Surface Water Hydrology
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Lecture 15
Modelling of Infiltration Capacity

In this week, we will discuss the modeling of infiltration capacity.

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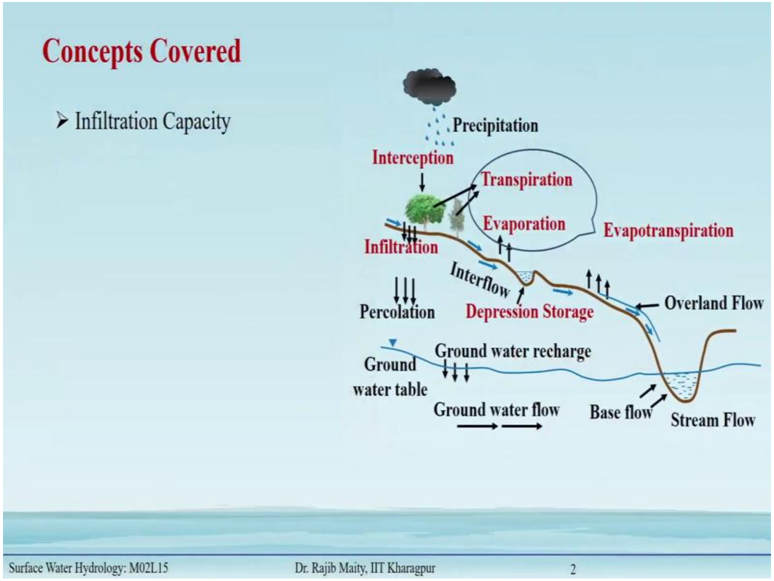

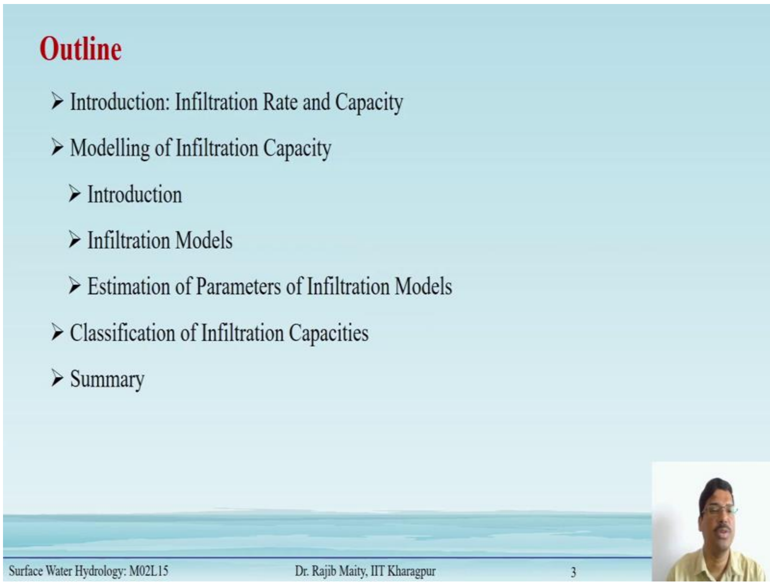

Under this concept cover, we will discuss the introduction of infiltration capacity, and we will learn two important terminologies, the starting one is infiltration rate and another one is infiltration capacity.

Next, we proceed to modelling of infiltration capacity with some background introduction, then we will discuss different infiltration models. And for those models, there are several parameters are there, we will discuss the estimation of these parameters for different infiltration models. Then we will discuss the classification of infiltration capacities before we proceed to a summary.

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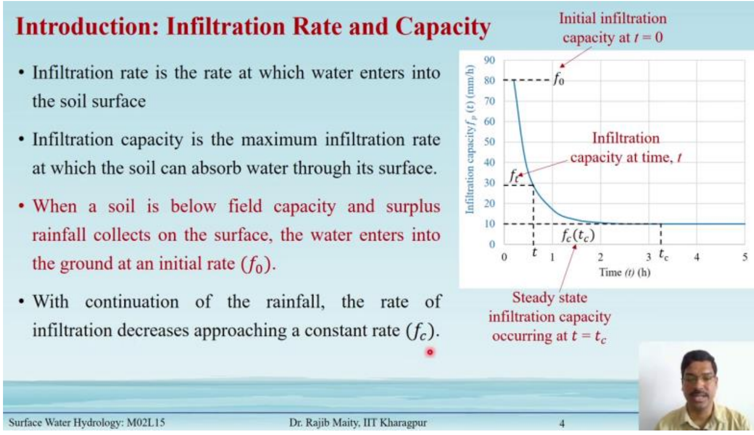

Introduction: Infiltration Rate and Capacity

Infiltration rate is the rate at which water enters the soil surface. Infiltration capacity is the maximum infiltration rate at which the soil can absorb water through its surface. When soil is below field capacity and surplus rainfall collects on the surface, the water enters into the ground at an initial rate f_o. With the continuation of the rainfall, the rate of infiltration decreases approaching a constant rate of f_c

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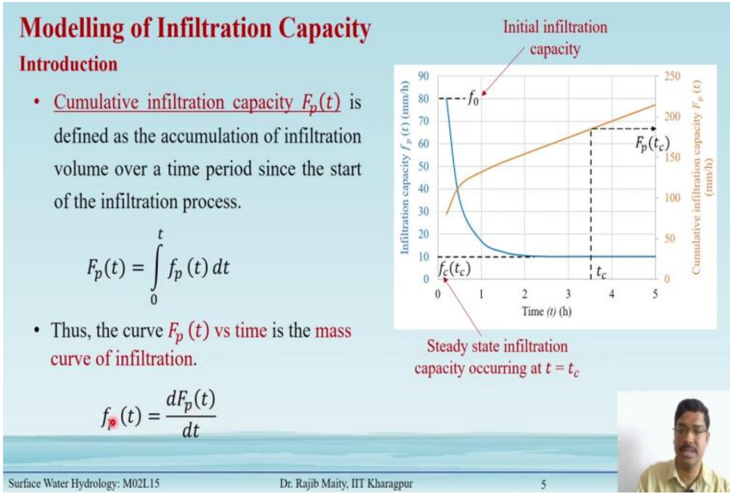

Modelling Infiltration Capacity

Introduction

Cumulative infiltration capacity $F_p(t)$ is defined as the accumulation of infiltration volume over a time period since the start of the infiltration process.

$$F_p(t) = \int_0^t f_p\,(t)\,dt$$

Thus, the curve $F_p(t)$ vs time is the mass curve of infiltration.

$$f_p\,(t) = \frac{dF_p(t)}{dt}$$

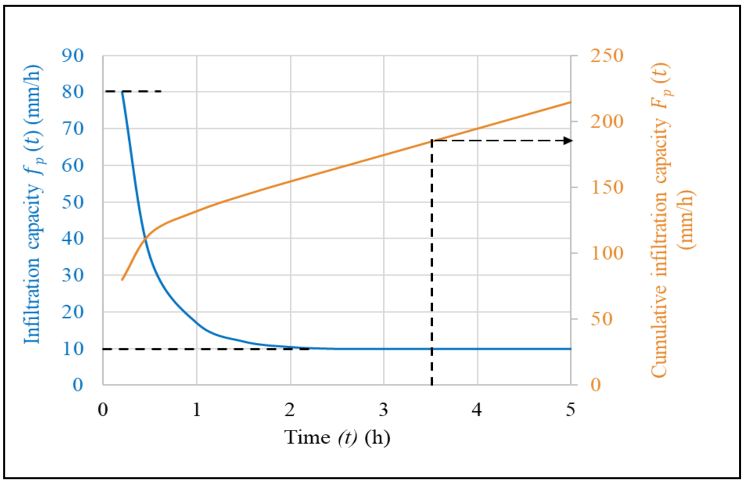

Fig.1 shows the curves of infiltration capacity and cumulative infiltration capacity

In fig.1, this blue line is the infiltration capacity that changes over this time, and this red line is the integrated form of this blue line which is the cumulative infiltration capacity. Thus, this $F_p(t)$ versus time is the mass curve of the infiltration and if we take a time derivative of that, we will get back this line for this infiltration capacity.

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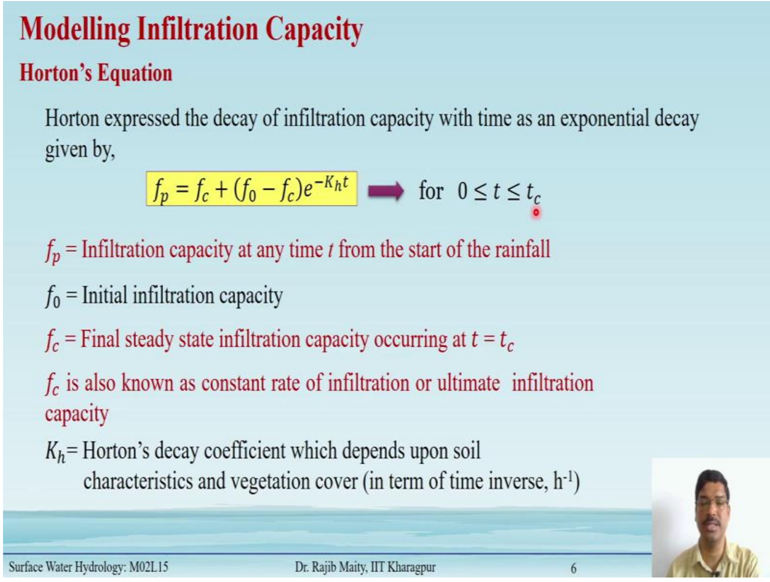

Horton's Equation

There are different modeling equations are available, one of the most important and frequently used equation is Horton's equation to model this infiltration capacity. Horton expressed the decay of infiltration capacity with time as an exponential decay given by,

$$f_p = f_c + (f_0 - f_c)e^{-K_h t}$$

for $0 \le t \le t_c$

f_p = Infiltration capacity at any time t from the start of the rainfall

f_0 = Initial infiltration capacity

f_c = Final steady-state infiltration capacity occurring at $t = t_c$

f_c is also known as constant rate of infiltration or ultimate infiltration capacity

K_h = Horton's decay coefficient which depends upon soil characteristics and vegetation cover (in terms of time inverse, h-1)

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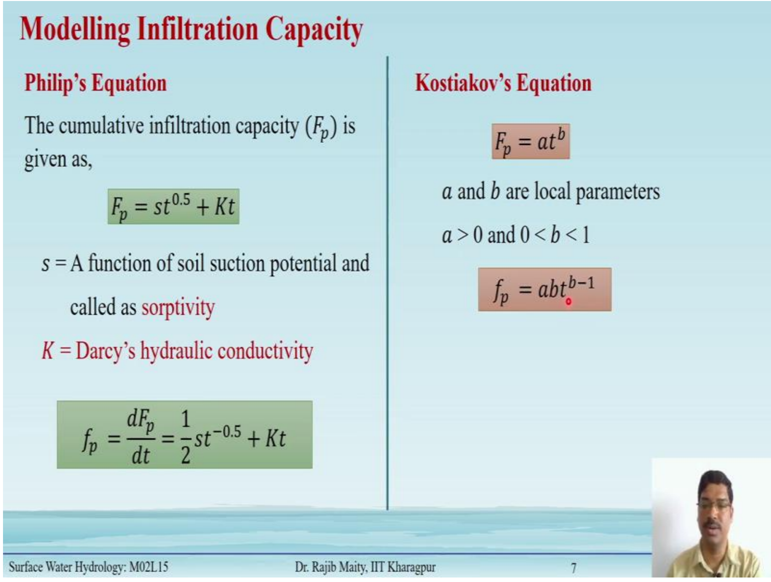

Philip's Equation

The cumulative infiltration capacity (F_p)is given as,

$$F_p = st^{0.5} + Kt$$

s = A function of soil suction potential and called as sorptivity

K = Darcy's hydraulic conductivity

$$f_p = \frac{dF_p}{dt} = \frac{1}{2}st^{-0.5} + Kt$$

Kostiakov's Equation

$$F_p = at^b$$

a and b are local parameters

$a > 0$ and $0 < b < 1$

$$f_p = abt^{b-1}$$

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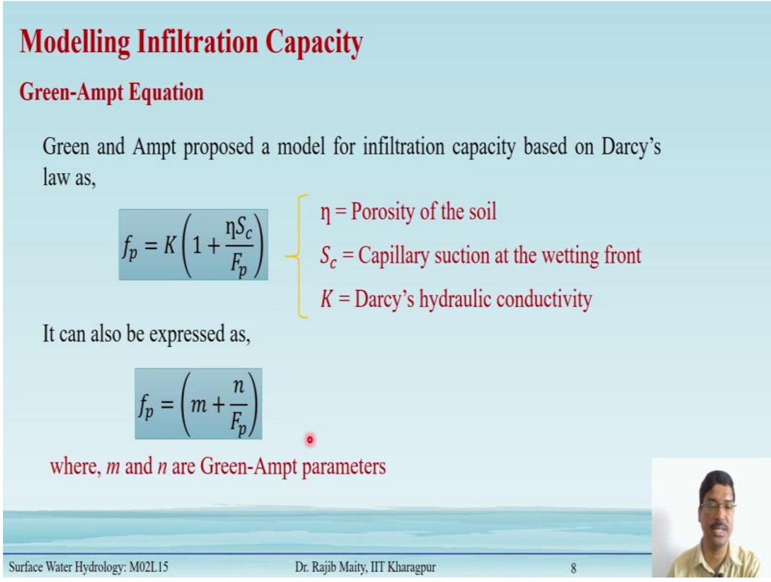

Green-Ampt Equation

Green and Ampt proposed a model for infiltration capacity based on Darcy's law as,

$$f_p = K\left(1+\frac{\eta S_c}{F_p}\right)$$

η = Porosity of the soil

S_c = Capillary suction at the wetting front

K = Darcy's hydraulic conductivity

It can also be expressed as,

$$f_p = \left(m+\frac{n}{F_p}\right)$$

Where m and n are Green-Ampt parameters

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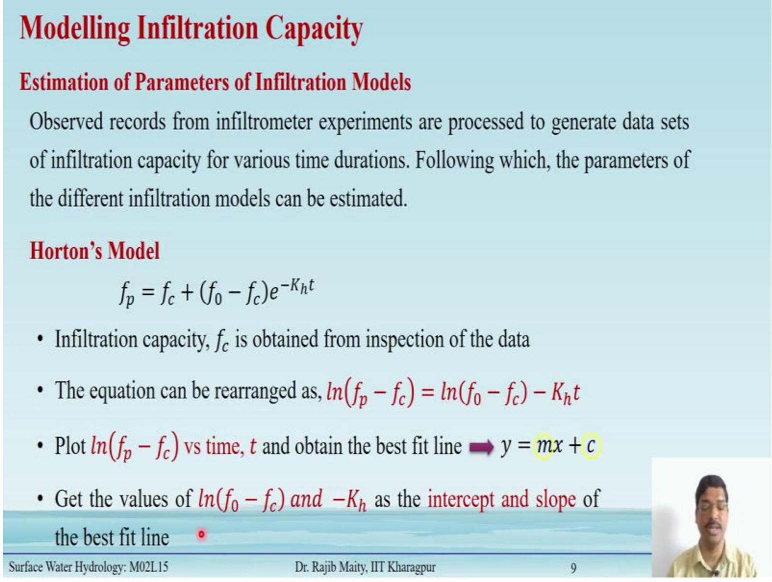

Estimation of Parameters of Infiltration Models

Observed records from infiltrometer experiments are processed to generate data sets of infiltration capacity for various time durations. Following this, the parameters of the different infiltration models can be estimated.

Horton's Model

$$f_p = f_c + (f_0 - f_c)e^{-K_h t}$$

Infiltration capacity, f_c is obtained from inspection of the data

The equation can be rearranged as, $ln(f_p - f_c) = ln(f_0 - f_c) - K_h t$

Plot $ln(f_p\text{-}f_c)$ vs time, t and obtain the best fit line, $y = mx + c$

Get the values of $ln(f_0\text{-}f_c)$ *and* $-K_h$ as the intercept and slope of the best-fit line.

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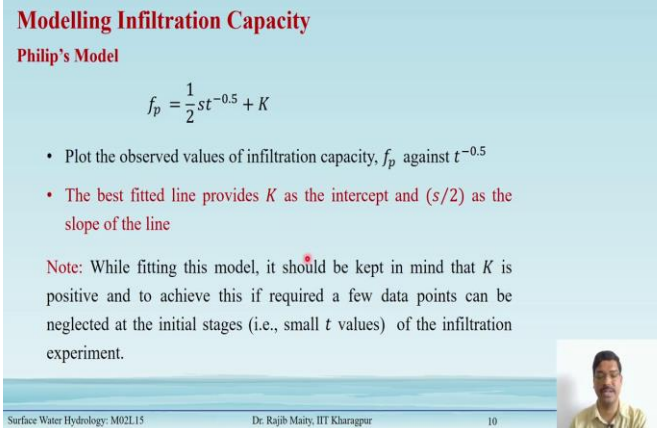

Philip's Model

$$f_p = \frac{1}{2} s t^{-0.5} + K$$

- Plot the observed values of infiltration capacity, f_p against $t^{-0.5}$
- The best-fitted line provides K as the intercept and $(s/2)$ as the slope of the line

It may be noted that while fitting this model, it should be kept in mind that K is positive, and to achieve this if required a few data points can be neglected at the initial stages (i.e., small t values) of the infiltration experiment.

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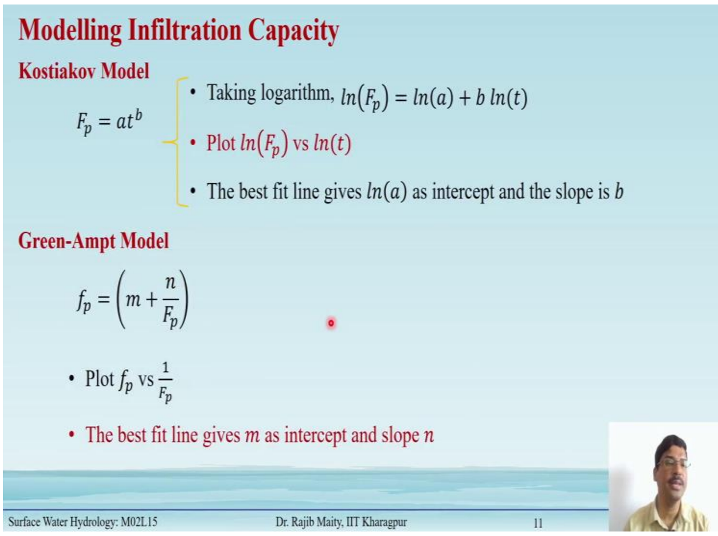

Kostiakov Model

$$F_p = at^b$$

- Taking logarithm, $ln\ (F_p) = ln(a) + b\ ln(t)$
- Plot $ln\ (F_p)$ vs $ln(t)$
- The best fit line gives $ln(a)$ as intercept and the slope is b

Green-Ampt Model

$$f_p = \left(m + \frac{n}{F_p}\right)$$

- Plot f_p vs $1/F_p$
- The best fit line gives m as intercept and slope n.

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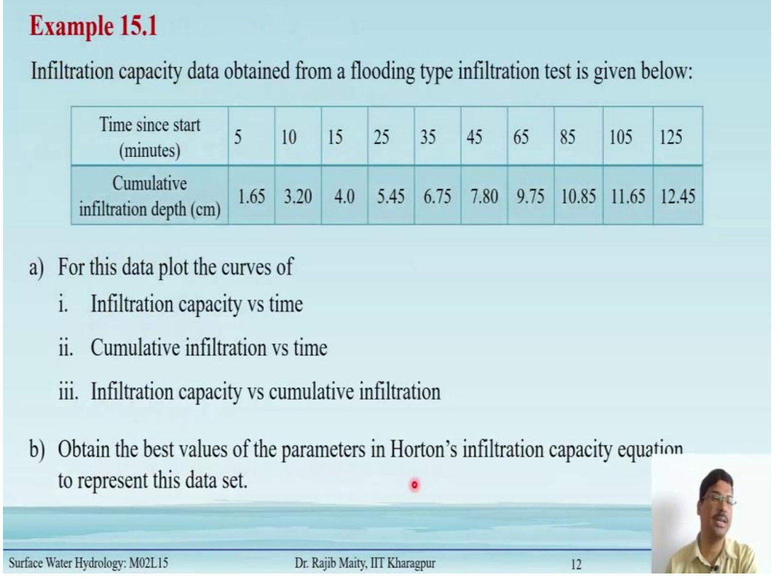

Example 15.1

Infiltration capacity data obtained from a flooding type infiltration test is given below:

Time since start (minutes)	5	10	15	25	35	45	65	85	105	125
Cumulative infiltration depth (cm)	1.65	3.20	4.0	5.45	6.75	7.80	9.75	10.85	11.65	12.45

a) For this data plot the curves of

I. Infiltration capacity vs time

II. Cumulative infiltration vs time

III. Infiltration capacity vs cumulative infiltration

b) Obtain the best values of the parameters in Horton's infiltration capacity equation to represent this data set.

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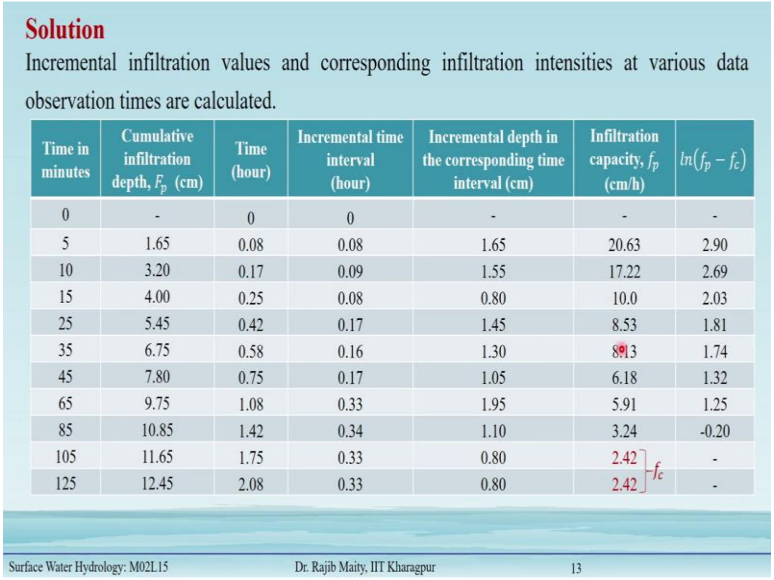
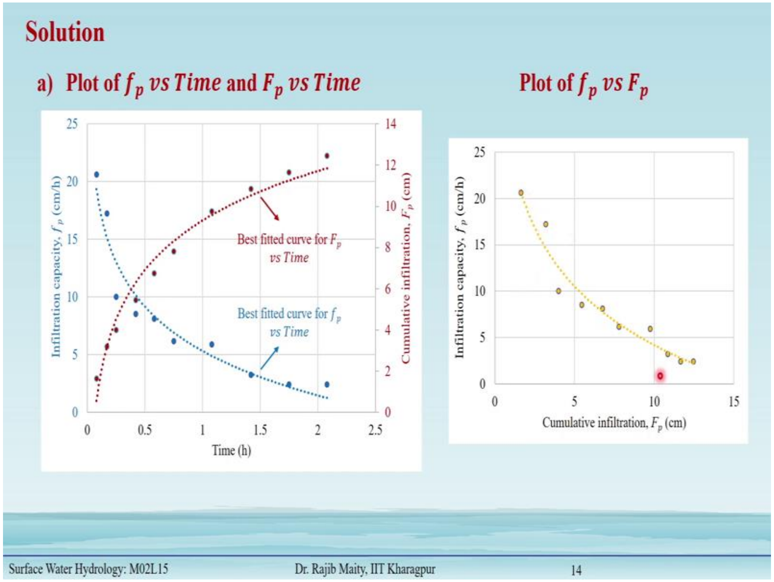

Solution

Incremental infiltration values and corresponding infiltration intensities at various data observation times are calculated.

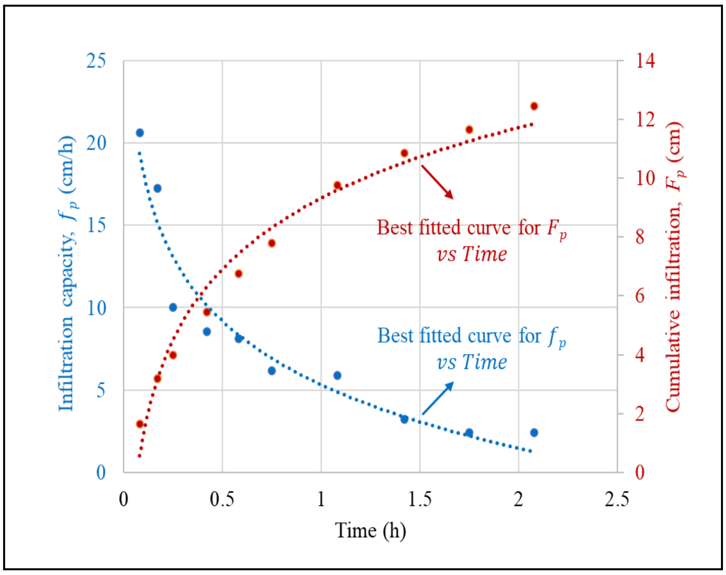
Time in minutes	Cumulative infiltration depth, F_p (cm)	Time (hour)	Incremental time interval (hour)	Incremental depth in the corresponding time interval (cm)	Infiltration capacity, f_p (cm/h)	$ln(f_p - f_c)$
0	-	0	0	-	-	-
5	1.65	0.08	0.08	1.65	20.63	2.90
10	3.20	0.17	0.09	1.55	17.22	2.69
15	4.00	0.25	0.08	0.80	10.0	2.03
25	5.45	0.42	0.17	1.45	8.53	1.81
35	6.75	0.58	0.16	1.30	8.13	1.74
45	7.80	0.75	0.17	1.05	6.18	1.32
65	9.75	1.08	0.33	1.95	5.91	1.25
85	10.85	1.42	0.34	1.10	3.24	-0.20
105	11.65	1.75	0.33	0.80	2.42	-
125	12.45	2.08	0.33	0.80	2.42	-



Solution

Incremental infiltration values and corresponding infiltration intensities at various data observation times are calculated.

Time in minutes	Cumulative infiltration depth, F_p (cm)	Time (hour)	Incremental time interval (hour)	Incremental depth in the corresponding time interval (cm)	Infiltration capacity, f_p (cm/h)	$ln(f_p - f_c)$
0	-	0	0	-	-	-
5	1.65	0.08	0.08	1.65	20.63	2.90
10	3.20	0.17	0.09	1.55	17.22	2.69
15	4.00	0.25	0.08	0.80	10.0	2.03
25	5.45	0.42	0.17	1.45	8.53	1.81
35	6.75	0.58	0.16	1.30	8.13	1.74
45	7.80	0.75	0.17	1.05	6.18	1.32
65	9.75	1.08	0.33	1.95	5.91	1.25
85	10.85	1.42	0.34	1.10	3.24	-0.20
105	11.65	1.75	0.33	0.80	2.42	-
125	12.45	2.08	0.33	0.80	2.42	-



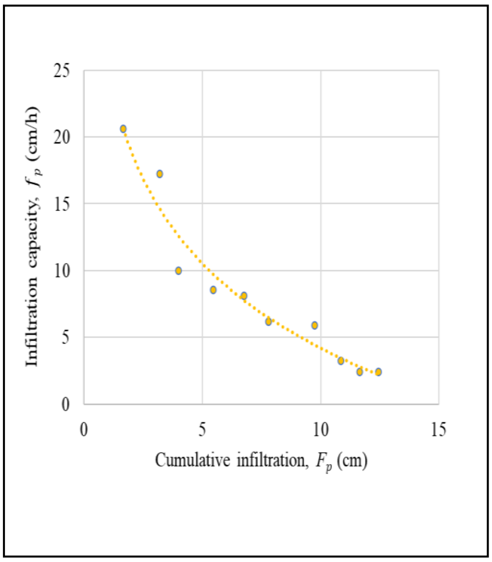


Fig.2 shows the Plot of $\boldsymbol{f_p\ vs\ Time}$ and $\boldsymbol{F_p\ vs\ Time}$

Fig.3 shows the Plot of $\boldsymbol{f_p\ vs\ F_p}$

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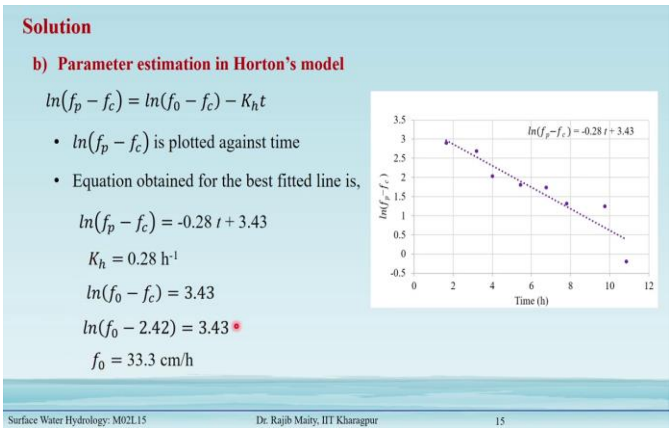

b) **Parameter estimation in Horton's model**

$$ln(f_p - f_c) = ln(f_0 - f_c) - K_h t$$

- $ln(f_p{-}f_c)$ is plotted against time
- Equation obtained for the best fitted line

$ln(f_p{-}f_c)$ = -0.28 t + 3.43

K_h = 0.28 h^{-1}

$ln(f_0{-}f_c)$ = 3.43

$ln(f_0{-}2.42)$ = 3.43

f_0 = 33.3 cm/h

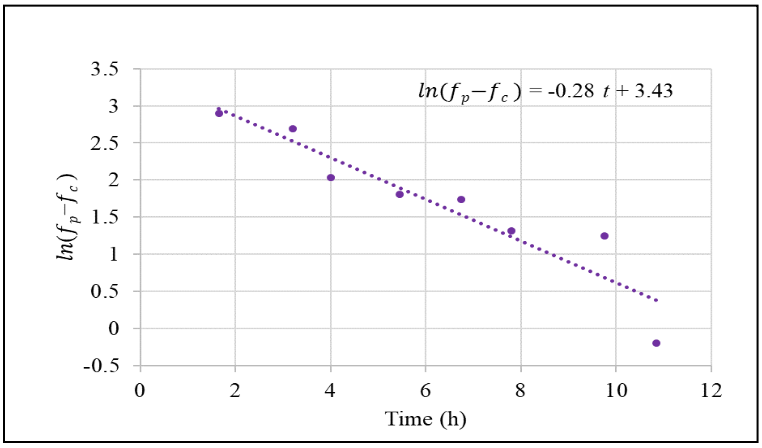

Fig.4 shows the Horton's Equation(The plot of $ln(f_p{-}f_c)$ vs Time)

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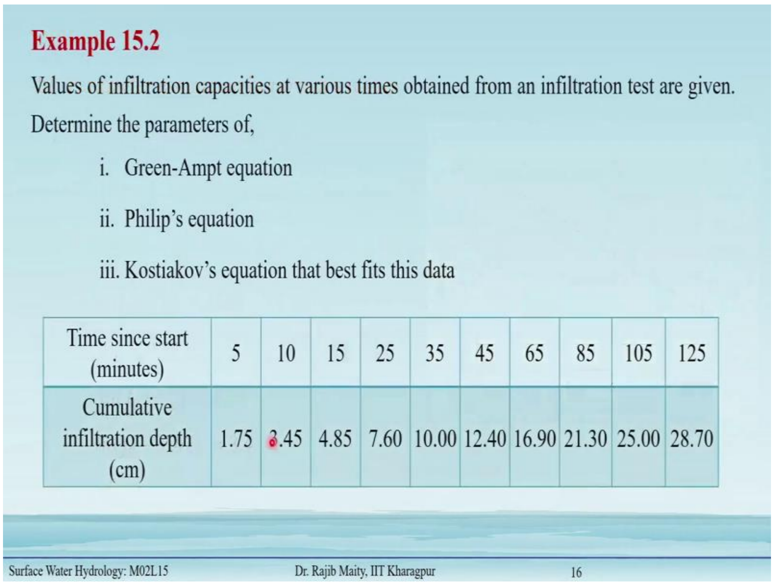

Example 15.2

Values of infiltration capacities at various times obtained from an infiltration test are given. Determine the parameters of,

i. Green-Ampt equation

ii. Philip's equation

iii. Kostiakov's equation that best fits this data

Time since start (minutes)	5	10	15	25	35	45	65	85	105	125
Cumulative infiltration depth (cm)	1.75	3.45	4.85	7.60	10.00	12.40	16.90	21.30	25.00	28.70

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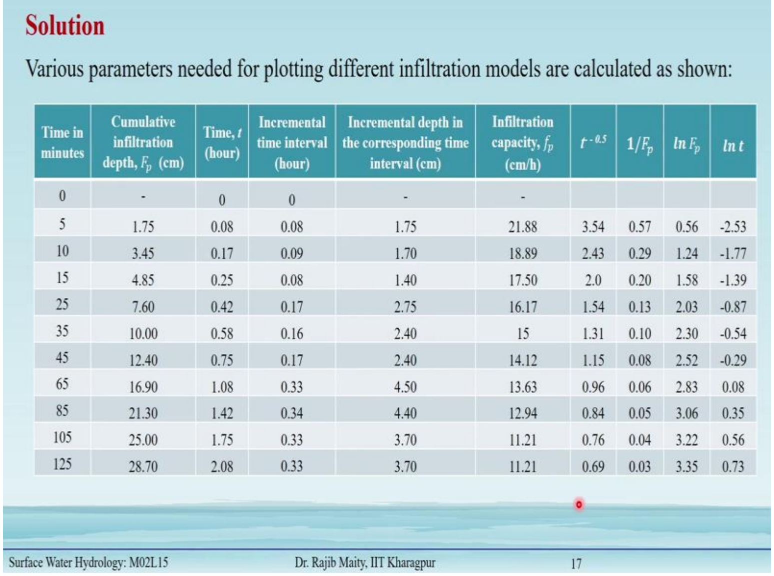
Solution

Various parameters needed for plotting different infiltration models are calculated as shown:

Time in minutes	Cumulative infiltration depth, F_p (cm)	Time, t (hour)	Incremental time interval (hour)	Incremental depth in the corresponding time interval (cm)	Infiltration capacity, f_p (cm/h)	$t^{-0.5}$	$1/F_p$	$\ln F_p$	$\ln t$
0	-	0	0	-	-				
5	1.75	0.08	0.08	1.75	21.88	3.54	0.57	0.56	-2.53
10	3.45	0.17	0.09	1.70	18.89	2.43	0.29	1.24	-1.77
15	4.85	0.25	0.08	1.40	17.50	2.0	0.20	1.58	-1.39
25	7.60	0.42	0.17	2.75	16.17	1.54	0.13	2.03	-0.87
35	10.00	0.58	0.16	2.40	15	1.31	0.10	2.30	-0.54
45	12.40	0.75	0.17	2.40	14.12	1.15	0.08	2.52	-0.29
65	16.90	1.08	0.33	4.50	13.63	0.96	0.06	2.83	0.08
85	21.30	1.42	0.34	4.40	12.94	0.84	0.05	3.06	0.35
105	25.00	1.75	0.33	3.70	11.21	0.76	0.04	3.22	0.56
125	28.70	2.08	0.33	3.70	11.21	0.69	0.03	3.35	0.73

Solution

Various parameters needed for plotting different infiltration models are calculated as shown:

Time in minutes	Cumulative infiltration depth, F_p (cm)	Time, t (hour)	Incremental time interval (hour)	Incremental depth in the corresponding time interval (cm)	Infiltration capacity, f_p (cm/h)	$t^{-0.5}$	$1/F_p$	$ln\ F_p$	$ln\ t$
0	-	0	0	-	-				
5	1.75	0.08	0.08	1.75	21.88	3.54	0.57	0.56	-2.53
10	3.45	0.17	0.09	1.70	18.89	2.43	0.29	1.24	-1.77
15	4.85	0.25	0.08	1.40	17.50	2.0	0.20	1.58	-1.39
25	7.60	0.42	0.17	2.75	16.17	1.54	0.13	2.03	-0.87
35	10.00	0.58	0.16	2.40	15	1.31	0.10	2.30	-0.54
45	12.40	0.75	0.17	2.40	14.12	1.15	0.08	2.52	-0.29
65	16.90	1.08	0.33	4.50	13.63	0.96	0.06	2.83	0.08
85	21.30	1.42	0.34	4.40	12.94	0.84	0.05	3.06	0.35
105	25.00	1.75	0.33	3.70	11.21	0.76	0.04	3.22	0.56
125	28.70	2.08	0.33	3.70	11.21	0.69	0.03	3.35	0.73

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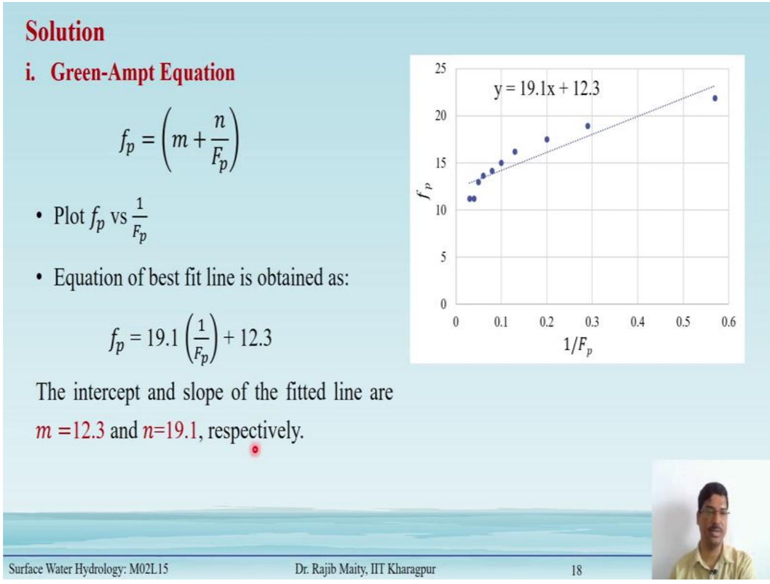

i. **Green-Ampt Equation**

$$f_p = \left(m + \frac{n}{F_p}\right)$$

- Plot f_p vs $1/F_p$
- Equation of best fit line is obtained as:

$$f_p = 19.1\left(\frac{1}{F_p}\right) + 12.3$$

The intercept and slope of the fitted line are m=12.3 and n=19.1, respectively.

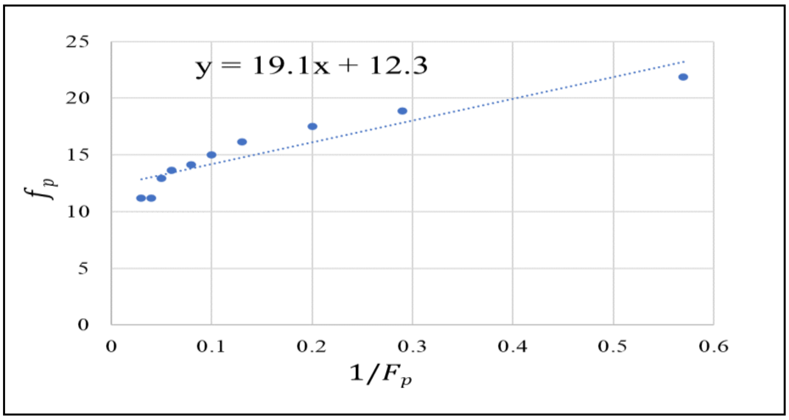

Fig.5 shows the fitting of Green-Ampt Equation - Example 15.2

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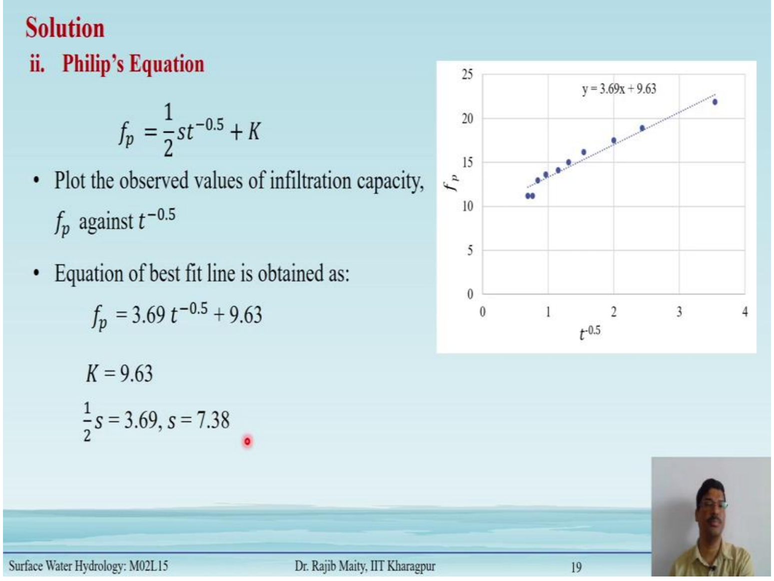

ii. **Philip's Equation**

$$f_p = \frac{1}{2}st^{-0.5} + K$$

Plot the observed values of infiltration capacity, f_p against $t^{-0.5}$

$f_p = 3.69\, t^{-0.5} + 9.63$

K=9.63

½ s=3.69, s=7.38

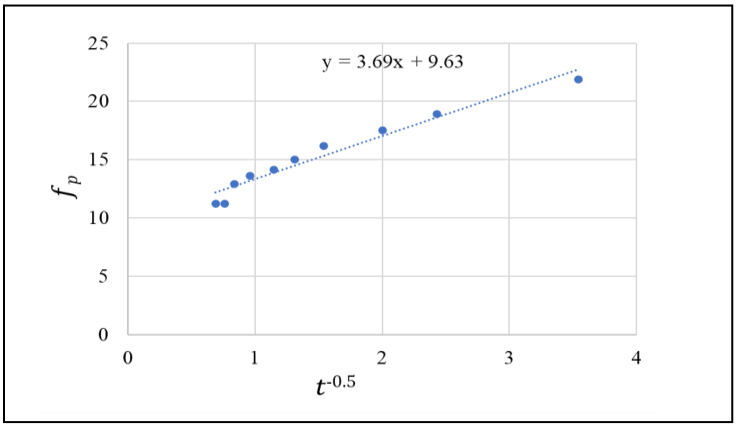

Fig.6 shows the fittings of Phillip's Equation -Example15.2

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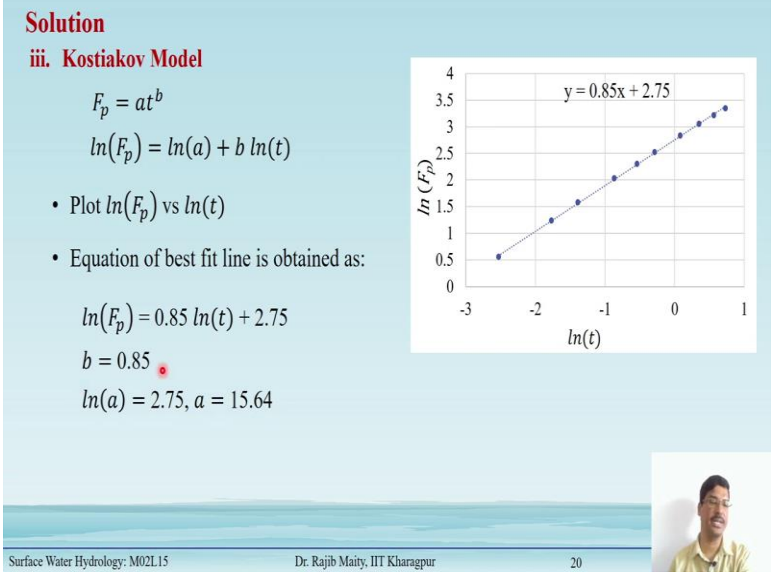

iii. **Kostiakov Model**

$$F_p = at^b$$

$$ln(F_p) = ln(a) + b\; ln(t)$$

- Plot $ln\, F_p$ vs $ln(t)$
- Equation of best fit line is obtained as:

$ln\,(Fp)$= 0.85 $ln(t)$ + 2.75

b=0.85

ln (a)= 2.75, a=15.64

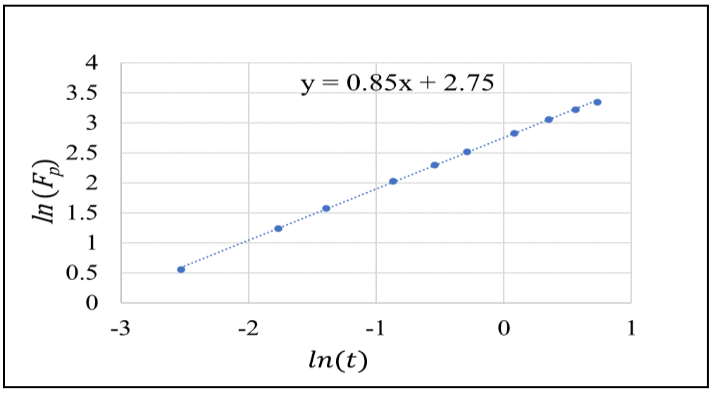

Fig.7 shows the fittings of Kostiakov Equation -Example 15.2

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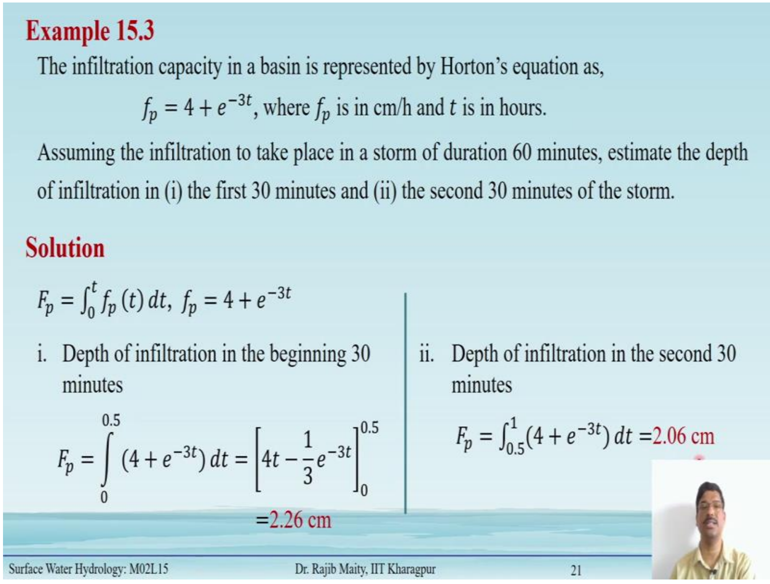

Example 15.3

The infiltration capacity in a basin is represented by Horton's equation as,

f_p=4+e^{-3t}, where f_p is in cm/h and t is in hours.

Assuming the infiltration to take place in a storm of duration 60 minutes, estimate the depth of infiltration in (i) the first 30 minutes and (ii) the second 30 minutes of the storm.

Solution

$$F_p = \int_0^t f_p\,(t)\,dt,\ f_p = 4 + e^{-3t}$$

i. Depth of infiltration in the beginning 30 minutes

$$F_p = \int_0^{0.5} (4 + e^{-3t})\,dt = \left[4t - \frac{1}{3}e^{-3t}\right]_0^{0.5}$$

$$= 2.26 \text{ cm}$$

ii. Depth of infiltration in the second 30 minutes

$$F_p = \int_{0.5}^{1} (4 + e^{-3t})\,dt = 2.06 \text{ cm}$$

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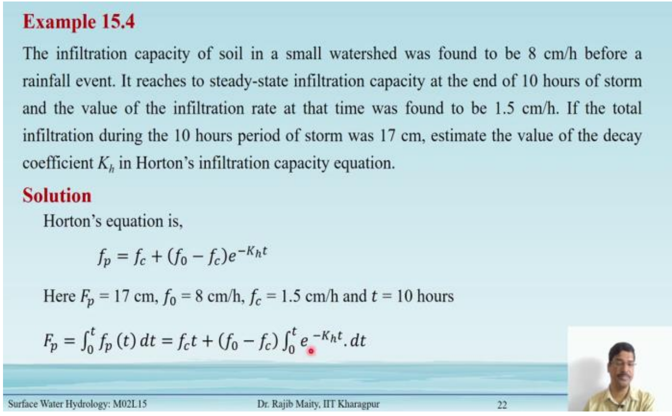

Example 15.4

The infiltration capacity of soil in a small watershed was found to be 8 cm/h before a rainfall event. It reaches steady-state infiltration capacity at the end of 10 hours of storm and the value of the infiltration rate at that time was found to be 1.5 cm/h. If the total infiltration during the 10 hours of the storm was 17 cm, estimate the value of the decay coefficient K_h in Horton's infiltration capacity equation.

Solution

Horton's equation is,

$$f_p = f_c + (f_0 - f_c)e^{-K_h t}$$

Here F_p = 17 cm, f_0 = 8 cm/h, f_c = 1.5 cm/h and t = 10 hours

$$F_p = \int_0^t f_p(t)\,dt = f_c t + (f_0 - f_c)\int_0^t e^{-K_h t}.\,dt$$

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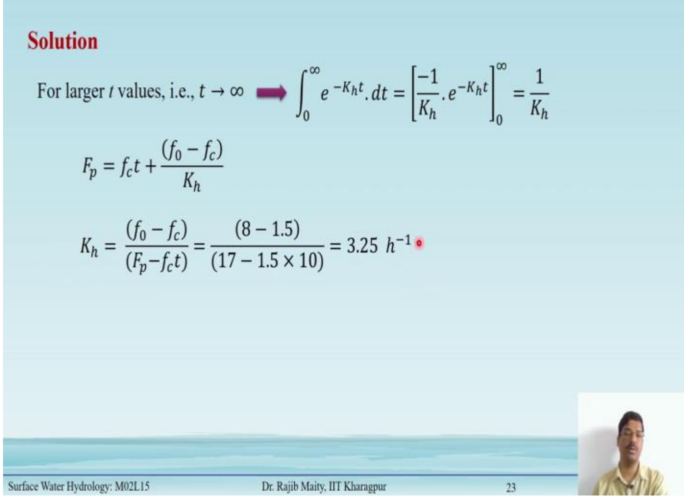

For larger t values, i.e., $t \to \infty$

$$\int_0^\infty e^{-K_h t}.dt = \left[\frac{-1}{K_h}.e^{-K_h t}\right]_0^\infty = \frac{1}{K_h}$$

$$F_p = f_c t + \frac{(f_0 - f_c)}{K_h}$$

$$K_h = \frac{(f_0 - f_c)}{(F_p - f_c t)} = \frac{(8 - 1.5)}{(17 - 1.5 \times 10)} = 3.25\ h^{-1}$$

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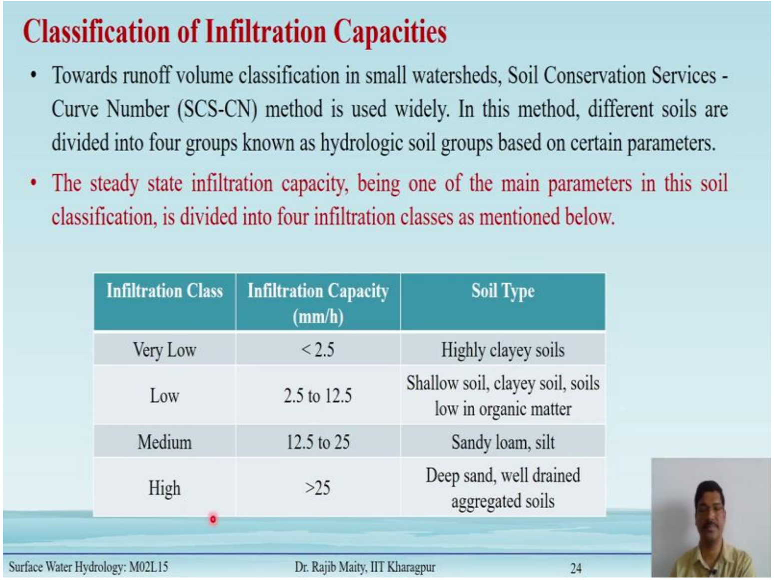

Classification of Infiltration Capacities

Towards runoff volume classification in small watersheds, Soil Conservation Services - Curve Number (SCS-CN) method is used widely. In this method, different soils are divided into four groups known as hydrologic soil groups based on certain parameters. The steady-state infiltration capacity, being one of the main parameters in this soil classification, is divided into four infiltration classes as mentioned below.

Infiltration Class	Infiltration Capacity (mm/h)	Soil Type
Very Low	< 2.5	Highly clayey soils
Low	2.5 to 12.5	Shallow soil, clayey soil, soils low in organic matter
Medium	12.5 to 25	Sandy loam, silt
High	>25	Deep sand, well drained aggregated soils

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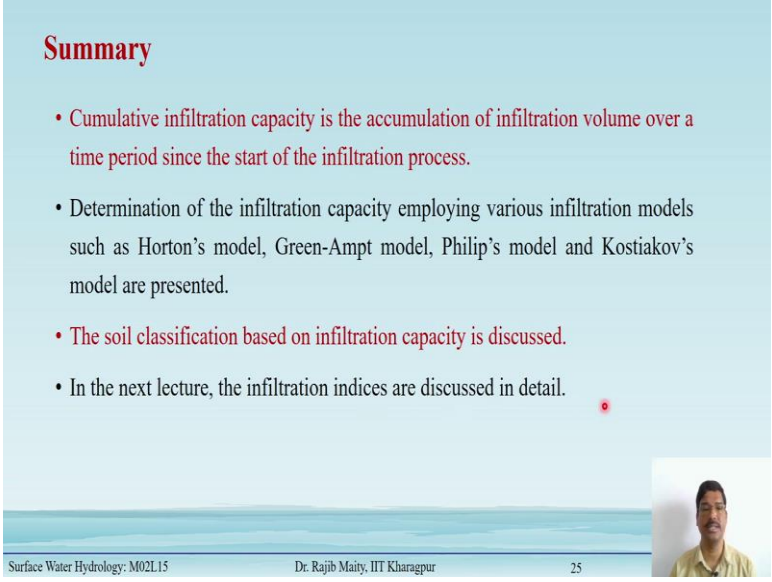

Summary

In summary, we learned the following points from this lecture:

- Cumulative infiltration capacity is the accumulation of infiltration volume over a time period since the start of the infiltration process.

- Determination of the infiltration capacity employing various infiltration models such as Horton's model, Green-Ampt model, Philip's model, and Kostiakov's model is presented.
- The soil classification based on infiltration capacity is discussed.
- In the next lecture, the infiltration indices are discussed in detail.