Surface Water Hydrology
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Lecture 43
Reservoir Routing: Goodrich Method and Runge-Kutta Method

In this lecture, we are covering two more methods one is the Goodrich method and the other one is the Runge-Kutta method.

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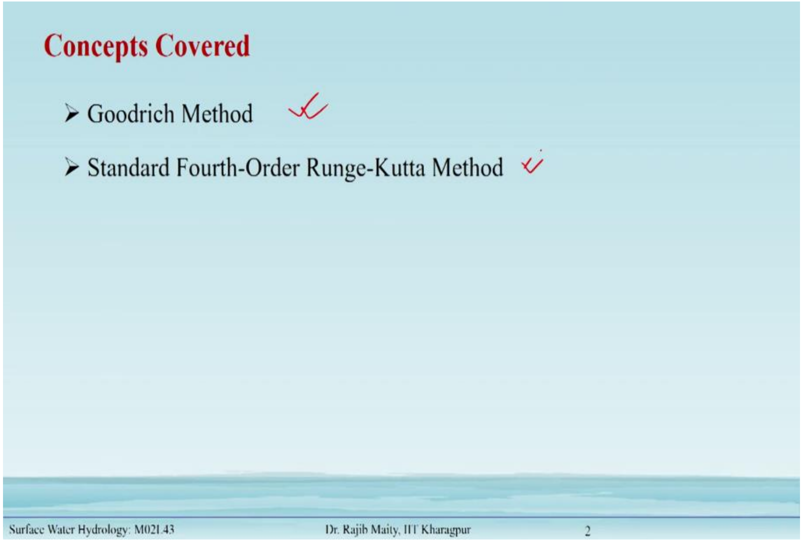

The concept covered that is Goodrich method and the Standard Fourth-Order Runge-Kutta method. Coming to the outline of this lecture, So, first, we will give the introduction to this Goodrich method its procedure and with the help of one solved example, will take up the same example that you have taken in the last class itself to show the to demonstrate the modified

Pul’s Method. And then we will discuss this Standard Fourth-Order Runge-Kutta method what is its benefit and what is its difficulties the things will be discussed finally. We will go to the summary.

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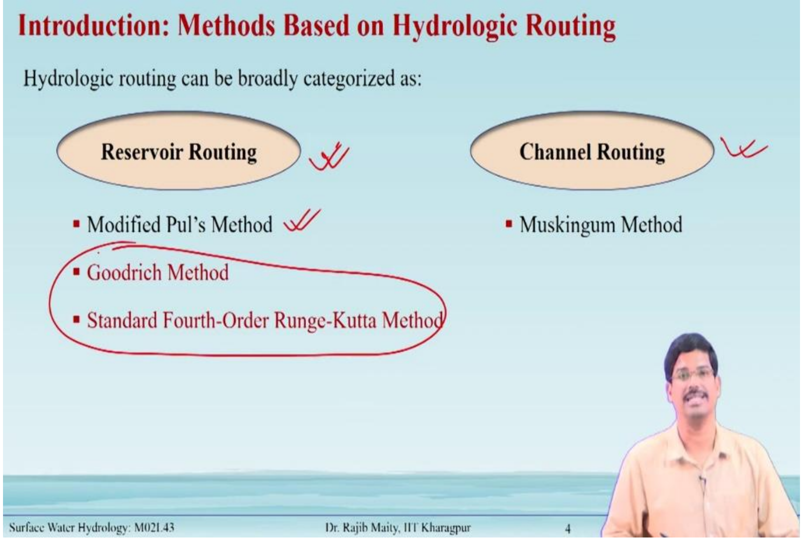

There are two categories are there one is Reservoir Routing other one is Channel Routing. In this Reservoir Routing, this modified Pul’s method was discussed in this last lecture, and we are taking up these two methods in this particular lecture Muskingum method will take in the next lecture.

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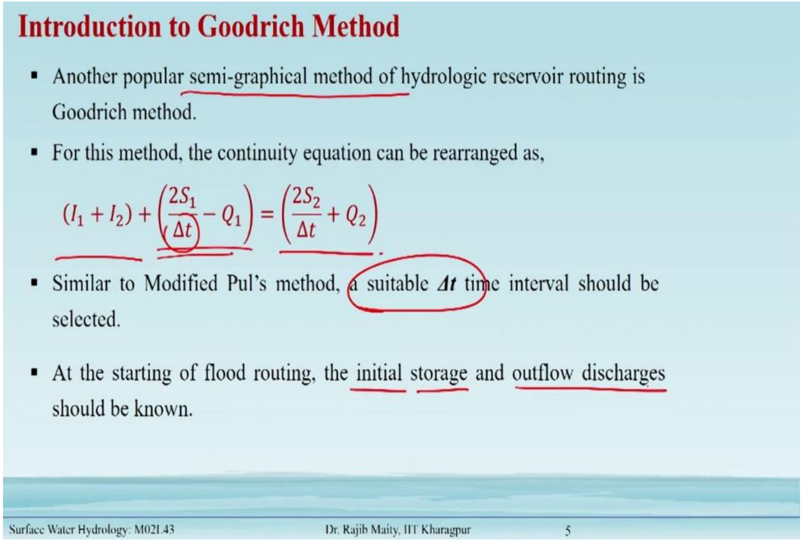

Introduction to Goodrich Method

Another popular semi-graphical method of hydrologic reservoir routing is the Goodrich method.

For this method, the continuity equation can be rearranged as,

$$(I_1 + I_2) + \left(\frac{2S_1}{\Delta t} - Q_1\right) = \left(\frac{2S_2}{\Delta t} + Q_2\right)$$

Similar to Modified Pul's method, a suitable Δt time interval should be selected. At the start of flood routing, the initial storage and outflow discharges should be known.

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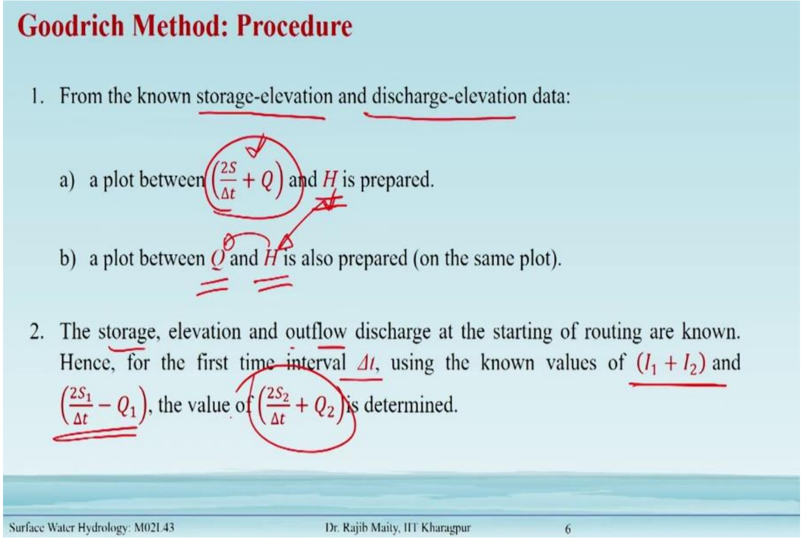

Goodrich Method: Procedure

Goodrich Method: Procedure

1. From the known storage-elevation and discharge-elevation data:

    a) A plot between $\left(\frac{2S}{\Delta t} + Q\right)$ and H is prepared.

    b) A plot between Q and H is also prepared (on the same plot).

2. The storage, elevation, and outflow discharge at the start of routing are known. Hence, for the first-time interval Δt, using the known values of $(I_1 + I_2)$ and $\left(\frac{2S_1}{\Delta t} - Q_1\right)$, the value of $\left(\frac{2S_2}{\Delta t} + Q_2\right)$ is determined.

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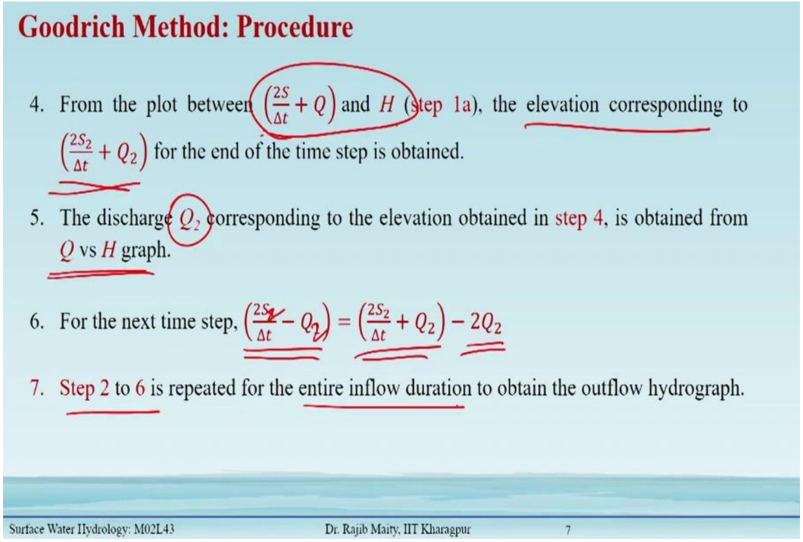

4. From the plot between $\left(\frac{2S}{\Delta t}+Q\right)$ and H (step 1a), the elevation corresponding to $\left(\frac{2S_2}{\Delta t}+Q_2\right)$ the end of the time step is obtained.

5. The discharge Q_2 corresponding to the elevation obtained in step 4, is obtained from Q vs H graph.

6. For the next time step, $\left(\frac{2S_1}{\Delta t}-Q_1\right)=\left(\frac{2S_2}{\Delta t}+Q_2\right)-2Q_2$

7. Step 2 to 6 is repeated for the entire inflow duration to obtain the outflow hydrograph.

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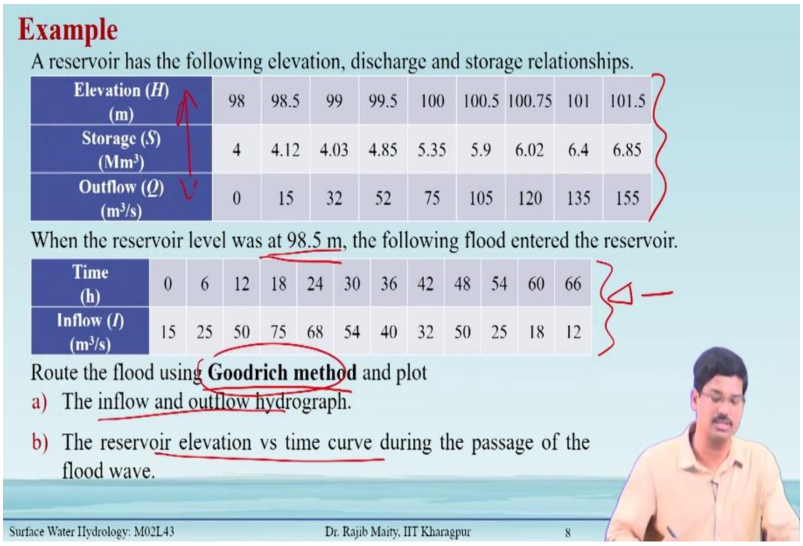

Example

A reservoir has the following elevation, discharge, and storage relationships.

Elevation (H) (m)	98	98.5	99	99.5	100	100.5	100.75	101	101.5
Storage (S) (Mm^3)	4	4.12	4.03	4.85	5.35	5.9	6.02	6.4	6.85
Outflow (Q) (m^3/s)	0	15	32	52	75	105	120	135	155

When the reservoir level was at 98.5 m, the following flood entered the reservoir.

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (I) (m^3/s)	15	25	50	75	68	54	40	32	50	25	18	12

Route the flood using the Goodrich method and plot

a) The inflow and outflow hydrograph.

b) The reservoir elevation vs time curve during the passage of the flood wave.

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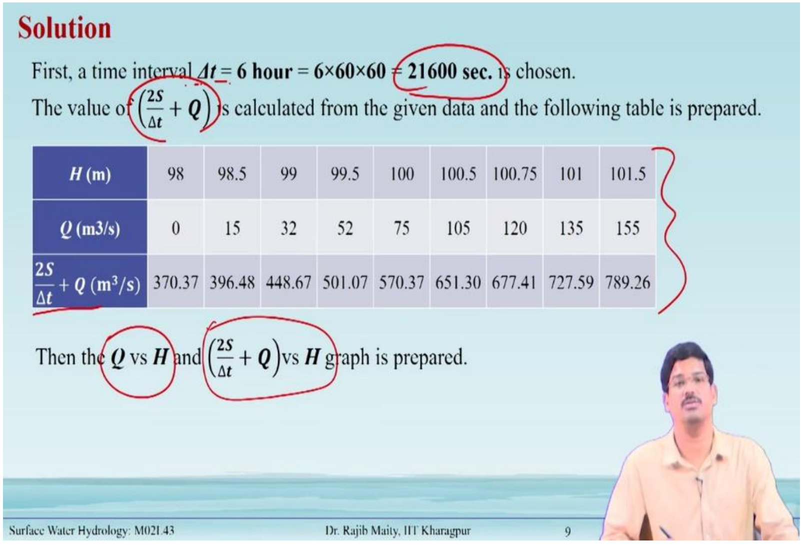

Solution

First, a time interval Δt = 6 hour = 6×60×60 = 21600 sec. is chosen.

The value $\left(\frac{2S}{\Delta t} + Q\right)$ is calculated from the given data and the following table is prepared.

H (m)	98	98.5	99	99.5	100	100.5	100.75	101	101.5
Q (m3/s)	0	15	32	52	75	105	120	135	155
$\frac{2S}{\Delta t} + Q$ (m³/s)	370.37	396.48	448.67	501.07	570.37	651.30	677.41	727.59	789.26

Then the Q vs. H and $\left(\frac{2S}{\Delta t} + Q\right)$vs. H graph is prepared.

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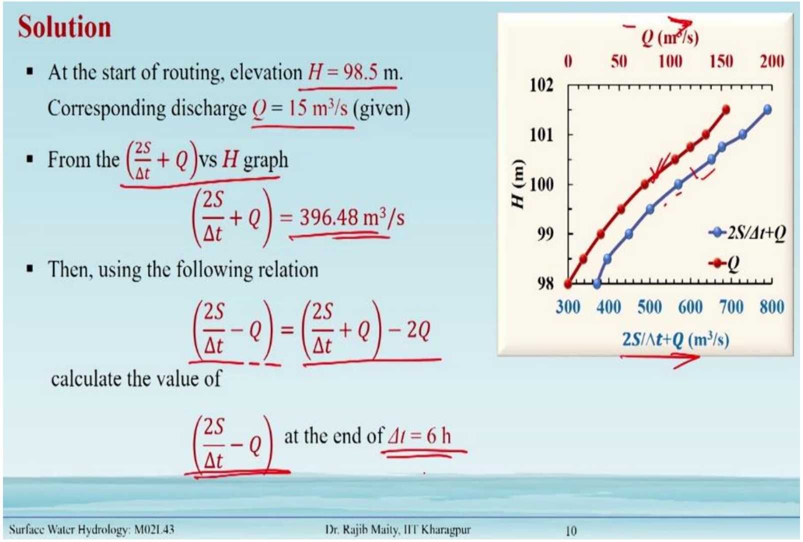

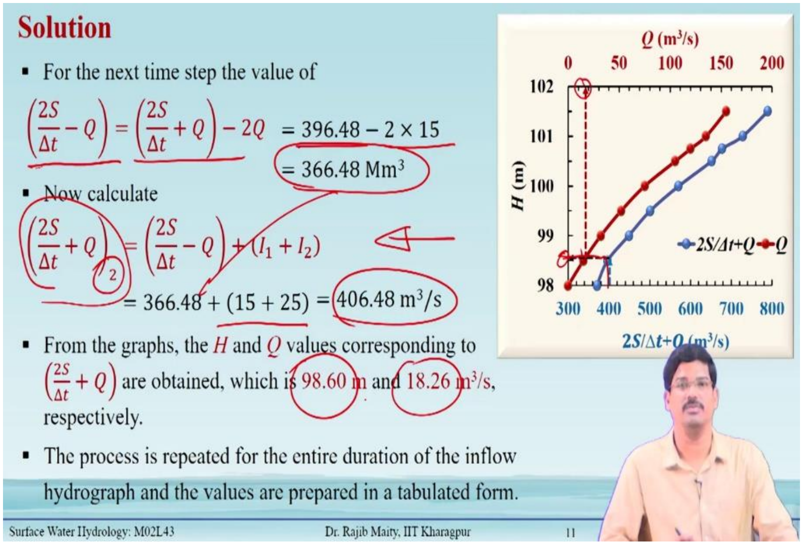

At the start of routing, elevation H = 98.5 m. Corresponding discharge Q = 15 m^3/s (given)

From the $\left(\frac{2S}{\Delta t} + Q\right)$vs. H graph

$$\left(\frac{2S}{\Delta t} + Q\right) = 396.48 \text{ m}^3/\text{s}$$

Then, using the following relation

$$\left(\frac{2S}{\Delta t} - Q\right) = \left(\frac{2S}{\Delta t} + Q\right) - 2Q$$

Calculate the value of

$\left(\frac{2S}{\Delta t} - Q\right)$at the end of Δt = 6 h

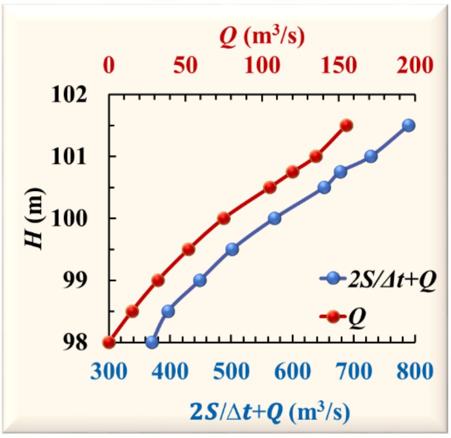

Figure 1: Goodrich Method of Storage Routing

For the next time step the value of

$$\left(\frac{2S}{\Delta t} - Q\right) = \left(\frac{2S}{\Delta t} + Q\right) - 2Q$$

=396.48−2×15 =366.48 Mm^3

Now calculate

$$\left(\frac{2S}{\Delta t} + Q\right)_2 = \left(\frac{2S}{\Delta t} - Q\right) + (I_1 + I_2)$$

=366.48+ (15+25) =406.48 m^3/s

From the graphs, the *H* and *Q* values corresponding to $\left(\frac{2S}{\Delta t} + Q\right)$ are obtained, which are 98.60 m and 18.26 m^3/s, respectively.

The process is repeated for the entire duration of the inflow hydrograph and the values are prepared in a tabulated form.

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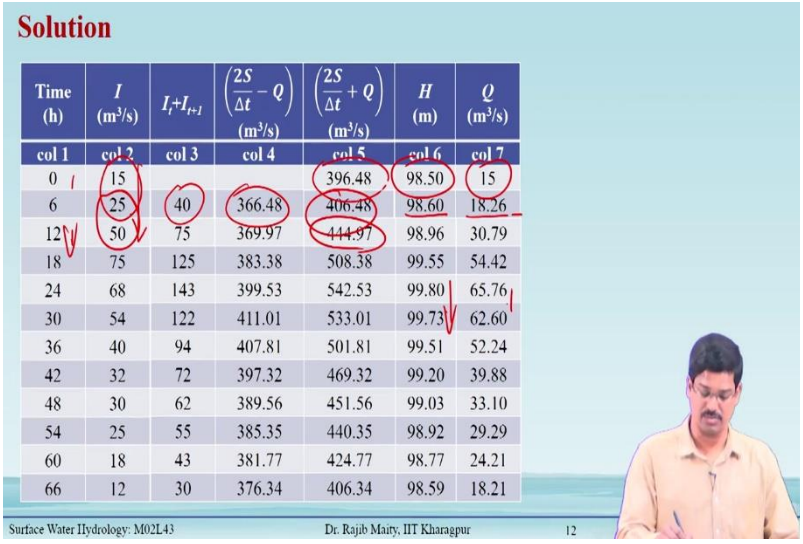

Time (h)	I (m³/s)	I_t+I_{t+1}	$\left(\frac{2S}{\Delta t}-Q\right)$ (m³/s)	$\left(\frac{2S}{\Delta t}+Q\right)$ (m³/s)	H (m)	Q (m³/s)
col 1	col 2	col 3	col 4	col 5	col 6	col 7
0	15			396.48	98.50	15
6	25	40	366.48	406.48	98.60	18.26
12	50	75	369.97	444.97	98.96	30.79
18	75	125	383.38	508.38	99.55	54.42
24	68	143	399.53	542.53	99.80	65.76
30	54	122	411.01	533.01	99.73	62.60
36	40	94	407.81	501.81	99.51	52.24
42	32	72	397.32	469.32	99.20	39.88
48	30	62	389.56	451.56	99.03	33.10
54	25	55	385.35	440.35	98.92	29.29
60	18	43	381.77	424.77	98.77	24.21
66	12	30	376.34	406.34	98.59	18.21

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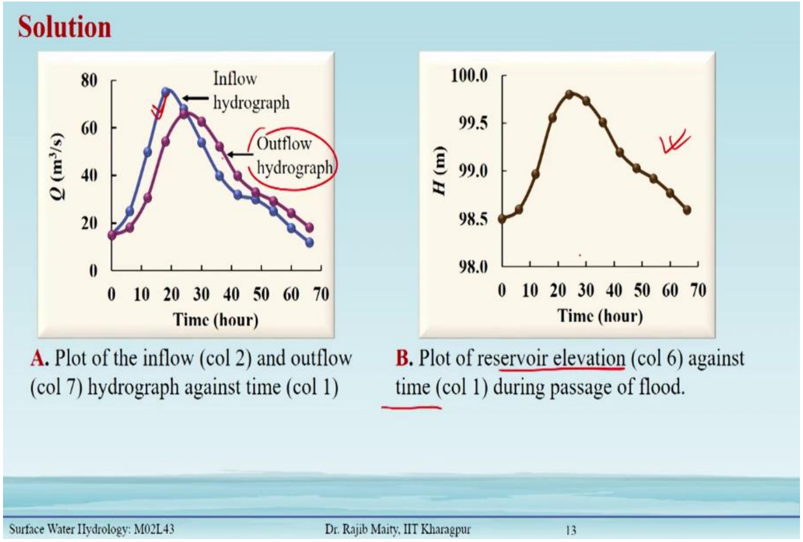
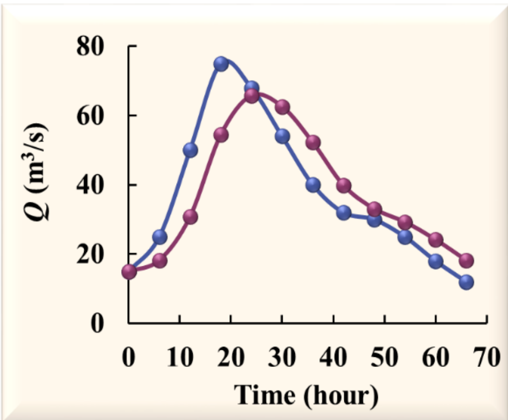
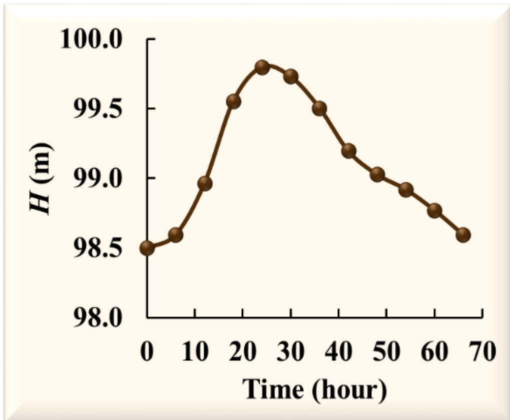




A. Plot of the inflow (col 2) and outflow (col 7) hydrograph against time (col 1)



B. Plot of reservoir elevation (col 6) against time (col 1) during the passage of flood.

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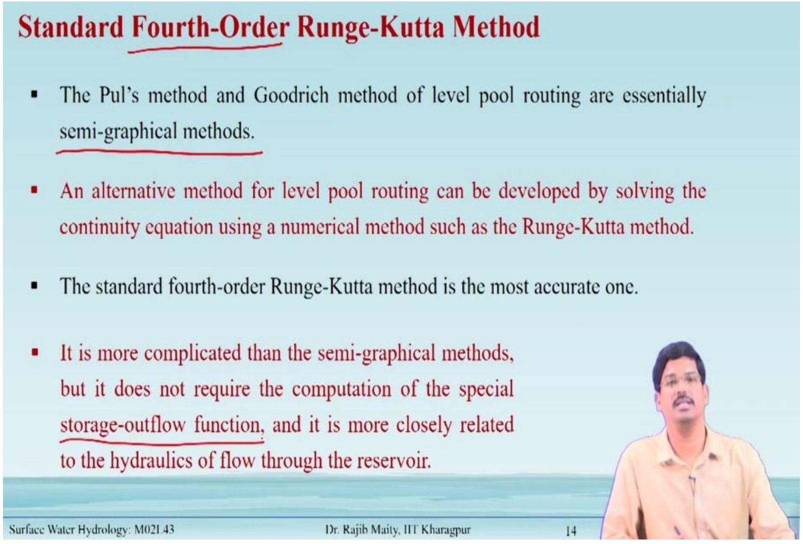

Standard Fourth-Order Runge-Kutta Method

Pul's method and Goodrich method of level pool routing are essentially semi-graphical methods. An alternative method for level pool routing can be developed by solving the continuity equation using a numerical method such as the Runge-Kutta method. The standard fourth-order Runge-Kutta method is the most accurate one.

It is more complicated than the semi-graphical methods, but it does not require the computation of the special storage-outflow function, and it is more closely related to the hydraulics of flow through the reservoir.

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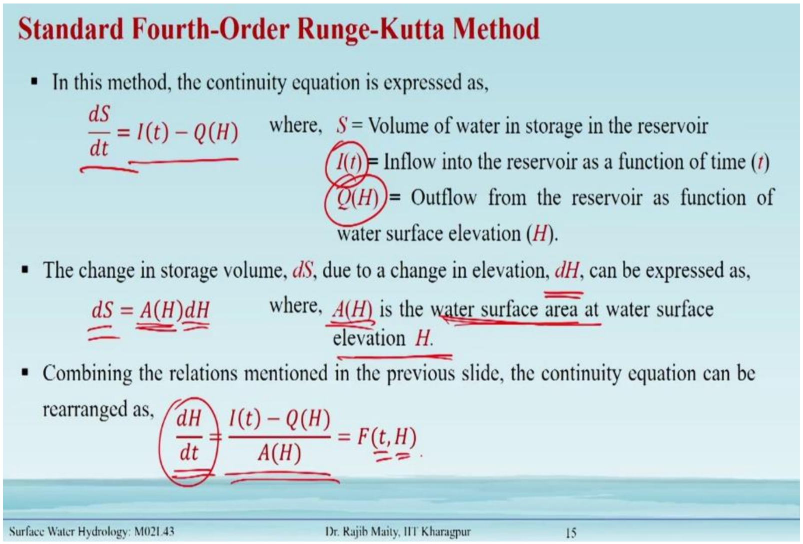

In this method, the continuity equation is expressed as,

$$\frac{dS}{dt} = I(t) - Q(H)$$

where,

S = Volume of water in storage in the reservoir

$I(t)$ = Inflow into the reservoir as a function of time (t)

$Q(H)$ = Outflow from the reservoir as a function of water surface elevation (H).

The change in storage volume, dS, due to a change in elevation, dH, can be expressed as,

$$dS = A(H)dH$$

where A(H) is the water surface area at water surface elevation H.

Combining the relations mentioned in the previous slide, the continuity equation can be rearranged as

$$\frac{dH}{dt} = \frac{I(t) - Q(H)}{A(H)} = F(t, H)$$

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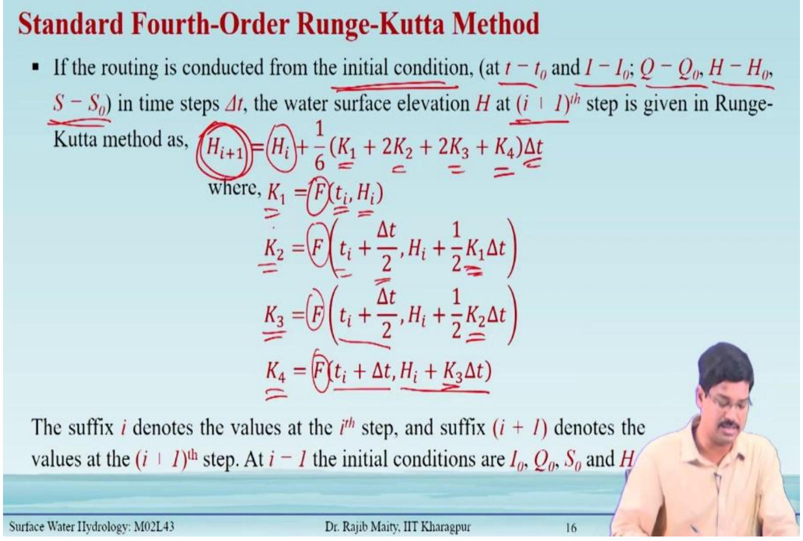

If the routing is conducted from the initial condition, (at $t = t_0$ and $I = I_0$; $Q = Q_0$, $H = H_0$, $S = S_0$) in time steps Δt, the water surface elevation H at $(i + 1)^{th}$ step is given in Runge-Kutta method as,

$$H_{i+1} = H_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)\Delta t$$

where,

$$K_1 = F(t_i,\ H_i)$$

$$K_2 = F\left(t_i + \frac{\Delta t}{2}, H_i + \frac{1}{2}K_1\Delta t\right)$$

$$K_3 = F\left(t_i + \frac{\Delta t}{2}, H_i + \frac{1}{2}K_2\Delta t\right)$$

$$K_4 = F(t_i + \Delta t, H_i + K_3\Delta t)$$

The suffix i denotes the values at the i^{th} step, and suffix $(i + 1)$ denotes the values at the $(i + 1)^{th}$ step. At $i = 1$ the initial conditions are I_0, Q_0, S_0, and H_0

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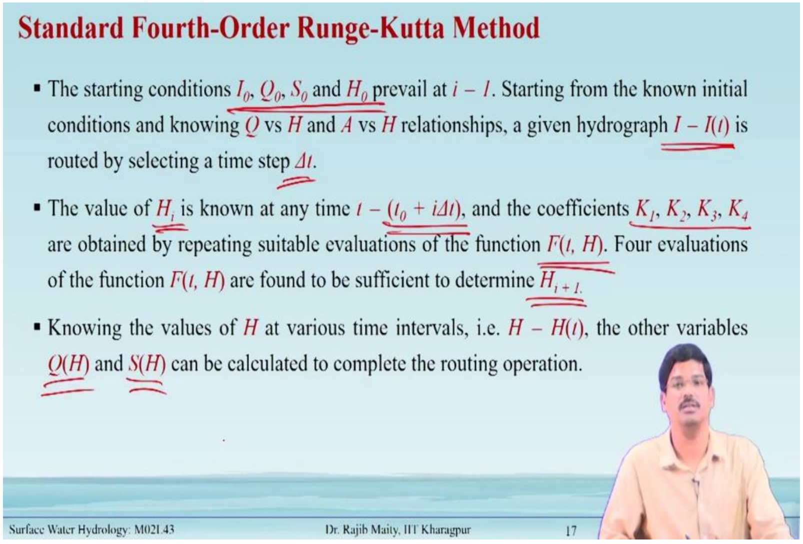

The starting conditions I_0, Q_0, S_0, and H_0 prevail at $i = 1$. Starting from the known initial conditions and knowing Q vs. H and A vs. H relationships, a given hydrograph $I = I(t)$ is routed by selecting a time step Δt.

The value of H_i is known at any time $t = (t_0 + i\Delta t)$, and the coefficients K_1, K_2, K_3, K_4 are obtained by repeating suitable evaluations of the function $F\ (t, H)$. Four evaluations of the function $F\ (t, H)$ are found to be sufficient to determine H_{i+1}.

Knowing the values of H at various time intervals, i.e. $H = H(t)$, the other variables $Q(H)$ and $S(H)$ can be calculated to complete the routing operation.

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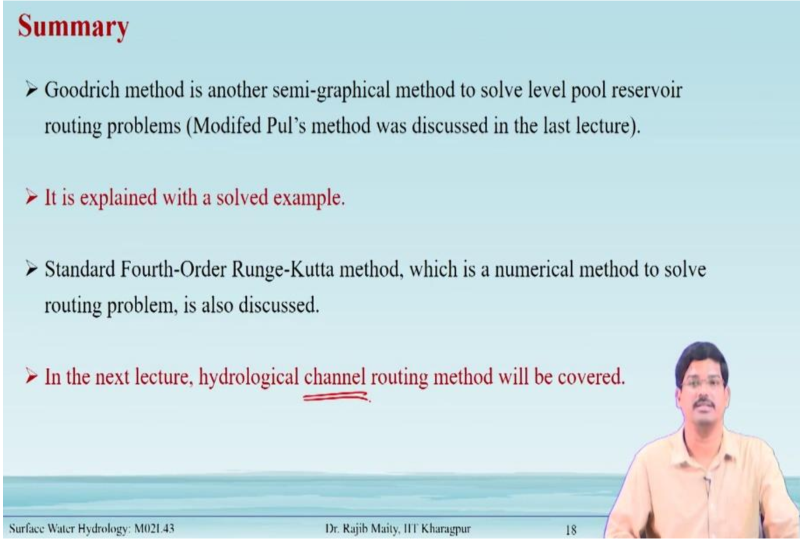

Summary

In summary, we learned the following points from this lecture:

- Goodrich method is another semi-graphical method to solve level pool reservoir routing problems (Modified Pul's method was discussed in the last lecture).
- It is explained with a solved example.
- Standard Fourth-Order Runge-Kutta method, which is a numerical method to solve routing problems, is also discussed.
- In the next lecture, the hydrological channel routing method will be covered.