative humidity etc. are needed

Statistical methods

- Hershfield method (1961,1965)

- Easy and quick estimate of PMP
- Long record of only precipitation data needed
- PMP estimates are found to be closely comparable with that from physical approaches

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Probable Maximum Precipitation: Application

There are two broad categories we can say. One is the physical method; other one is the statistical method. Under the physical methods, maximization of the major historical rainstorm, the storms transportation method, moisture maximization technique, these are the some of the techniques are available. Under the statistical method, the Hershfield method is there and there are some advancements in the recent years of that method. So, pros and cons wise so far as the physical methods are concerned, the detailed and the more complex meteorological analysis is required.

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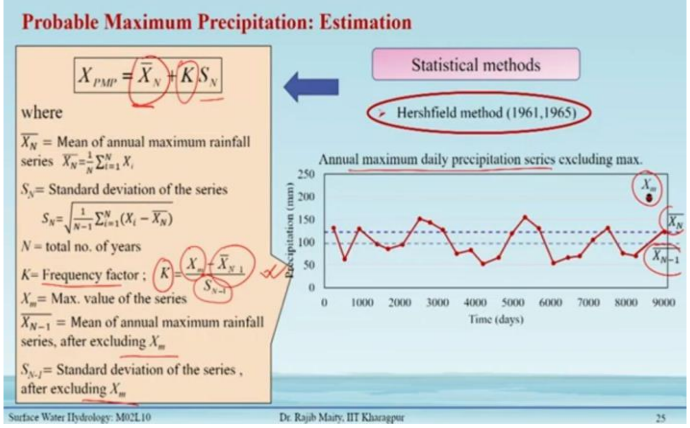

Probable Maximum Precipitation: Estimation

$$\boxed{X_{PMP} = \overline{X}_N + K S_N}$$

where

$\overline{X_N}$ = Mean of annual maximum rainfall series $\overline{X_N} = \frac{1}{N}\sum_{i=1}^{N} X_i$

S_N = Standard deviation of the series

$$S_N = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X_N})}$$

N = total no. of years

K = Frequency factor ; $K = \frac{X_m - \overline{X}_{N-1}}{S_{N-1}}$

X_m = Max. value of the series

$\overline{X_{N-1}}$ = Mean of annual maximum rainfall series, after excluding X_m

S_{N-1} = Standard deviation of the series , after excluding X_m

Statistical methods

- Hershfield method (1961,1965)

Annual maximum daily precipitation series excluding max.

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$$X_{PMP} = \overline{X}_N + K\,S_N$$

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X_N=Mean of annual maximum rainfall series $\overline{X_N}=\frac{1}{N}\sum_{i=1}^{N} X_i$

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$$K = \frac{X_m - \overline{X}_{N-1}}{S_{N-1}}$$

X_m= Max. Value of the series

X_{N-1}=Mean of annual maximum rainfall series, after excluding X_m

S_{N-1}= Standard deviation of the series, after excluding X_m

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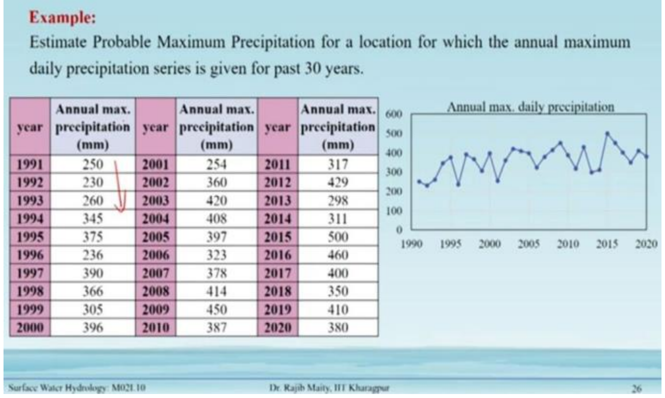

Example:

Estimate Probable Maximum Precipitation for a location for which the annual maximum daily precipitation series is given for past 30 years.

year	Annual max. precipitation (mm)	year	Annual max. precipitation (mm)	year	Annual max. precipitation (mm)
1991	250	2001	254	2011	317
1992	230	2002	360	2012	429
1993	260	2003	420	2013	298
1994	345	2004	408	2014	311
1995	375	2005	397	2015	500
1996	236	2006	323	2016	460
1997	390	2007	378	2017	400
1998	366	2008	414	2018	350
1999	305	2009	450	2019	410
2000	396	2010	387	2020	380

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Example:

Estimate Probable Maximum Precipitation for a location for which the annual maximum daily precipitation series is given for past 30 years.

year	Annual max. precipitation (mm)	year	Annual max. precipitation (mm)	year	Annual max. precipitation (mm)
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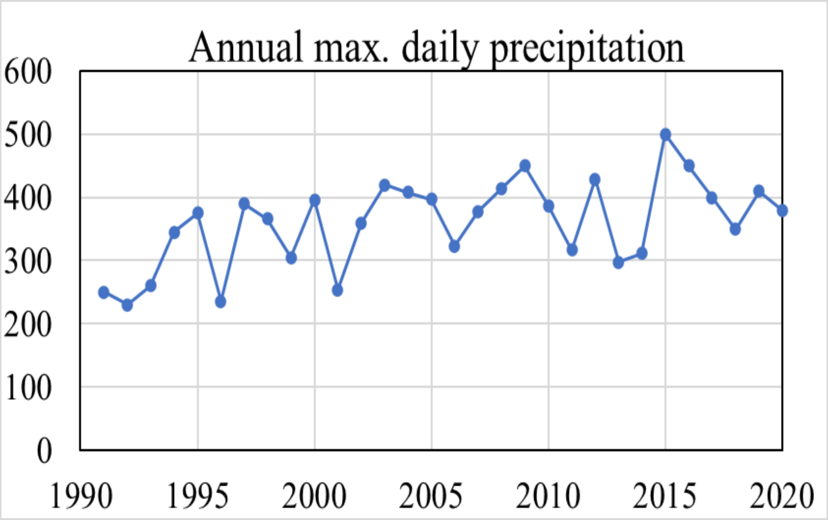

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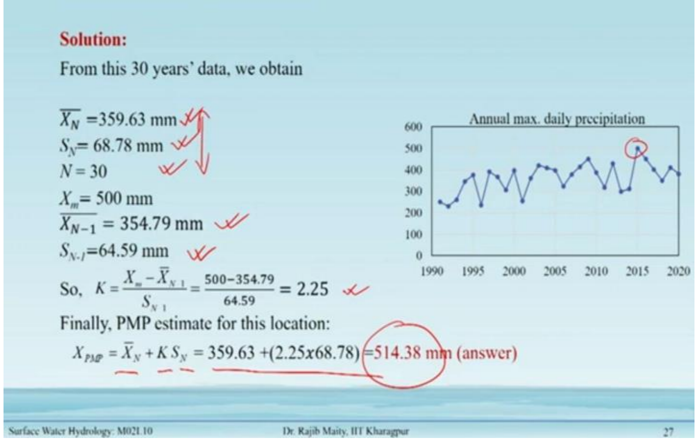

Solution:

From this 30 years' data, we obtain

$\overline{X_N}$ =359.63 mm
S_N= 68.78 mm
$N = 30$
X_m= 500 mm
$\overline{X_{N-1}}$ = 354.79 mm
S_{N-1}=64.59 mm

So, $K=\frac{X_m-\overline{X}_{N-1}}{S_{N-1}}=\frac{500-354.79}{64.59}=2.25$

Finally, PMP estimate for this location:

$X_{PMP}=\overline{X}_N+K\,S_N$ = 359.63 +(2.25x68.78) =514.38 mm (answer)

Solution:

From this 30 years' data, we obtain

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S_N= 68.78 mm
$N = 30$
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So, $=\frac{500-354.79}{64.59}=2.25$

Finally, PMP estimate for this location:

= 359.63 +(2.25x68.78) =514.38 mm (answer)

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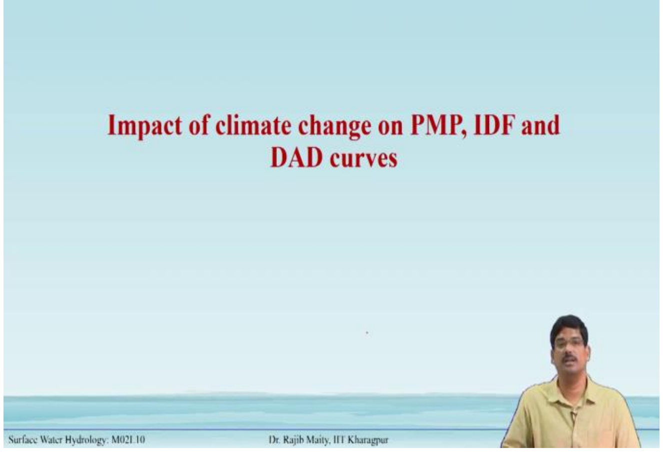

Impact of climate change on PMP, IDF and DAD curves

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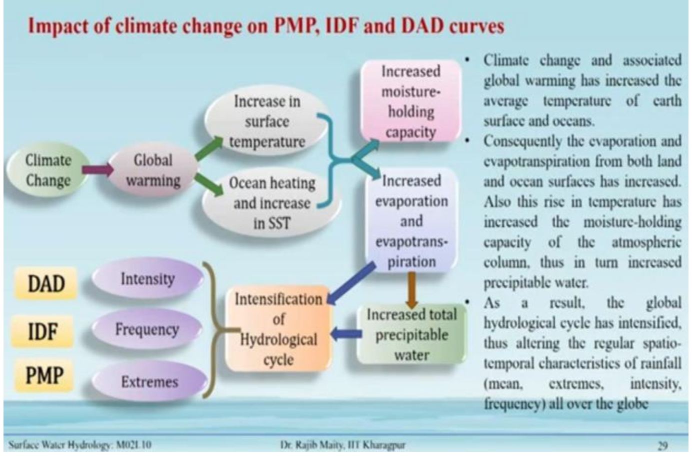

Impact of climate change on PMP, IDF and DAD curves

Climate Change

Global warming

Increase in surface temperature

Ocean heating and increase in SST

Increased moisture-holding capacity

Increased evaporation and evapotrans-piration

Increased total precipitable water

Intensification of Hydrological cycle

Intensity

Frequency

Extremes

DAD

IDF

PMP

- Climate change and associated global warming has increased the average temperature of earth surface and oceans.
- Consequently the evaporation and evapotranspiration from both land and ocean surfaces has increased. Also this rise in temperature has increased the moisture-holding capacity of the atmospheric column, thus in turn increased precipitable water.
- As a result, the global hydrological cycle has intensified, thus altering the regular spatio-temporal characteristics of rainfall (mean, extremes, intensity, frequency) all over the globe

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Impact of climate change on PMP, IDF and DAD curves

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- ➢ Consequently the evaporation and evapotranspiration from both land and ocean surfaces has increased. Also this rise in temperature has increased the moisture-holding capacity of the atmospheric column, thus in turn increased precipitable water.
- ➢ As a result, the global hydrological cycle has intensified, thus altering the regular spatio-temporal characteristics of rainfall (mean, extremes, intensity, frequency) all over the globe

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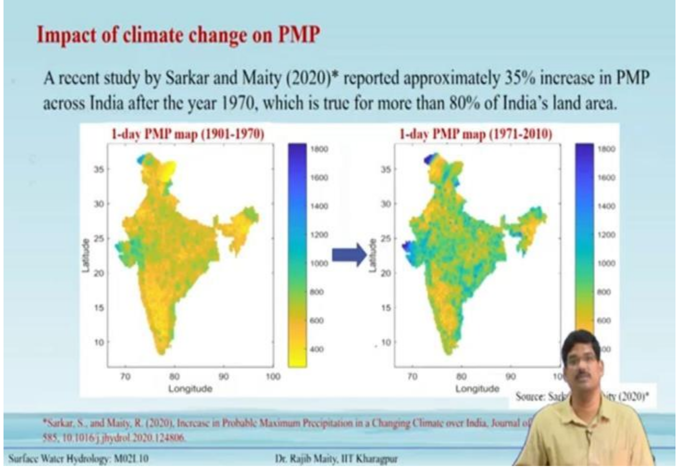

Impact of climate change on PMP

A recent study by Sarkar and Maity (2020)* reported approximately 35% increase in PMP across India after the year 1970, which is true for more than 80% of India's land area.

1-day PMP map (1901-1970)

1-day PMP map (1971-2010)

Source: Sa

ty (2020)*

*Sarkar, S., and Maity, R. (2020). Increase in Probable Maximum Precipitation in a Changing Climate over India, Journal of

585, 10.1016/j.jhydrol.2020.124806.

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Impact of climate change on PMP

A recent study by Sarkar and Maity (2020) reported approximately 35% increase in PMP across India after the year 1970, which is true for more than 80% of India's land area. This is one typical example that has been shown in fig.6. If we see the PMP map from 1901 to 1970, then it looks like this. Whereas, if I do the same estimates from the 1971 to 2010, we can see at the most of the places these magnitudes are are increased, at some places by several fold.

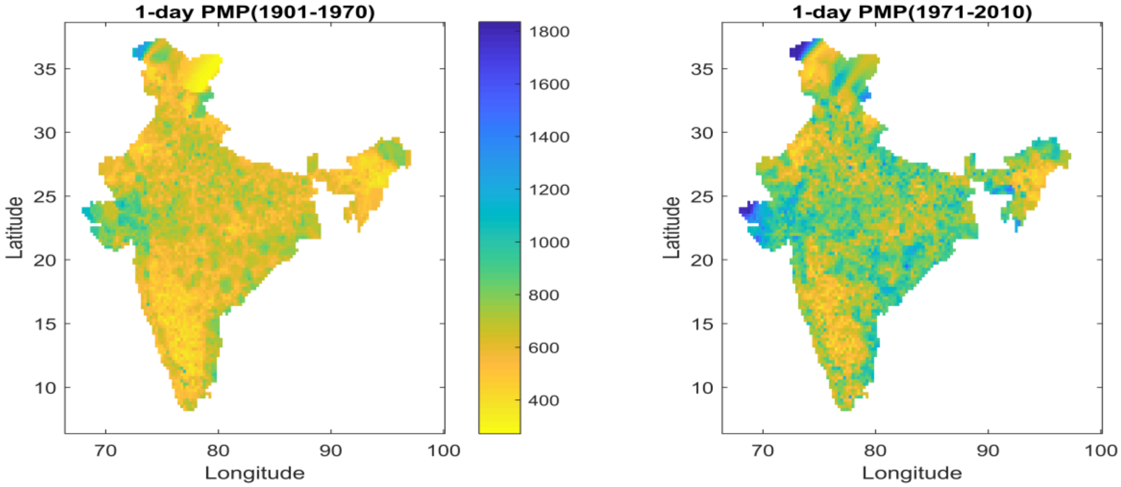

Fig.6 shows the 1-day PMP from 1901-1970 and 1971-2010

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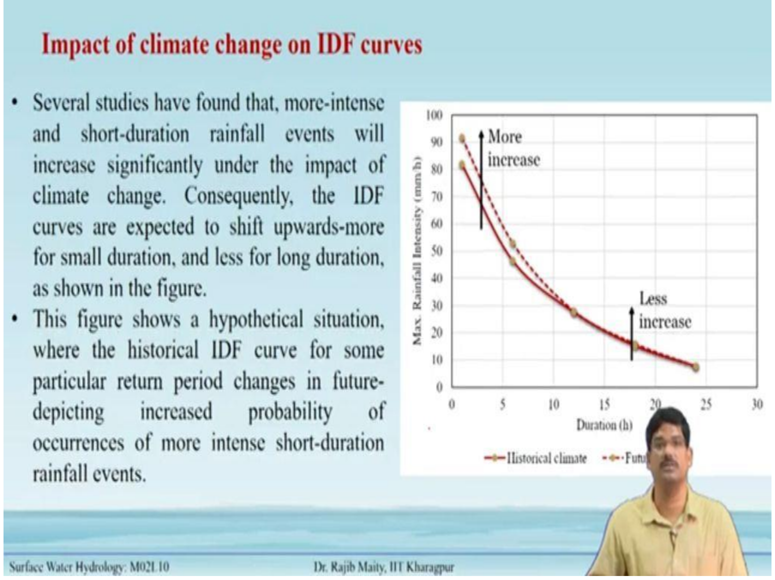

Impact of climate change on IDF curves

- Several studies have found that, more-intense and short-duration rainfall events will increase significantly under the impact of climate change. Consequently, the IDF curves are expected to shift upwards-more for small duration, and less for long duration, as shown in the figure.
- This figure shows a hypothetical situation, where the historical IDF curve for some particular return period changes in future-depicting increased probability of occurrences of more intense short-duration rainfall events.

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- This fig.7 shows a hypothetical situation, where the historical IDF curve for some particular return period changes in future-depicting increased probability of occurrences of more intense short-duration rainfall events.

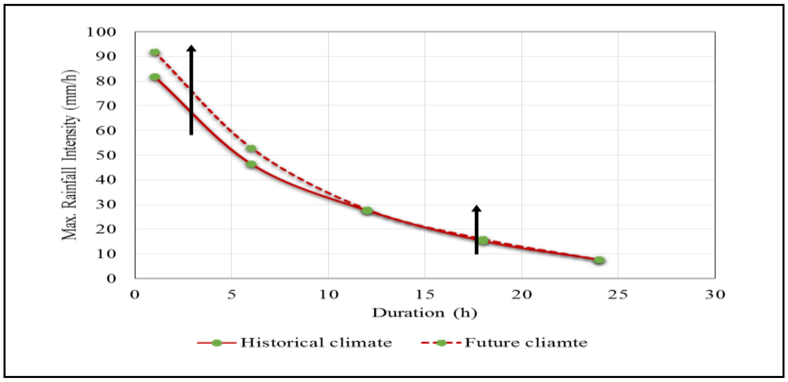

Fig.7 shows the IDF curve for the historical climate and the future climate

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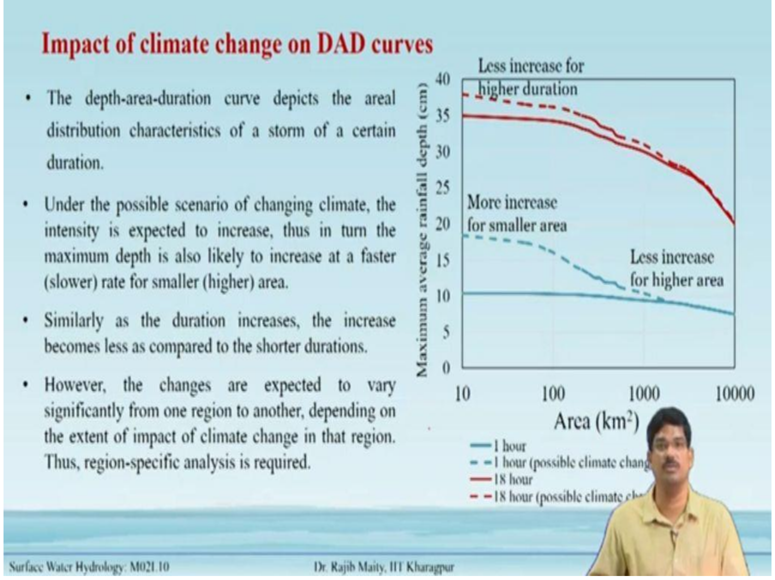

Impact of climate change on DAD curves

- The depth-area-duration curve depicts the areal distribution characteristics of a storm of a certain duration.
- Under the possible scenario of changing climate, the intensity is expected to increase, thus in turn the maximum depth is also likely to increase at a faster (slower) rate for smaller (higher) area.
- Similarly as the duration increases, the increase becomes less as compared to the shorter durations.
- However, the changes are expected to vary significantly from one region to another, depending on the extent of impact of climate change in that region. Thus, region-specific analysis is required.

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Impact of climate change on DAD curves

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- Under the possible scenario of changing climate, the intensity is expected to increase, thus in turn the maximum depth is also likely to increase at a faster (slower) rate for the smaller (higher) area shown in fig.8.
- Similarly, as the duration increases, the increase becomes less as compared to the shorter durations shown in fig.8.
- However, the changes are expected to vary significantly from one region to another, depending on the extent of the impact of climate change in that region. Thus, region-specific analysis is required.

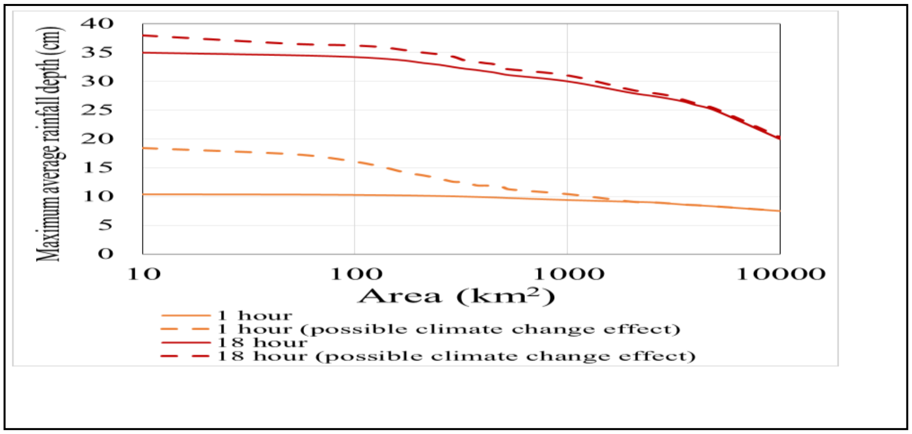

Fig.8 shows the Impact of climate change on DAD curves

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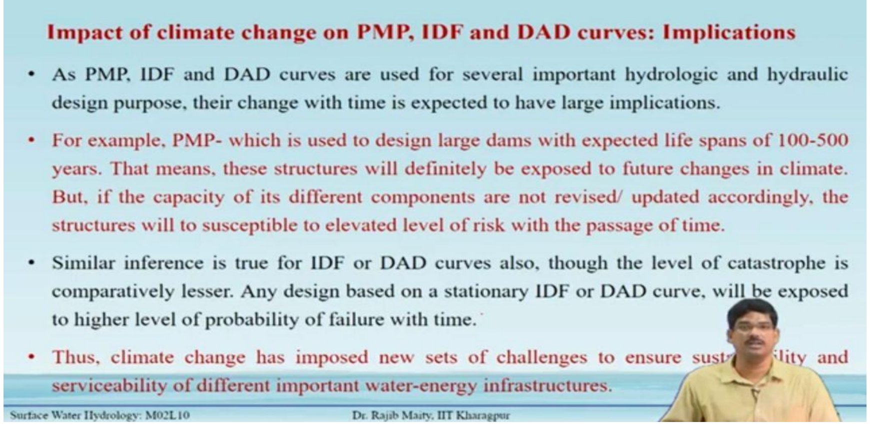

Impact of climate change on PMP, IDF and DAD curves: Implications

- As PMP, IDF and DAD curves are used for several important hydrologic and hydraulic design purpose, their change with time is expected to have large implications.
- For example, PMP- which is used to design large dams with expected life spans of 100-500 years. That means, these structures will definitely be exposed to future changes in climate. But, if the capacity of its different components are not revised/ updated accordingly, the structures will to susceptible to elevated level of risk with the passage of time.
- Similar inference is true for IDF or DAD curves also, though the level of catastrophe is comparatively lesser. Any design based on a stationary IDF or DAD curve, will be exposed to higher level of probability of failure with time.
- Thus, climate change has imposed new sets of challenges to ensure sust[illegible]ility and serviceability of different important water-energy infrastructures.

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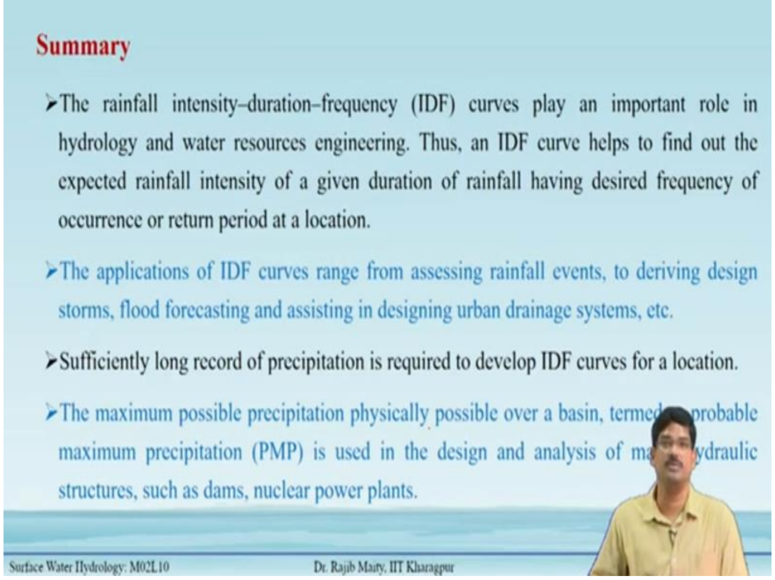

Summary

- The rainfall intensity–duration–frequency (IDF) curves play an important role in hydrology and water resources engineering. Thus, an IDF curve helps to find out the expected rainfall intensity of a given duration of rainfall having desired frequency of occurrence or return period at a location.
- The applications of IDF curves range from assessing rainfall events, to deriving design storms, flood forecasting and assisting in designing urban drainage systems, etc.
- Sufficiently long record of precipitation is required to develop IDF curves for a location.
- The maximum possible precipitation physically possible over a basin, termed probable maximum precipitation (PMP) is used in the design and analysis of ma ydraulic structures, such as dams, nuclear power plants.

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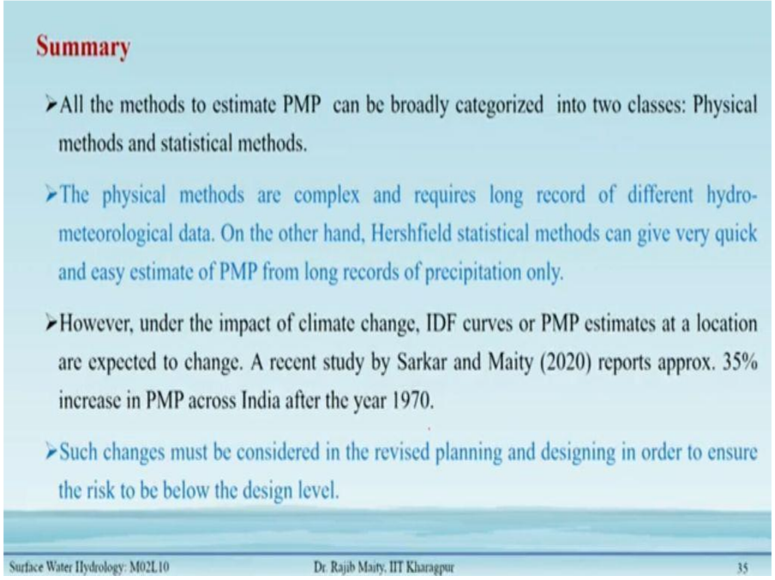

Summary

- All the methods to estimate PMP can be broadly categorized into two classes: Physical methods and statistical methods.
- The physical methods are complex and requires long record of different hydro-meteorological data. On the other hand, Hershfield statistical methods can give very quick and easy estimate of PMP from long records of precipitation only.
- However, under the impact of climate change, IDF curves or PMP estimates at a location are expected to change. A recent study by Sarkar and Maity (2020) reports approx. 35% increase in PMP across India after the year 1970.
- Such changes must be considered in the revised planning and designing in order to ensure the risk to be below the design level.

Summary

In summary, we learned the following points from this lecture:

- The rainfall intensity–duration–frequency (IDF) curves play an important role in hydrology and water resources engineering. Thus, an IDF curve helps to find out the expected rainfall intensity of a given duration of rainfall having desired frequency of occurrence or return period at a location.
- The applications of IDF curves range from assessing rainfall events to deriving design storms, flood forecasting, and assisting in designing urban drainage systems, etc.
- A sufficiently long record of precipitation is required to develop IDF curves for a location.
- The maximum possible precipitation physically possible over a basin, termed as probable maximum precipitation (PMP) is used in the design and analysis of major hydraulic structures, such as dams, nuclear power plants.
- All the methods to estimate PMP can be broadly categorized into two classes: Physical methods and statistical methods.
- The physical methods are complex and requires long record of different hydro-meteorological data. On the other hand, Hershfield statistical methods can give very quick and easy estimate of PMP from long records of precipitation only.

- However, under the impact of climate change, IDF curves or PMP estimates at a location are expected to change. A recent study by Sarkar and Maity (2020) reports approx. 35% increase in PMP across India after the year 1970.
- Such changes must be considered in the revised planning and design in order to ensure the risk is below the design level.