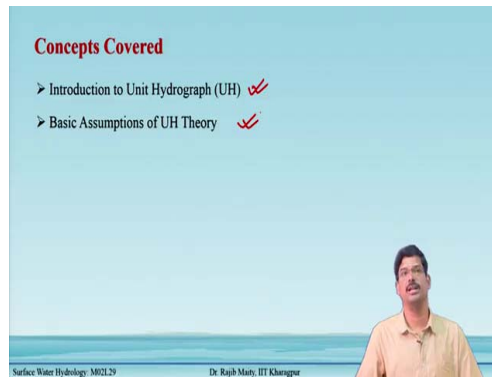


Surface Water Hydrology
Professor Rajib Maity
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture – 29
Introduction to Unit Hydrographs

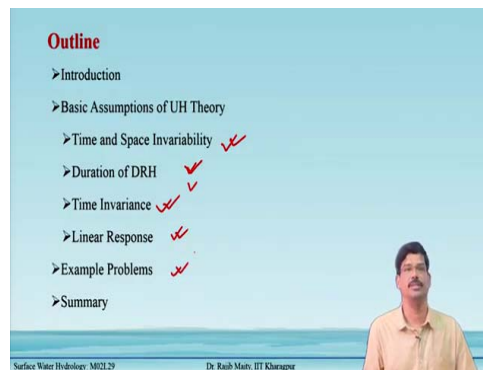
In this lecture, we will give the Introduction to Unit Hydrograph.

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So, under this concept covered two things will be considered the first one is an introduction to unit hydrograph and the second one there are some basic assumptions are there on which the unit hydrograph theory is based.

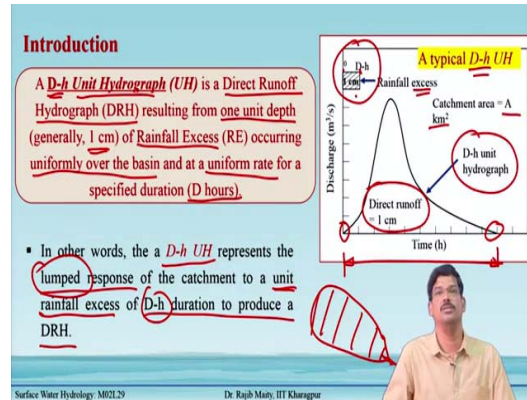
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The outline for this lecture is first some introduction then basic assumptions of the unit hydrograph theory that includes different things specifically for specific things. The first one is the time and space in variability. The second one is the duration of the direct runoff hydrograph

the third one is the time invariance and then its linear response. We will learn these background assumptions through some example problems before going to the summary.

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Introduction

A D hour unit hydrograph is a direct runoff hydrograph resulting from one-unit depth generally one centimeter of rainfall excess occurring uniformly over the basin and at a uniform rate for the specified duration of D hours.

In the D-hour unit hydrograph, D can be 2 hours, 3 hours, 4 hours, 8 hours, 12 hours. After separating the base flow whatever is there that we call the DRH. Now, DRH always corresponds to some effective rainfall hydrograph. The DRH is resulting from the one-unit depth of rainfall excess generally consider always as 1 centimeter of rainfall excess that is not the end, two more things after this is occurring uniformly over the basin and at a uniform rate throughout this duration of D hours.

Again, considering the all-other properties of this hydrograph there is a rising limb there is a crest segment there is a falling limb, but the starting and ending at 0 give a completely enclosed area with respect to the time axis which is in the horizontal axis. So, below this curve up to this time axis the direct runoff that comes that is that should be equal to the effective rainfall hydrograph.

So, here another popular interpretation is that the D hour unit hydrograph represents the lump response of the catchment to a unit rainfall excess of D hour duration to produce a DRH.

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Introduction

- It relates only direct runoff to the rainfall excess. Since only 1 cm depth of rainfall excess is considered, the area of the unit hydrograph is equal to a volume given by 1 cm over the catchment.
- The rainfall is considered to have a constant average intensity of excess rainfall of 1/D cm/h for the duration of D-h storm.
- The spatial distribution of storm is considered to be uniform all over the catchment.

A typical D-h UH graph shows Discharge (m³/s) on the y-axis and Time (h) on the x-axis. The curve is labeled 'D-h unit hydrograph'. A horizontal line at the top of the curve is labeled 'Rainfall excess' with a height of '1 cm'. The area under this line is labeled 'Catchment area = A km²'. The duration of the storm is labeled 'D-h storm' and 'Direct runoff'.

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It relates only direct runoff to the rainfall excess. Since only 1 cm depth of rainfall excess is considered, the area of the unit hydrograph is equal to a volume given by 1 cm over the catchment.

The rainfall is considered to have a constant average intensity of excess rainfall of $1/D$ cm/h for the duration of the D-h storm. The spatial distribution of storms is considered to be uniform all over the catchment.

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Basic Assumptions of UH Theory

- Time invariability of Rainfall**
This assumption states that the effective rainfall or rainfall excess is occurring at a constant intensity ($1/D$ cm/h) during the D-h duration for a D-h UH.
- Spatial invariability of Rainfall**
This assumption states that the effective rainfall or rainfall excess is uniformly distributed over the entire basin.
- Time base of the DRH**
This assumption states that the DRH is dependent only on the duration of the effective rainfall and independent of the intensity of effective rainfall.

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Basic Assumptions of UH Theory

➤ Time invariability of Rainfall

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➤ The spatial invariability of Rainfall

This assumption states that the effective rainfall or rainfall excess is uniformly distributed over the entire basin.

➤ **The time base of the DRH**

This assumption states that the DRH is dependent only on the duration of the effective rainfall and independent of the intensity of effective rainfall.

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Basic Assumptions of UH Theory

- **Time Invariance**
This assumption states that the UH will remain same irrespective of when the effective rainfall occurs (i.e. any season, month, day or year) over the basin. Thus, it can be said that direct runoff response to a given effective rainfall in a catchment is time invariant.
- **Linear Response**
The direct runoff response to the rainfall excess is assumed to be linear. Linear response can be explained as follows:
 - If an input $x_1(t)$ causes an output $y_1(t)$ and $x_2(t)$ causes an output $y_2(t)$ then, an input $x_1(t) + x_2(t)$ gives an output $y_1(t) + y_2(t)$.
 - If an input $x_1(t)$ causes an output $y_1(t)$, then $Cx_1(t)$ causes an output $Cy_1(t)$.

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➤ **Time Invariance**

This assumption states that the UH will remain the same irrespective of when the effective rainfall occurs (i.e., any season, month, day, or year) over the basin. Thus, it can be said that direct runoff response to a given effective rainfall in a catchment is time-invariant.

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If an input $x_1(t)$ causes an output $y_1(t)$, then $Cx_1(t)$ causes an output $Cy_1(t)$.

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Basic Assumptions of UH Theory

- Principle of Proportionality**
 This assumption states that the ordinate of the DRH are proportional to the magnitude of rainfall.

Thus, if rainfall excess in a D -h duration is r times the unit depth, the resulting DRH will have ordinates bearing ratio r to those of the corresponding D -h unit hydrograph. Also, the base of the DRH will be same as that of the unit hydrograph since the area of DRH will increase by the ratio r .

So, if $x_2(t) = r(x_1(t))$, then $y_2(t) = r(y_1(t))$

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➤ **Principle of Proportionality**

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$$\text{So, if } x_2(t) = r(x_1(t)), \text{ then } y_2(t) = r(y_1(t))$$

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Example 29.1:

The ordinates of 8-h Unit Hydrograph (UH) of a catchment are as follows:

Time (h)	0	4	8	12	16	20	24	32	40	48	56	64	72	80	92
UH ordinates (m ³ /s)	0	35	60	95	135	170	195	170	120	70	46	35	26	18	0

Calculate the ordinate of the DRH that will result from an excess rainfall of 2.5 cm in 8 hours.

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Example 29.1:

The ordinates of an 8-h Unit Hydrograph (UH) of a catchment are as follows:

Time (h)	0	4	8	12	16	20	24	32	40	48	56	64	72	80	92
UH ordinates (m ³ /s)	0	35	60	95	135	170	195	170	120	70	46	35	26	18	0

Calculate the ordinate of the DRH that will result from an excess rainfall of 2.5 cm in 8 hours.

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Solution

The ordinates of DRH are obtained by multiplying the ordinate of the 8-h UH by a factor of 2.5 as shown in Table.

Following observation can be made from the table,

- The intervals of the ordinates of the UH are not related to the duration of the rainfall excess in anyway and can be any convenient value.

Time (h)	Ordinates of 8-h UH (m ³ /s)	Ordinates of 2.5 cm DRH (m ³ /s)
0	0	0
4	35	87.5
8	60	150
12	95	237.5
16	135	337.5
20	170	425
24	195	487.5
32	170	425
40	120	300
48	70	175
56	46	115
64	35	87.5
72	26	65
80	18	45
92	0	0

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Solution

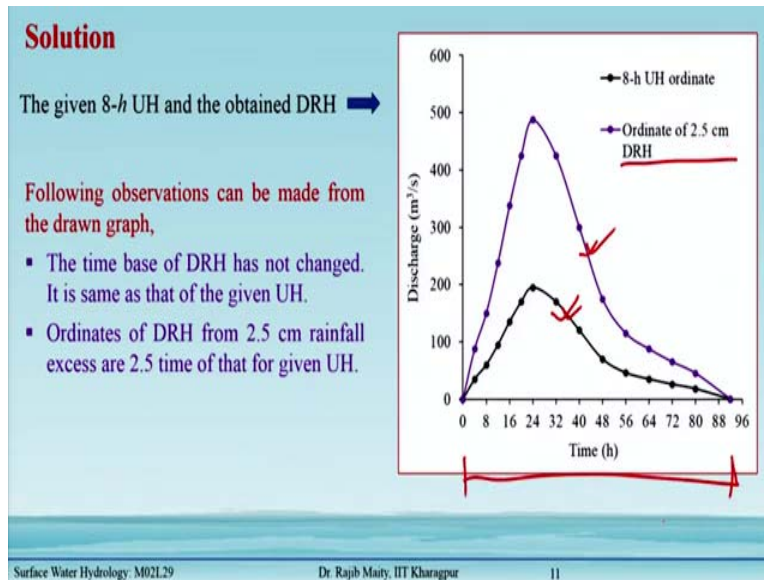
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48	70	175
56	46	115
64	35	87.5
72	26	65
80	18	45
92	0	0

Following observation can be made from the table,

The intervals of the ordinates of the UH are not related to the duration of the rainfall excess in any way and can be any convenient value.

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The given 8-h UH and the obtained DRH

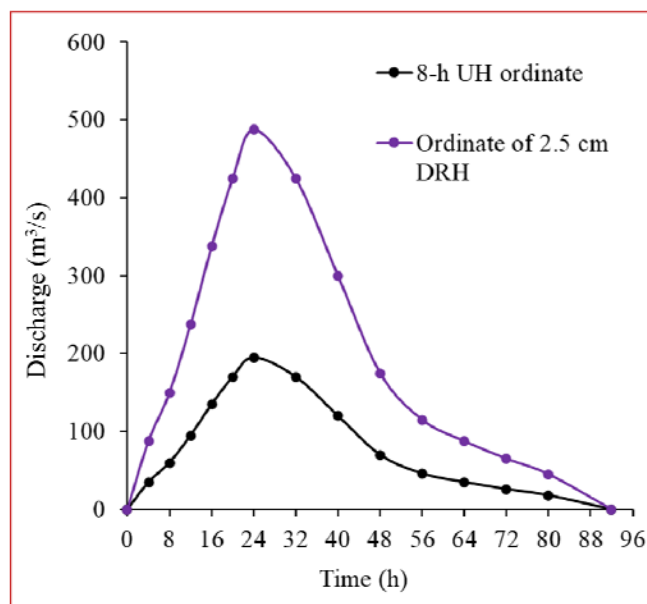


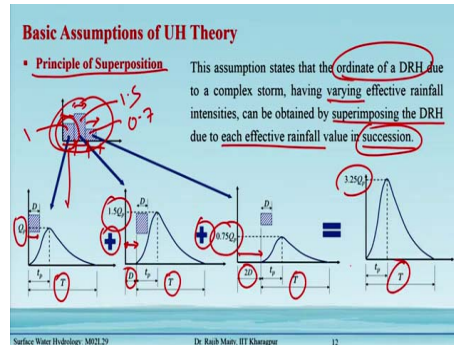
Fig.1 shows the UH of example 29.1

Following observations can be made from the drawn graph,

The time base of DRH has not changed. It is the same as that of the given UH.

Ordinates of DRH from 2.5 cm rainfall excess are 2.5 times that for given UH.

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➤ Principle of Superposition

This assumption states that the ordinate of a DRH due to a complex storm, having varying effective rainfall intensities, can be obtained by superimposing the DRH due to each effective rainfall value in succession.

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Example 29.2:

The ordinates of a 8-h unit hydrograph (UH) of a catchment are as follows

Time (h)	0	4	8	12	16	20	24	32	40	48	56	64	72	80	92
UH ordinates (m ³ /s)	0	35	60	95	135	170	195	170	120	70	46	35	26	18	0

Calculate the ordinate of the DRH resulting from an excess rainfall of 5.0 cm and 2.0 cm, each of 8-h duration, occurring successively.

Example 29.2:

The ordinates of an 8-h unit hydrograph (UH) of a catchment are as follows

Time (h)	0	4	8	12	16	20	24	32	40	48	56	64	72	80	96
UH ordinates (m ³ /s)	0	35	60	95	135	170	195	170	120	70	46	35	26	18	0

Calculate the ordinate of the DRH resulting from an excess rainfall of 5.0 cm and 2.0 cm, each of 8-h duration, occurring successively.

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Solution

- It may be noted that 2.0 cm DRH occurs after 5.0 cm DRH.
- Therefore, the ordinates of 5.0 cm DRH will be lagged (delayed) by 8-h as shown in the table.

Time (h)	Ordinates of 8-h UH (m ³ /s)	Ordinates of 5.0 cm DRH (m ³ /s)	Ordinates of 2.0 cm DRH lagged by 8-h (m ³ /s)
0	0	0	--
4	35	175	--
8	60	300	0
12	95	475	70
16	135	675	120
20	170	850	190
24	195	975	270
32	170	850	340
40	120	600	390
48	70	350	340
56	46	230	240
64	35	175	140
72	26	130	92
80	18	90	70
92	0	0	52
100	--	--	36
108	--	--	0

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Solution

It may be noted that 2.0 cm DRH occurs after 5.0 cm DRH.

Therefore, the ordinates of 5.0 cm DRH will be lagged (delayed) by 8-h as shown in the table.

Time (h)	Ordinates of 8-h UH (m ³ /s)	Ordinates of 5.0 cm DRH (m ³ /s)	Ordinates of 2.0 cm DRH lagged by 8-h (m ³ /s)
0	0	0	-
4	35	175	-
8	60	300	0
12	95	475	70
16	135	675	120
20	170	850	190
24	195	975	270
28	183	913	340
32	170	850	390
40	120	600	340
48	70	350	240
56	46	230	140
64	35	175	92
72	26	130	70
80	18	90	52
88	6	30	36
96	0	0	12
104	-	-	0

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Solution

Now, using the principle of superposition the resulting DRH from the two successive excess rainfalls are obtained by adding the ordinates of both the DRHs, maintaining the time correspondence.

Time (h)	Ordinates of 8-h UH (m ³ /s)	Ordinates of 5.0 cm DRH (m ³ /s)	Ordinates of 2.0 cm DRH lagged by 8-h (m ³ /s)	Resulting ordinates of 7.0 cm DRH (m ³ /s)
0	0	0	--	0
4	35	175	--	175
8	60	300	0	300
12	95	475	70	545
16	135	675	120	795
20	170	850	190	1040
24	195	975	270	1245
28	183	913	340	1253
32	170	850	390	1240
40	120	600	340	940
48	70	350	240	590
56	46	230	140	370
64	35	175	92	267
72	26	130	70	200
80	18	90	52	142
88	9	45	36	66
96	0	0	12	12
100	--	--	36	36
108	--	--	0	0

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Now, using the principle of superposition the resulting DRH from the two successive excess rainfalls are obtained by adding the ordinates of both the DRHs, maintaining the time correspondence.

Time (h)	Ordinates of 8-h UH (m ³ /s)	Ordinates of 5.0 cm DRH (m ³ /s)	Ordinates of 2.0 cm DRH lagged by 8-h (m ³ /s)	Resulting ordinates of 7.0 cm DRH (m ³ /s)
0	0	0	-	0
4	35	175	-	175
8	60	300	0	300
12	95	475	70	545
16	135	675	120	795
20	170	850	190	1040
24	195	975	270	1245
28	183	913	340	1253
32	170	850	390	1240
40	120	600	340	940
48	70	350	240	590
56	46	230	140	370
64	35	175	92	267
72	26	130	70	200
80	18	90	52	142
88	9	45	36	66
96	0	0	12	12
104	0	0	0	0

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Solution

This figure shows the obtained DRH corresponding to 2.0 cm ER, 5.0 cm ER and the final composite DRH resulting from the two ERs.

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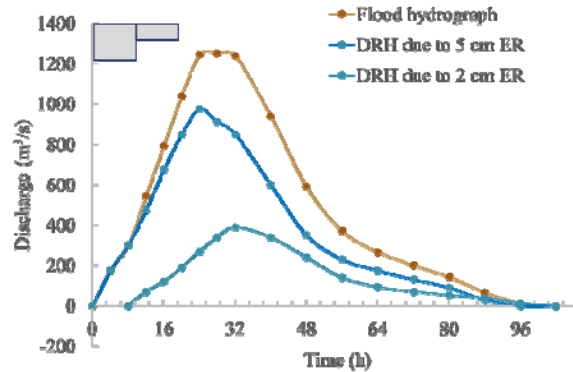



Fig.2 shows the obtained DRH corresponding to 2.0 cm ER, 5.0 cm ER, and the final composite DRH resulting from the two ERs of example 29.2.

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Summary

- A D - h UH is a DRH resulting from 1 unit (generally, 1 cm) of Rainfall Excess (RE) occurring uniformly over the basin and at a uniform rate for the specified D - h duration.
- Basic assumptions of the UH theory are space and time invariability, time-invariance, linear response and fixed time base of the DRH.
- Using the concept of linear response, two important principles, namely principle of proportionality and principle of superposition are explained.
- In the next lecture, applications of unit hydrograph will be discussed.



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Summary

In summary, we learned the following points from this lecture:

- A D - h UH is a DRH resulting from 1 unit (generally, 1 cm) of Rainfall Excess (RE) occurring uniformly over the basin and at a uniform rate for the specified D - h duration.
- Basic assumptions of the UH theory are space and time invariability, time-invariance, linear response, and fixed time base of the DRH.
- Using the concept of linear response, two important principles, namely the principle of proportionality and the principle of superposition are explained.
- In the next lecture, applications of unit hydrographs will be discussed.