**Surface Water Hydrology**
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**Lecture 21**
**Stage-Discharge Relationship and Rating Curve**

In this lecture, we will discuss the state discharge relationship and rating curve, which is one of the most important techniques to be applied in the field and measuring the streamflow at different locations.

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In this lecture, the two major concepts will be covered, the first one is a measurement of the stage, and the second thing is how this stage is related to the discharge that is called the stage-discharge relationship, which is also known as the rating curve.

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The outline goes like this, the introduction, then we know how to measure the stage and there are two types of gauges are there to measure the stage, one is the non-recording type and the other one is the recording type. And then with that stage, how to develop the stage-discharge relationship. And again, there are two different categories are there, one is called the Permanent control, and the other one is called the Shifting control. And finally, we will see some summary.

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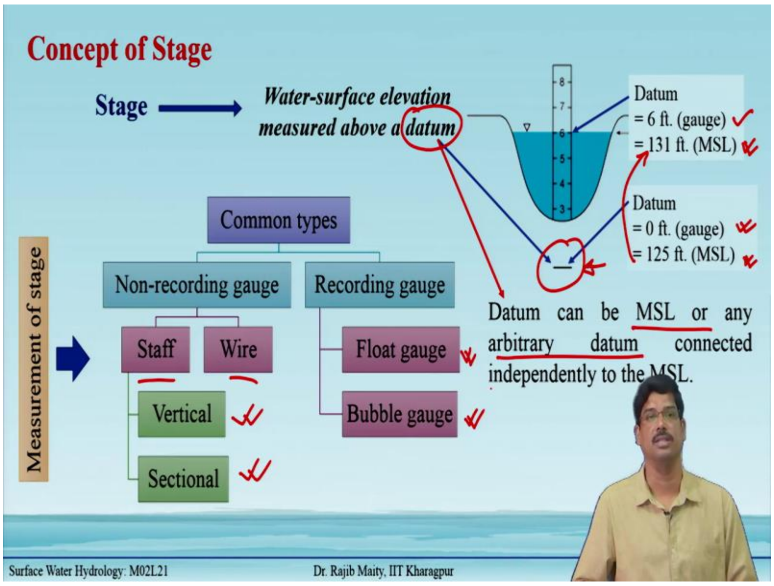


**Concept of Stage**

The stage is the water surface elevation measured above a datum. Now, this datum can be in MSL or any other arbitrary datum can also be used which is connected independently to the mean sea level as we have seen in this example.

Now, when we measure this stage, two common types are there, one is non-recording gauge and the other one is Recording gauge, under this non-recording gauge there are two things that there, one is the Staff other one is the Wire gauge. And in the staff can be again two types, one can be vertical and one can be sectional and in the recording gauge, it can be Float gauge and it can be Bubble gauge, these two can be the recording gauge under the category of recording gauge.

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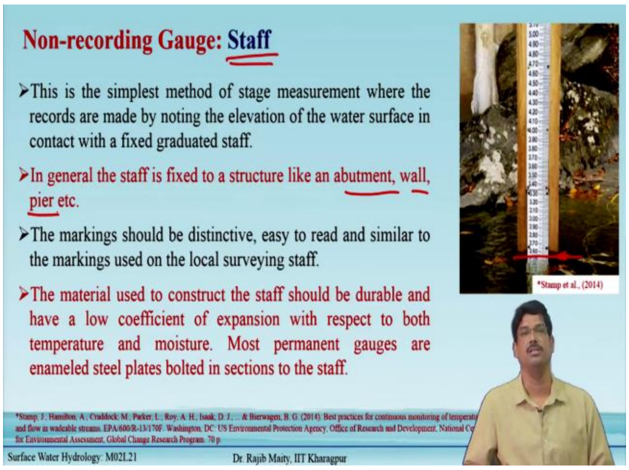


**Non-recording Gauge: Staff**

This is the simplest method of stage measurement where the records are made by noting the elevation of the water surface in contact with a fixed graduated staff. In general, the staff is fixed to a structure like an abutment, wall, pier, etc. The markings should be distinctive, easy to read, and similar to the markings used on the local surveying staff.

The material used to construct the staff should be durable and have a low coefficient of expansion with respect to both temperature and moisture. Most permanent gauges are enameled steel plates bolted in sections to the staff.

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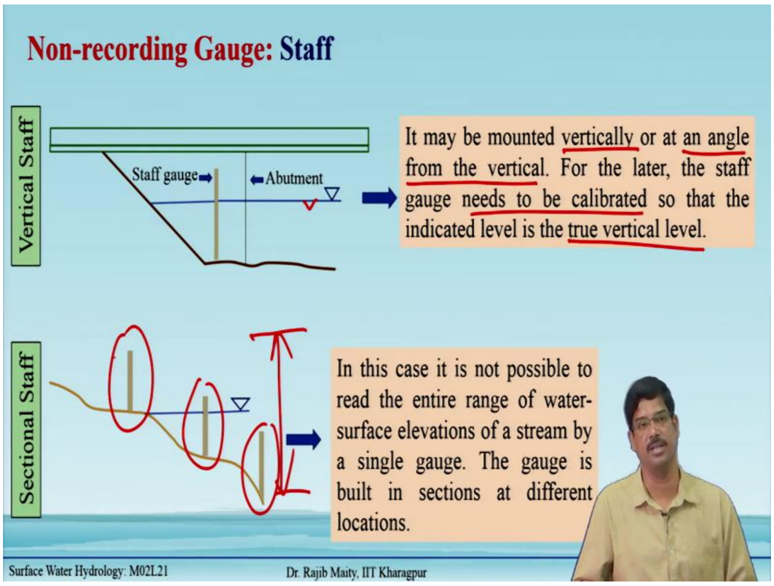

There are two types, one is the vertical staff and the second category is the sectional staff.

*Vertical Staff:* It may be mounted vertically or at an angle from the vertical. For the later, the staff gauge needs to be calibrated so that the indicated level is the true vertical level.

*Sectional Staff:* In this case, it is not possible to read the entire range of water-surface elevations of a stream by a single gauge. The gauge is built in sections at different locations.

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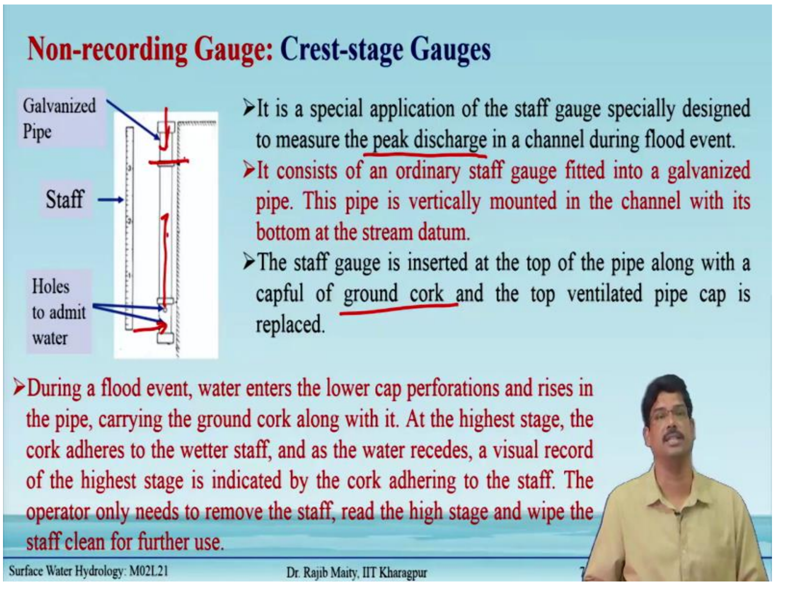


**Non-recording Gauge: Crest-stage Gauges**

It is a special application of the staff gauge specially designed to measure the peak discharge in a channel during a flood event. It consists of an ordinary staff gauge fitted into a galvanized pipe. This pipe is vertically mounted in the channel with its bottom at the stream datum.

The staff gauge is inserted at the top of the pipe along with a capful of ground cork and the top ventilated pipe cap is replaced.

During a flood event, water enters the lower cap perforations and rises in the pipe, carrying the ground cork along with-it and. At the highest stage, the cork adheres to the wetter staff, and as the water recedes, a visual record of the highest stage is indicated by the cork adhering to the staff. The operator only needs to remove the staff, read the high stage and wipe the staff clean for further use.

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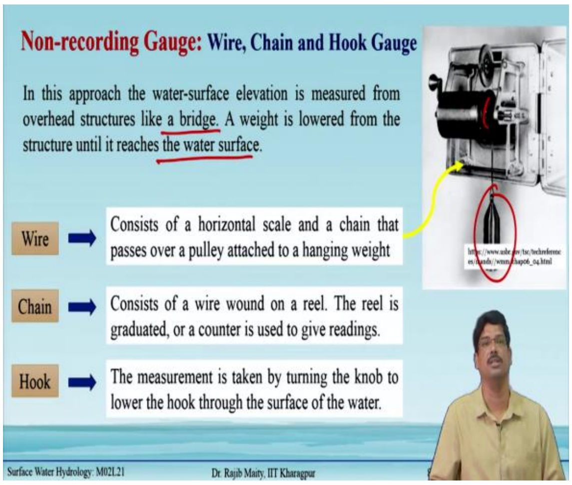


**Non-recording Gauge: Wire, Chain and Hook Gauge**

In this approach, the water-surface elevation is measured from overhead structures like a bridge. A weight is lowered from the structure until it reaches the water surface.

Another type of this Non-recording Gage is called the Wire, Chain, and Hook Gauge.

*Wire:* Consists of a horizontal scale and a chain that passes over a pulley attached to a hanging weight.

*Chain:* Consists of a wire wound on a reel. The reel is graduated, or a counter is used to give readings.

*Hook:* The measurement is taken by turning the knob to lower the hook through the surface of the water.

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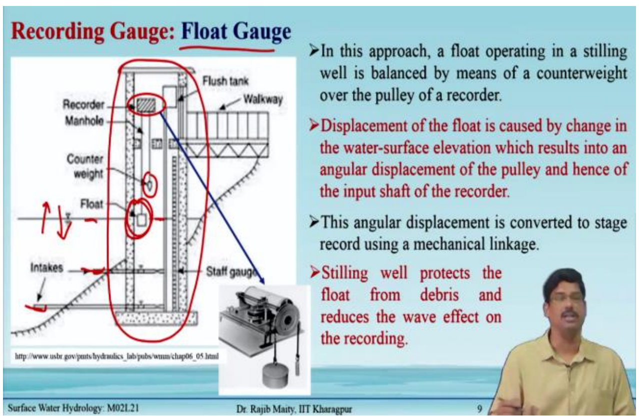

**Recording Gauge: Float Gauge**

In this approach, a float operating in a stilling well is balanced using a counterweight over the pulley of a recorder. Displacement of the float is caused by the change in the water-surface elevation which results in an angular displacement of the pulley and hence of the input shaft of the recorder.

This angular displacement is converted to a stage record using a mechanical linkage. Stilling well protects the float from debris and reduces the wave effect on the recording. It is not affected by the local waves or the debris in the flow, and you can see that there is an intake here, there is an intake at the different levels, which takes to water so that water level inside the well and outside the well is same.

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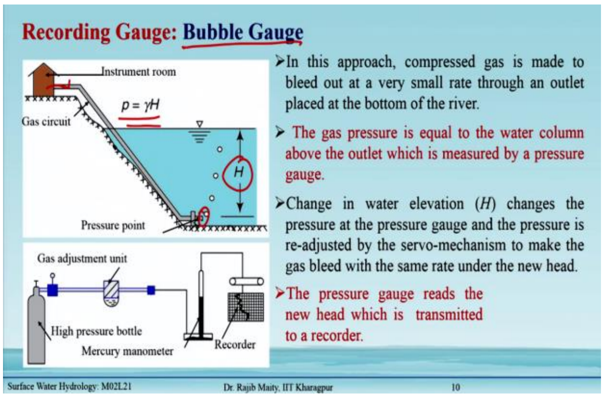


**Recording Gauge: Bubble Gauge**

The second type of recording gauge is called the Bubble gauge and it is based on the principle that how much pressure we need to put from an instrument from here to make the air bubble bleed out. If the height is more the pressure here will be more. So, if the height is less then the pressure will be less. So, there is a pressure sensor that is there to measure how much pressure is being created here and that is related to this one.

So, in this approach, the compressed gas is made to bleed out at a very small rate through an outlet placed at the bottom of the river. The gas pressure is equal to the water column above the outlet which is measured by a pressure gauge. Change in the water elevation changes the pressure at the pressure gauge and the pressure is readjusted by the servo-mechanism to make the gas bleed at the same rate under the new head. So, this pressure gauge reads a new head which is transmitted to a recorder.

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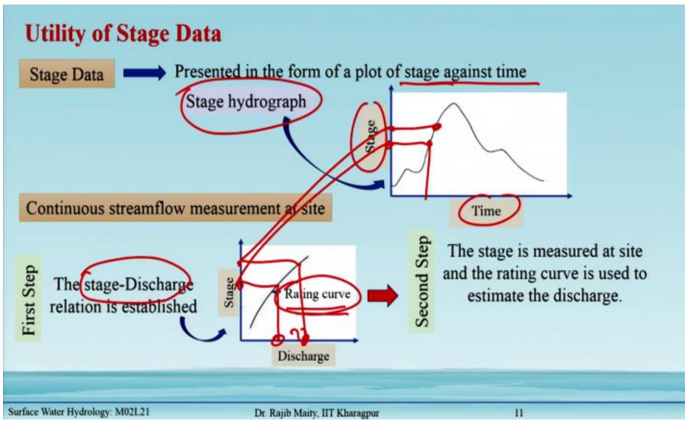


## Utility of Stage Data

When stage against time what we get is called the Stage hydrograph. The State hydrograph looks like this in the y axis we put that put the stage and the x-axis we put the time. So, now, the second thing is the continuous streamflow measurement at the site, if we need to know that for a particular stage what the discharge is for that particular stage at that particular point of time. So, we have to develop some relationship between the stage and the discharge.

The stage is measured at the site and the rating curve is used to estimate the discharge. So, this is the utility of the Rating curve that we can utilize to link the stage data to the discharge data.

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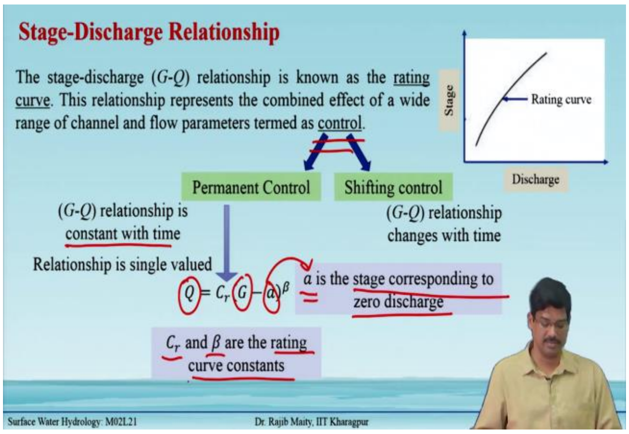

**Stage-Discharge Relationship**

The stage-discharge (G-Q) relationship is known as the rating curve. This relationship represents the combined effect of a wide range of channel and flow parameters termed as control. There are two categories, this control so far as this control is concerned one is called the Permanent control other one is called the Sifting control.

**Permanent Control**

Under this permanent control, this relationship Stage-discharge relationship is constant with the time it does not change. (G-Q) a relationship is constant with time where the discharge is Q and G is the stage and we have to just relate these two and the functional form is

$$Q = C_r(G - a)^{\beta}$$

$C_r$ and $\beta$ are the rating curve constants and $a$ are the stages corresponding to zero discharge. Now, in a site, we cannot measure it directly that this a, that what is the stage for which the discharge is 0. So, whatever the data that we have, we only prepare one thing and from there, we have to estimate what could be the value of the a.

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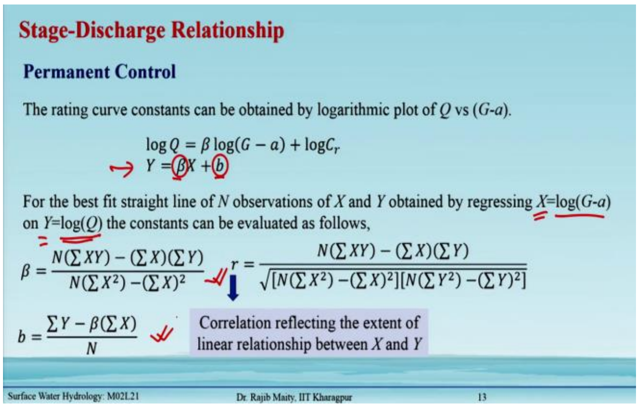


The rating curve constants can be obtained by a logarithmic plot of *Q* vs. (*G*-*a*).

$$\log Q = \beta \log(G - a) + \log C_r$$
$$Y = \beta X + b$$

For the best fit straight line of *N* observations of *X* and *Y* obtained by regressing *X*=log(*G*-*a*) on *Y*=log(*Q*), the constants can be evaluated as follows,

$$\beta = \frac{N(\sum XY) - (\sum X)(\sum Y)}{N(\sum X^2) - (\sum X)^2}$$

$$b = \frac{\sum Y - \beta(\sum X)}{N}$$

$$r = \frac{N(\sum XY) - (\sum X)(\sum Y)}{\sqrt{[N(\sum X^2) - (\sum X)^2][N(\sum Y^2) - (\sum Y)^2]}}$$

Where r is the Correlation reflecting the extent of a linear relationship between $X$ and $Y$

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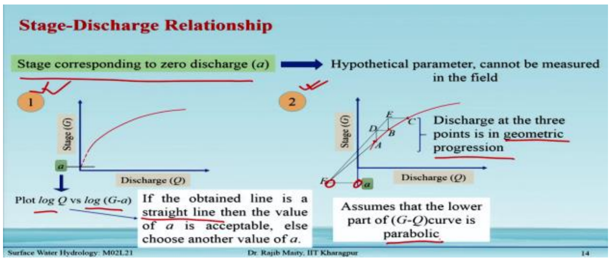


**Stage-Discharge Relationship**

To measure the stage corresponding to the 0 discharge, there are a couple of methods are there. It is a hypothetical parameter that cannot be measured in the field. So, here is the in the y axis shown in fig.1, that stage and the discharge are shown and there are different methods.

In the first method, we just extend this part in a dotted line and see where it cuts the y axis, so, that we just use this one as a provided that if we just plot this log (Q) vs. log (G – a) and if it turns out to be a straight line on a normal arithmetic graph paper, then we can say that our estimate of this a is fine here.

In the second method, we find out three different points on the curve that we got A, B, and C in such a way that they are in the geometric progression. We draw the horizontal lines and vertical lines for all these A, B, and C points and they cut E and D.

So, we join this A and B and extended it to the backward D and E we extend to the backward there are some points F where these two straight-line cuts, and this y-axis intercept we use as an a. So, in this process, it is assumed that the lower part of this rating curve is parabolic, and based on that one we can get this value of a.

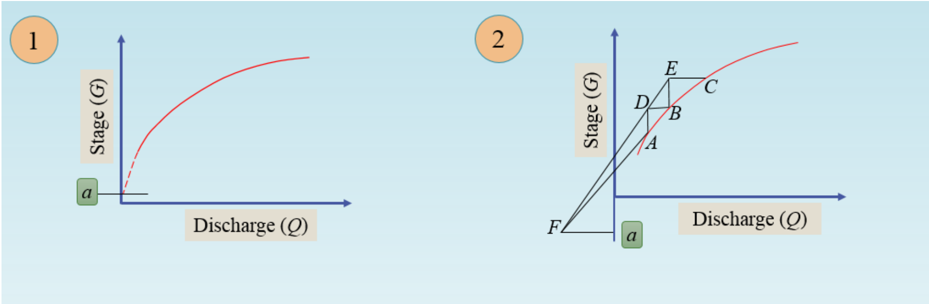


Fig.1 shows the stage-discharge graph for calculation of a

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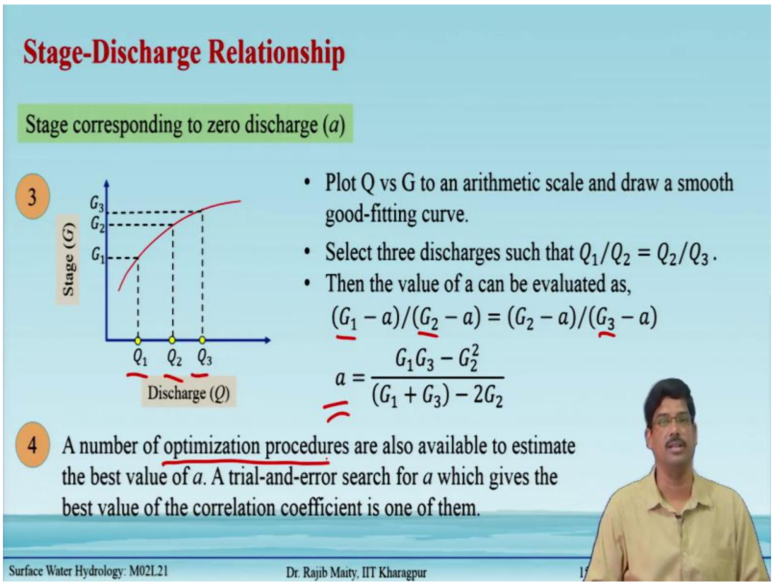


## Stage corresponding to zero discharge (*a*)

We plot this Q and G as shown in fig.2 and then we select three different discharges this $Q_1$, $Q_2$, and $Q_3$ in such a way that they are in the geometric progression that is the relationship that $Q_1/Q_2$ goes to $Q_2/Q_3$.

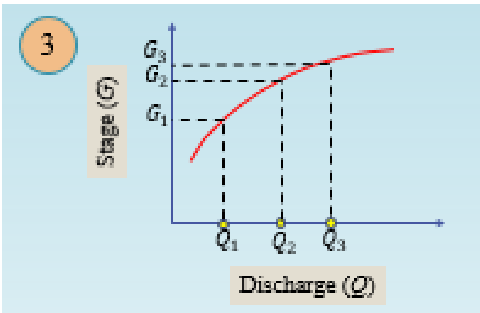


Fig.2 shows the stage-discharge relationship

Then the value of a can be evaluated as,

$$(G_1 - a)/(G_2 - a) = (G_2 - a)/(G_3 - a)$$

$$a = \frac{G_1 G_3 - G_2^2}{(G_1 + G_3) - 2G_2}$$

Many optimization procedures are also available to estimate the best value of $a$. A trial-and-error search for $a$ which gives the best value of the correlation coefficient is one of them.

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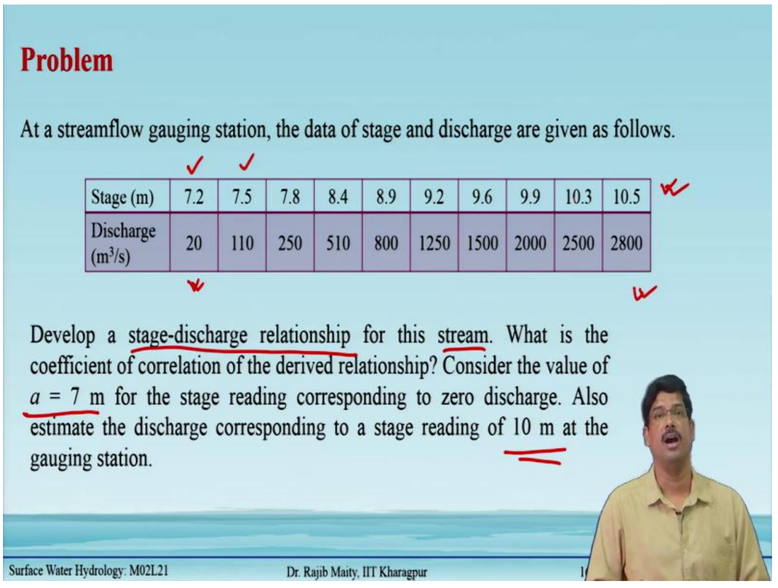


**Problem**

At a streamflow gauging station, the data of stage and discharge are given as follows.

| Stage (m) | 7.2 | 7.5 | 7.8 | 8.4 | 8.9 | 9.2 | 9.6 | 9.9 | 10.3 | 10.5 |
|---|---|---|---|---|---|---|---|---|---|---|
| Discharge ($m^3/s$) | 20 | 110 | 250 | 510 | 800 | 1250 | 1500 | 2000 | 2500 | 2800 |

Develop a stage-discharge relationship for this stream. What is the coefficient of correlation of the derived relationship? Consider the value of $a$ = 7 m for the stage reading corresponding to zero discharge. Also, estimate the discharge corresponding to a stage reading of 10 m at the gauging station.

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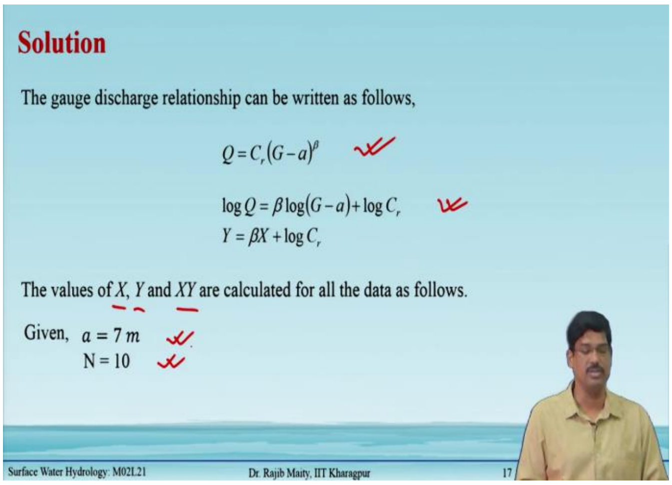


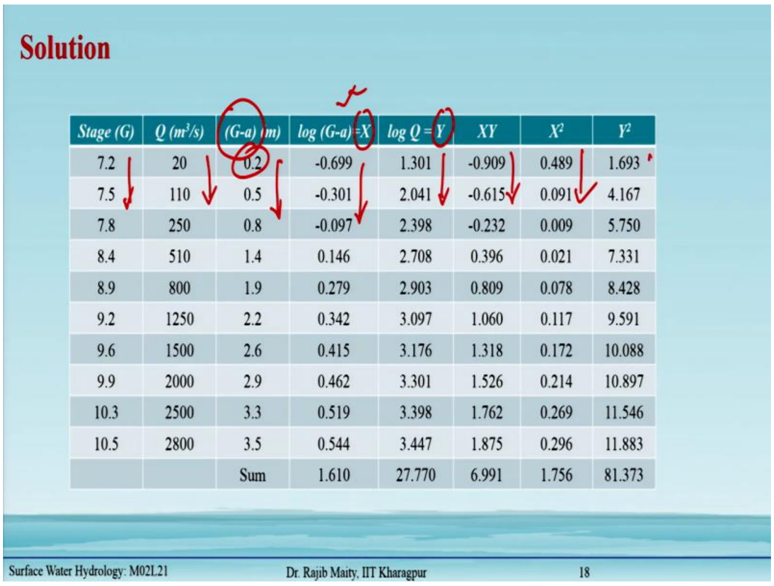
Solution

| Stage (G) | Q ($m^3$/s) | (G-a) (m) | log (G-a) = X | log Q = Y | XY | $X^2$ | $Y^2$ |
|---|---|---|---|---|---|---|---|
| 7.2 | 20 | 0.2 | -0.699 | 1.301 | -0.909 | 0.489 | 1.693 |
| 7.5 | 110 | 0.5 | -0.301 | 2.041 | -0.615 | 0.091 | 4.167 |
| 7.8 | 250 | 0.8 | -0.097 | 2.398 | -0.232 | 0.009 | 5.750 |
| 8.4 | 510 | 1.4 | 0.146 | 2.708 | 0.396 | 0.021 | 7.331 |
| 8.9 | 800 | 1.9 | 0.279 | 2.903 | 0.809 | 0.078 | 8.428 |
| 9.2 | 1250 | 2.2 | 0.342 | 3.097 | 1.060 | 0.117 | 9.591 |
| 9.6 | 1500 | 2.6 | 0.415 | 3.176 | 1.318 | 0.172 | 10.088 |
| 9.9 | 2000 | 2.9 | 0.462 | 3.301 | 1.526 | 0.214 | 10.897 |
| 10.3 | 2500 | 3.3 | 0.519 | 3.398 | 1.762 | 0.269 | 11.546 |
| 10.5 | 2800 | 3.5 | 0.544 | 3.447 | 1.875 | 0.296 | 11.883 |
| | | Sum | 1.610 | 27.770 | 6.991 | 1.756 | 81.373 |



**Solution**

The gauge discharge relationship can be written as follows,

$$Q = C_r (G-a)^{\beta}$$

$$\log Q = \beta \log(G-a) + \log C_r$$

$$Y = \beta X + \log C_r$$

The values of *X*, *Y*, and *XY* are calculated for all the data as follows.

Given, a= 7m

N= 10

| Stage (G) | Q ($m^3/s$) | (G-a) (m) | log (G-a)=X | log Q = Y | XY | $X^2$ | $Y^2$ |
|---|---|---|---|---|---|---|---|
| 7.2 | 20 | 0.2 | -0.699 | 1.301 | -0.909 | 0.489 | 1.693 |
| 7.5 | 110 | 0.5 | -0.301 | 2.041 | -0.615 | 0.091 | 4.167 |
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| 10.5 | 2800 | 3.5 | 0.544 | 3.447 | 1.875 | 0.296 | 11.883 |
| | | Sum | 1.610 | 27.770 | 6.991 | 1.756 | 81.373 |

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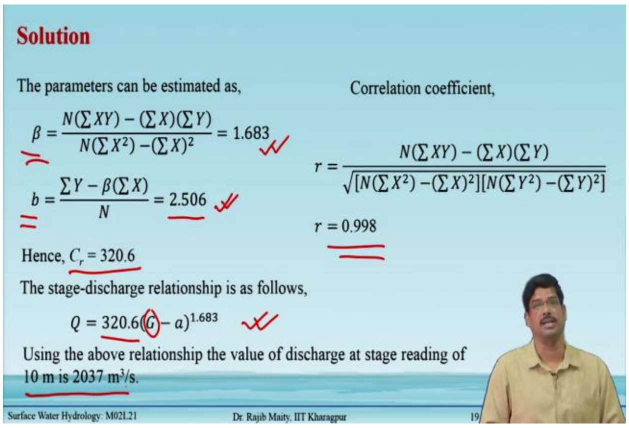


The parameters can be estimated as,

$$\beta = \frac{N(\sum XY) - (\sum X)(\sum Y)}{N(\sum X^2) - (\sum X)^2} = 1.683$$

$$b = \frac{\sum Y - \beta(\sum X)}{N} = 2.506$$

Correlation coefficient,

$$r = \frac{N(\sum XY) - (\sum X)(\sum Y)}{\sqrt{[N(\sum X^2) - (\sum X)^2][N(\sum Y^2) - (\sum Y)^2]}}$$

$$r = 0.998$$

Hence, $C_r = 320.6$

The stage-discharge relationship is as follows,

$$Q = 320.6(G - a)^{1.683}$$

Using the above relationship, the value of discharge at a stage reading of 10 m is 2037 $m^3/s$.

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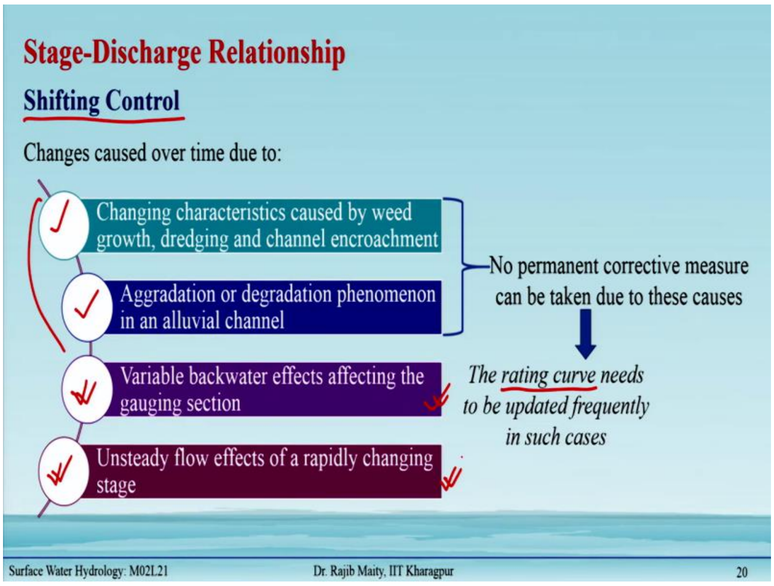


**Stage-Discharge Relationship**

**Shifting Control**

In the permanent control, the stage-discharge relationship rating curve is in a single line. Now, when it is a shifting control that relationship need not be unique and different causes are there which basically in which leads to the sifting control. Here are the four major reasons :

- Changing characteristics caused by weed growth, dredging, and channel encroachment
- Aggradation or degradation phenomenon in an alluvial channel
- Variable backwater effects affecting the gauging section
- Unsteady flow effects of a rapidly changing stage

So far as the first two things are there two are there is no such permanent corrective measures are there. We need to redevelop the rating curve again and again and we need to update it as when the cross-sectional property changes we have to redevelop the rating column. So, far as the third and the fourth is concerned there are some mathematical approaches to take care of that sifting control.

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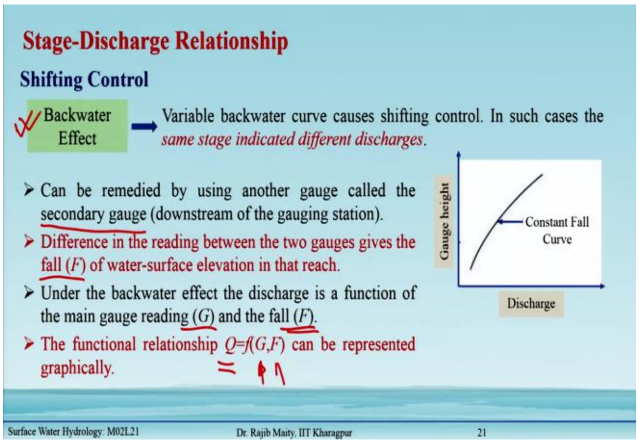


*Backwater effect:* There are some structures downstream that are causing some sort of backwater effect and that is causing the shifting control in the stage-discharge relationship. So, variable backwater curves cause the sifting control in such cases, the same stage indicates different discharges. In this case, another gauge that is called the secondary gauge is also installed along with the main primary engage.

The secondary gauge is generally downstream of the primary gauging station the difference between the reading between the two gauges gives the fall between two gauges that is F of the water surface elevation in that reach. Under the backwater effect, the discharge is a function of the main gauge reading (G) and the fall (F). Now, the functional relationship that is the Q is not only the function of the stage of the primary gauge, it is also a function of the fall. So, this can be presented graphically.

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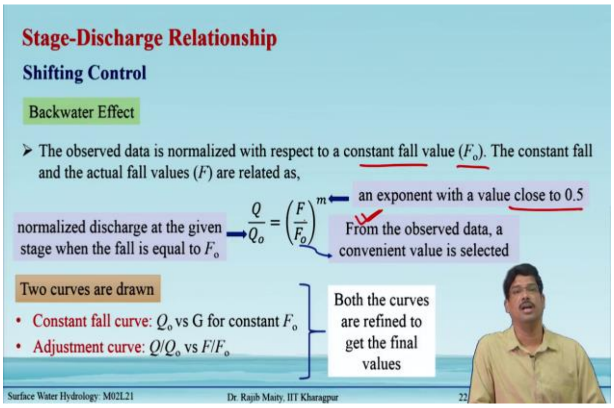

The observed data is normalized with respect to a constant fall value (Fo). The constant fall and the actual fall values (F) are related as,

$$\frac{Q}{Q_o} = \left(\frac{F}{F_o}\right)^m$$

Where, $F_{o=}$ From the observed data, a convenient value is selected

$Q_o$= Normalized discharge at the given stage when the fall is equal to $F_o$

m= an exponent with a value close to 0.5

So, we know a particular fall $F_o$ which is from the observed data a convenient value is selected and we develop this kind of relationship, that is non dimensionalized both the side with respect to a, in respect to a fixed discharge and the fixed fall for the observed from the observed record. And now, there are two curves, one is that constant fall curve $Q_0$ vs. G for the constant $F_0$ and adjustment curve that is Q /$Q_0$ versus F / $F_0$ and both the curves are refined to get the final values.

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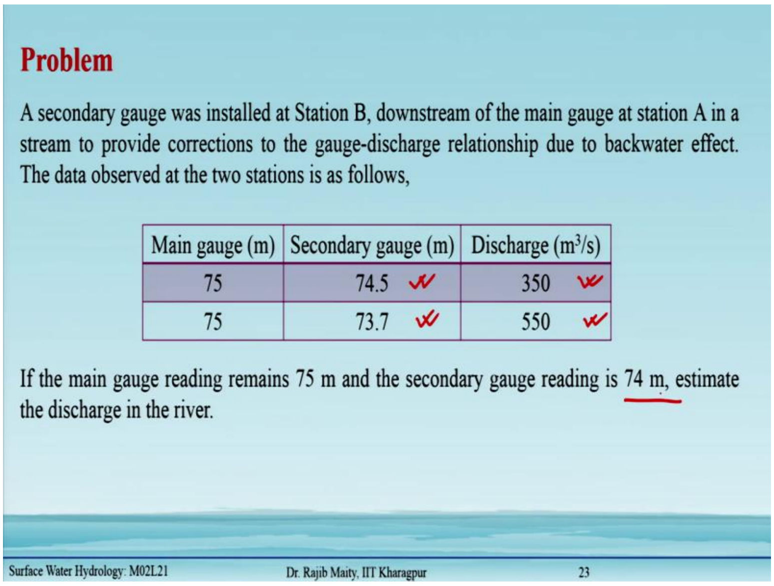


**Problem**

A secondary gauge was installed at Station B, downstream of the main gauge at station A in a stream to provide corrections to the gauge-discharge relationship due to the backwater effect. The data observed at the two stations is as follows,

| Main gauge (m) | Secondary gauge (m) | Discharge ($m^3/s$) |
|---|---|---|
| 75 | 74.5 | 350 |
| 75 | 73.7 | 550 |

If the main gauge reading remains 75 m and the secondary gauge reading is 74 m, estimate the discharge in the river.

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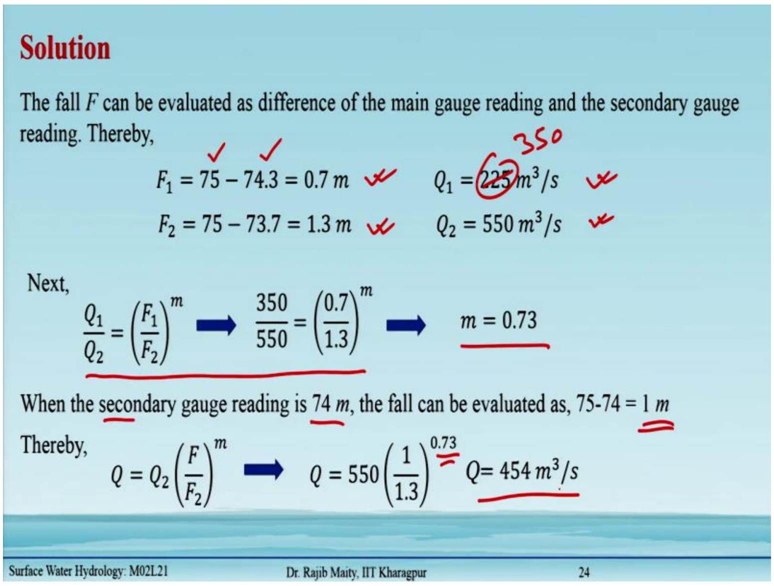


**Solution**

The fall $F$ can be evaluated as the difference between the main gauge reading and the secondary gauge reading. Thereby,

$$F_1 = 75 - 74.3 = 0.7\ m \qquad\qquad Q_1 = 225\ m^3/s$$
$$F_2 = 75 - 73.7 = 1.3\ m \qquad\qquad Q_2 = 550\ m^3/s$$

Next,

$$\frac{Q_1}{Q_2} = \left(\frac{F_1}{F_2}\right)^m \Rightarrow \frac{350}{550} = \left(\frac{0.7}{1.3}\right)^m \Rightarrow m = 0.73$$

When the secondary gauge reading is 74 $m$, the fall can be evaluated as 75-74 = 1 $m$

Thereby,

$$Q = Q_2\left(\frac{F}{F_2}\right)^m \Rightarrow Q = 550\left(\frac{1}{1.3}\right)^{0.73} \quad Q = 454\ m^3/s$$

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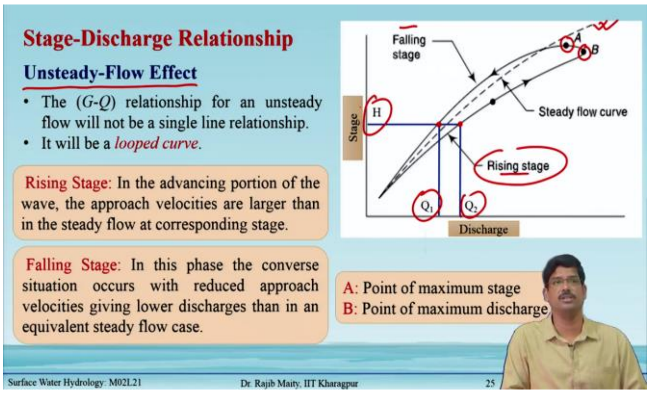


## Unsteady-Flow Effect

Next comes the shifting control due to unsteady flow effects. Unsteady flow effect means when this stage-discharge relationship for an unsteady flow will not be a single line relationship. So, when the sum flood is coming and passing through a stage, there will not be a single line as we have seen in the dotted line is the single line relationship. Whereas, in the case of the unsteady flow during a flood, it can looped curve as it is shown in fig.3.

So, in the rising stage the advancing portion of this web, the approach velocities are larger than in case of the steady flow at the corresponding stage and in the falling stage when in this phase if the converse situation occurs that is reduced approach velocity giving the lower discharge than in an equivalent steady flow state. In fig.3, there are some points A and B. This point B is the point of maximum discharge and point A which is the point of the maximum stage.

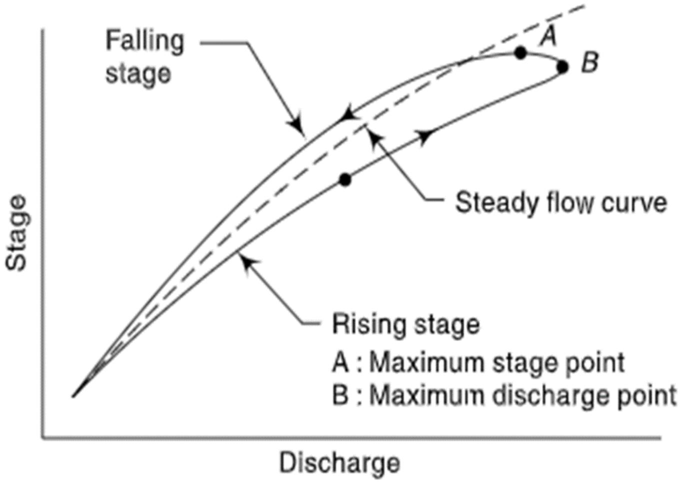


Fig.3 shows the loop rating curve

Now, if I just take a stage H, it cuts the looped curve in two points. One is that $Q_1$ other one is a $Q_2$ which means, this is the lower one is rising in stage, so, in the rising states the discharge is higher and for the fallings stage when it is the when it is receding, that time the discharge is lower for the same stage. So, this can happen during a flood flooding event. This is called the looped rating curve.

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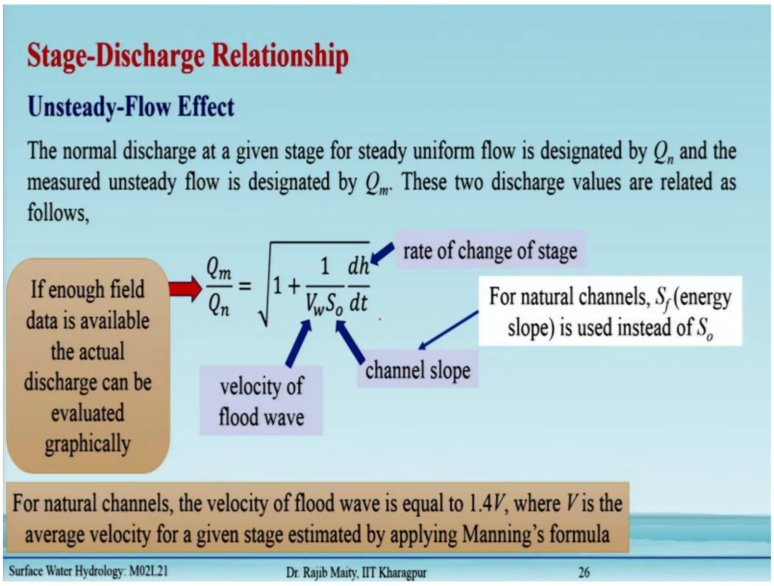


The normal discharge at a given stage for steady uniform flow is designated by $Q_n$ and the measured unsteady flow is designated by $Q_m$. These two discharge values are related as follows,

$$\frac{Q_m}{Q_n} = \sqrt{1 + \frac{1}{V_w S_o}\frac{dh}{dt}}$$

Where, $V_w$= velocity of the flood wave

$S_o$ = channel slope

$dh/dt$ = rate of change of stage

For natural channels, the velocity of the flood wave is equal to 1.4$V$, where $V$ is the average velocity for a given stage estimated by applying Manning's formula. However, in enough field data is available the actual discharge can be evaluated graphically from that expression.

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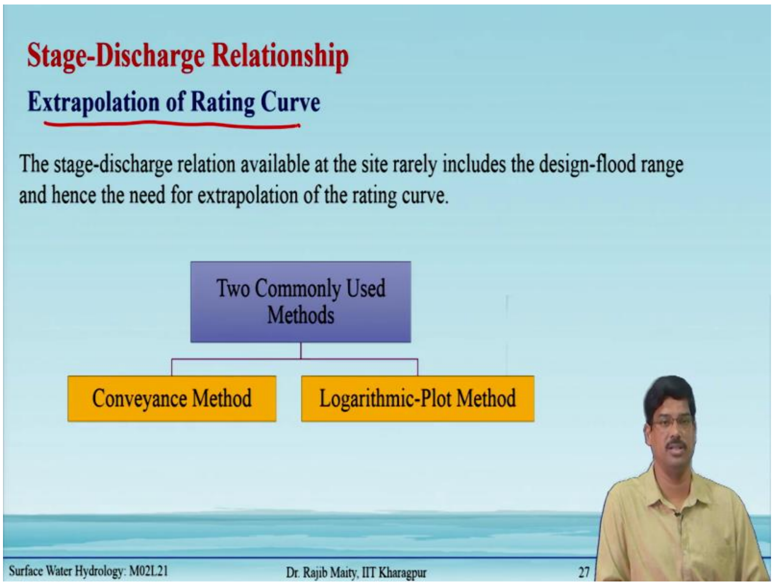


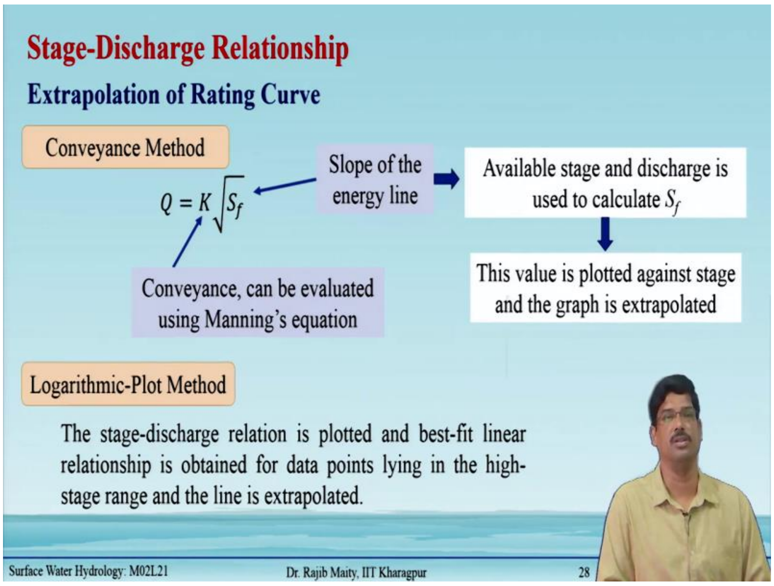


**Extrapolation of Rating Curve**

The Extrapolation of the rating curve in the case of any studies, we have to practice some sort of caution, we cannot extend it we cannot extrapolate it without knowing the ground condition. So, there is a thorough knowledge of the site condition is needed before we extrapolate any rating curve.

Stage-discharge relation available at the site rarely includes the design-flood range, so, we sometimes need to extend it to get for some very extreme flow cases. There are two commonly used methods are there, one is the conveyance method other one is the logarithmic-plot method.

**Conveyance Method**

$$Q = K\sqrt{S_f}$$

Where $S_f$ = The slope of the energy line

K = The conveyance can be calculated using manning's equation.

And the slope is available stage-discharge is used to calculate this $S_f$ and this value is plotted against the graph and that graph is being extrapolated to find out the value which is beyond the range when need to when the rating curve was developed.

**Logarithmic-Plot Method**

The stage-discharge relation is plotted and a best-fit linear relationship is obtained for data points lying in the high-stage range and the line is extrapolated

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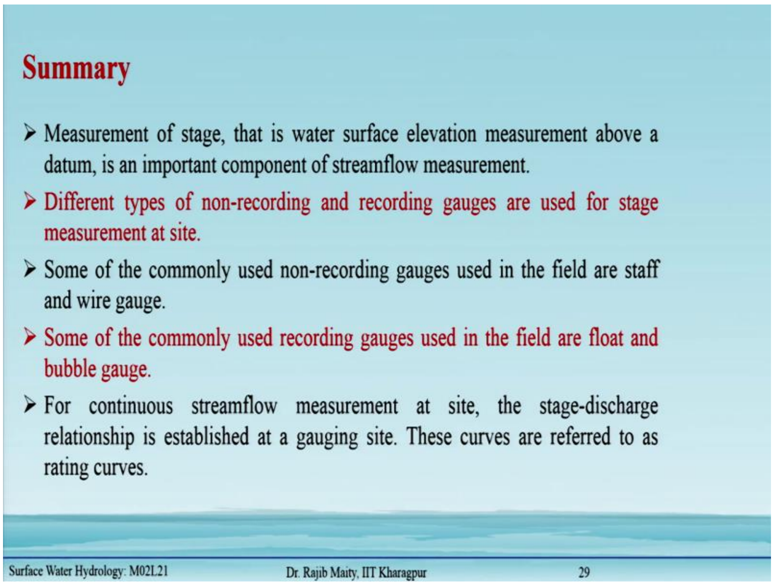


**Summary**

In summary, we learned the following points from this lecture:

- Measurement of the stage, that is water surface elevation measurement above a datum, is an important component of streamflow measurement.

- Different types of non-recording and recording gauges are used for stage measurement at the site.
- Some of the commonly used non-recording gauges used in the field are staff and wire gauges.
- Some of the commonly used recording gauges used in the field are float and bubble gauges.
- For continuous streamflow measurement at the site, the stage-discharge relationship is established at a gauging site. These curves are referred to as rating curves.