Surface Water Hydrology
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Lecture 46
Concept of Routing in IUH Development & Clark's Method

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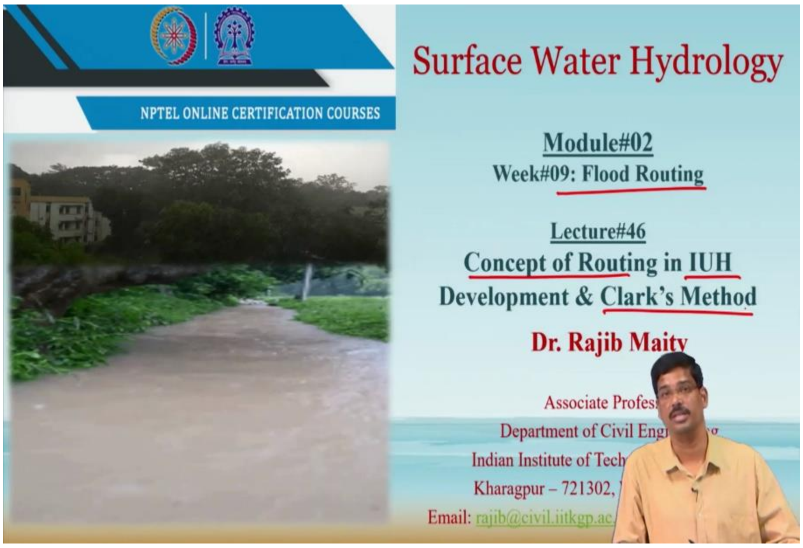

In this particular lecture, we will be discussing the concept of routing in the instantaneous unit hydrograph development and the Clark method.

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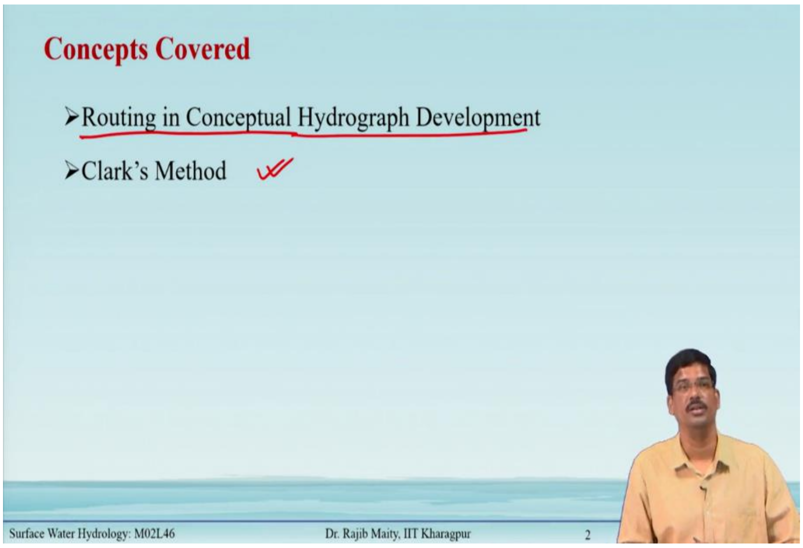

The concept covered in this lecture is the routing in conceptual hydrograph development and there are two methods there will be discussed. The first one is Clark's method and the second one is the Nash model.

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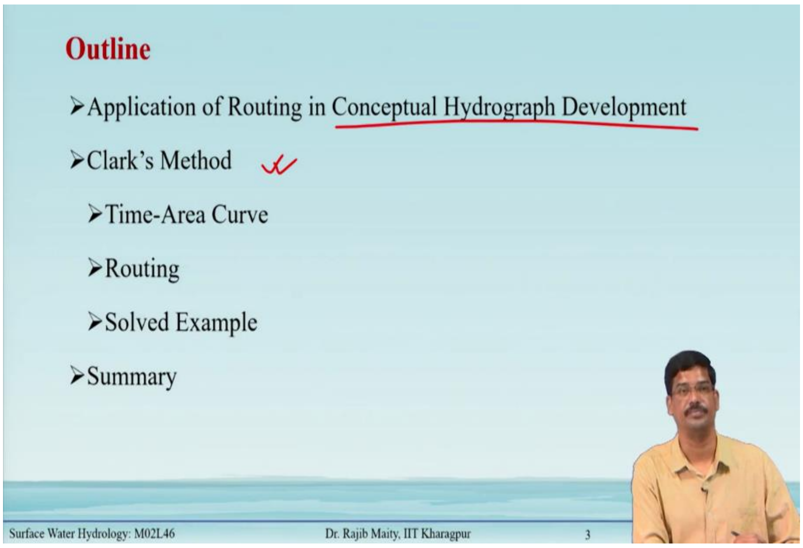

The hydrograph analysis and the unit hydrograph concepts can be utilized to develop the conceptual unit, conceptual hydrograph. So, this Clark's method is one of them and that we will discuss in detail today.

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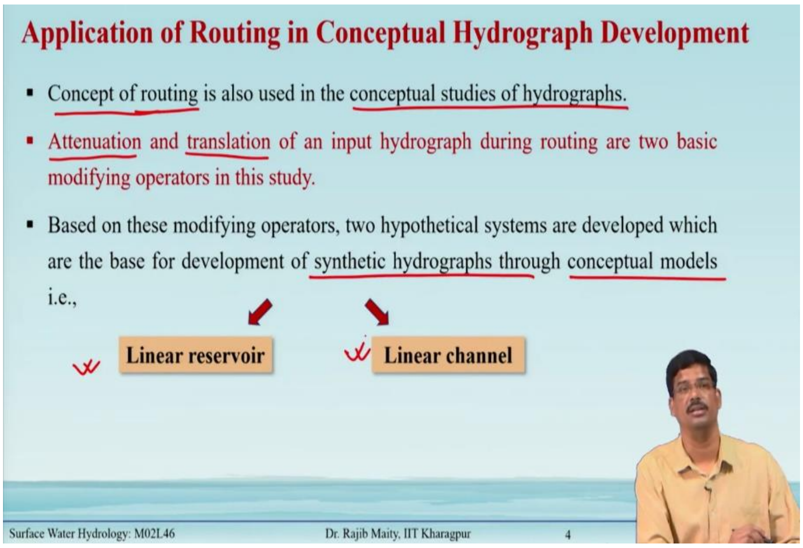

Application of Routing in Conceptual Hydrograph Development

The concept of routing is also used in the conceptual studies of hydrographs. The attenuation and translation of an input hydrograph during the routing are the two basic modifying operators in this in this study. So, based on these modifying operators that are attenuation and translation, two hypothetical systems are developed which are the base for the development of these synthetic hydrographs through the conceptual approach, two conceptual models that we will consider. One is the linear reservoir; the other one is a linear channel.

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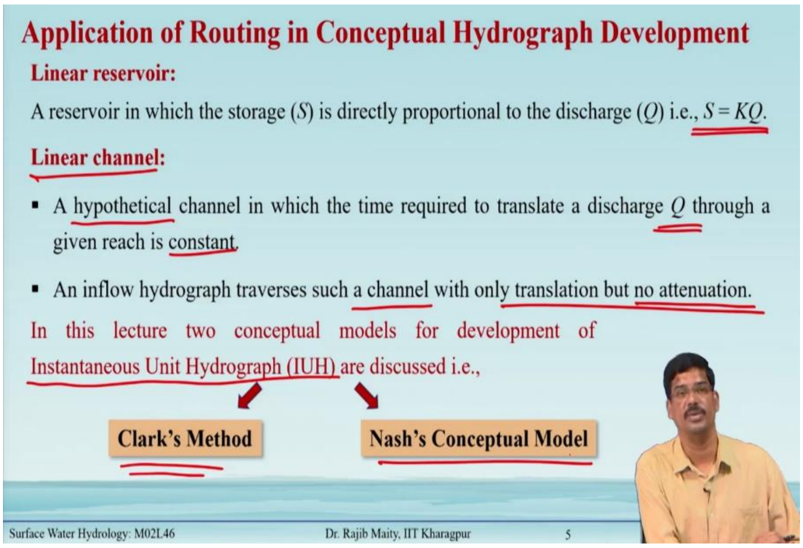

Application of Routing in Conceptual Hydrograph Development

Linear reservoir:

A reservoir in which the storage (S) is directly proportional to the discharge (Q) i.e., $S = KQ$.

Linear channel:

A hypothetical channel in which the time required to translate a discharge Q through a given reach is constant. An inflow hydrograph traverses such a channel with only translation but no attenuation.

In this lecture two conceptual models for the development of Instantaneous Unit Hydrograph (IUH) are discussed i.e., i) Clark's Method; ii) Nash's Conceptual Model

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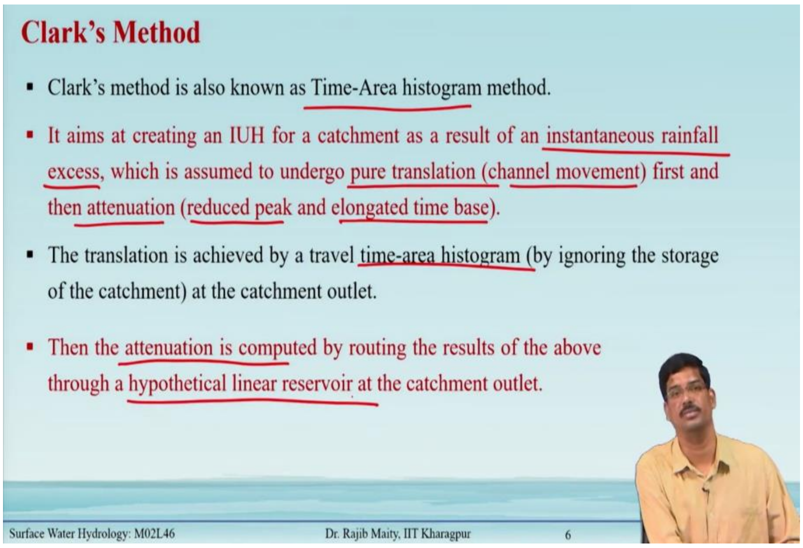

Clark's Method

Clark's method is also known as another name is called the Time-Area histogram method and its target is to create an Instantaneous Unit Hydrograph for a catchment as a result of an instantaneous rainfall excess, which is assumed to undergo the pure translation first, that is the channel moment and then the attenuation the reduced peak and elongated time base.

The translation is achieved by a travel time-area histogram. And when it is computing the translation at that time the storage in the catchment is completely ignored. And then when we come to this attenuation, this attenuation is computed by routing the result of the above through the hypothetical linear reservoir at the catchment outlet.

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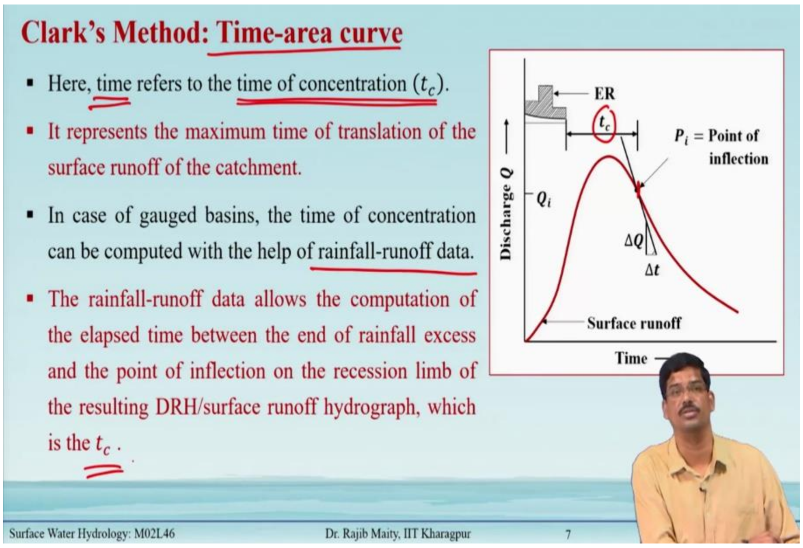

Clark's Method: time-area curve

Here the time is referred to as the time of concentration (t_c). Once the excess rainfall stops and from there up to the inflection point on the recession limb, the limb from this hydrograph is called the time of concentration. In other words, it is the time that takes for water to reach from the farthest point of the watershed to the outlet.

After the rainfall starts and thus once the time of concentration is reached and then the entire catchment is contributing to the outlet. So, it represents the maximum time of the translation of the surface runoff of this catchment. And in the case of a gauged basin, the time of concentration can be computed with the help of the rainfall-runoff data.

But when the rainfall-runoff data allows the computation of this elapsed time between the end of the rainfall excess and the point of inflection on the recession limb of the resulting direct runoff hydrograph or surface runoff hydrograph which is the t_c.

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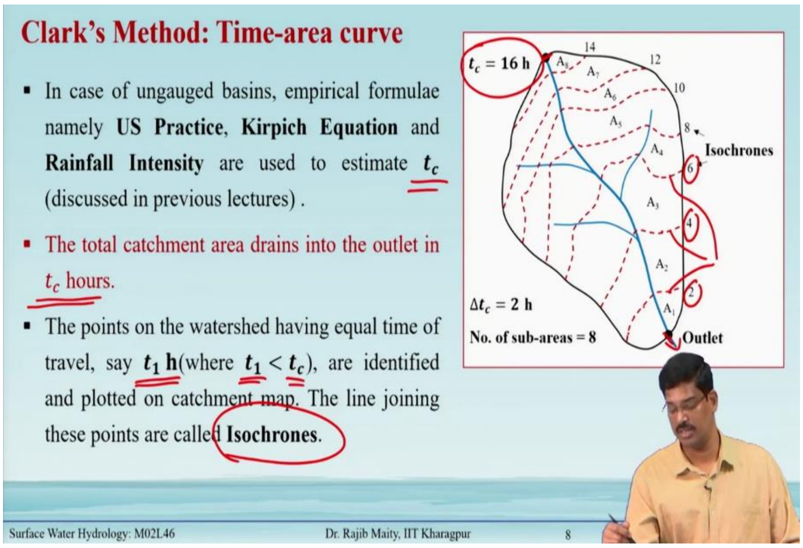

In the case of an ungauged basin, the empirical formulae: the US practice, the Kirpich Equation, and the Rainfall Intensity are used to estimate the time of concentration if it is the ungauged basin.

The total catchment area drains into the outlet in tc hours. That means when the t_c hours are elapsed from the furthest point it comes to this outlet. So, the entire catchment starts contributing. There is one example is given in Fig.1. For example, in this watershed in fig.1 that t_c is 16 hours. So, from this farthest point, it will take that 16 hours' time to reach the outlet.

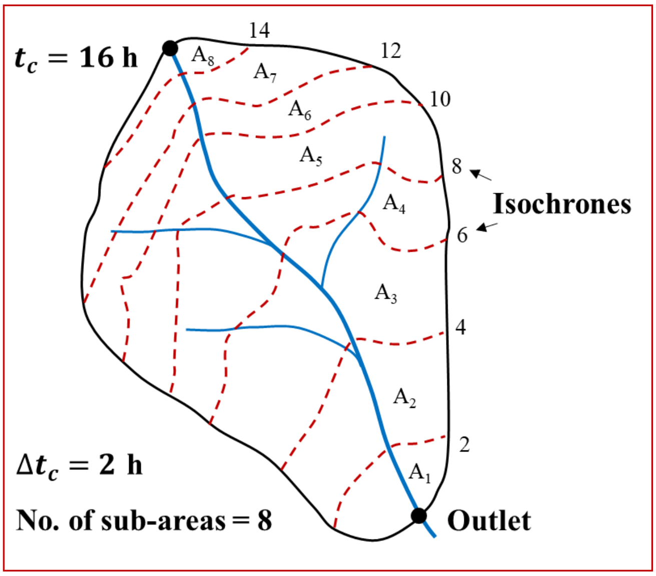

Figure 1: Isochrones in a Catchment

The points on the watershed having an equal time of travel, say t_1 hour where t_1 is less than t_c are identified and plotted on the catchment map and this line joins these points, points of the equal time of travel it is known as isochrones.

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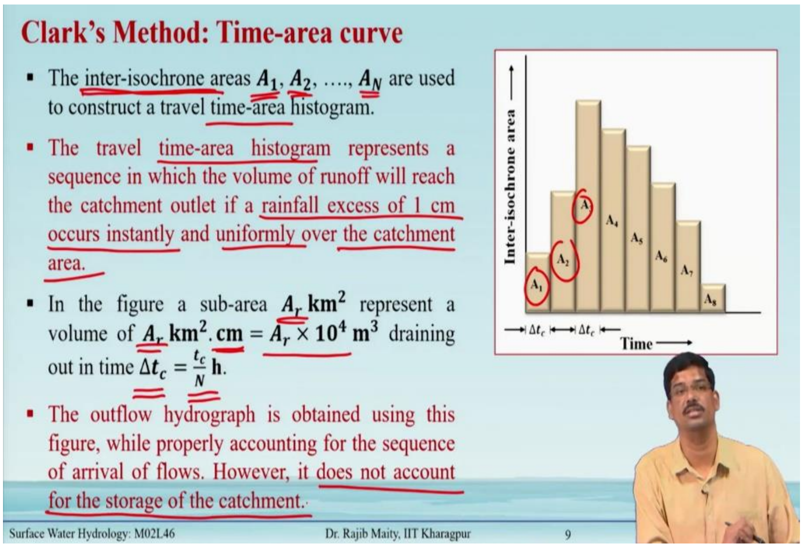

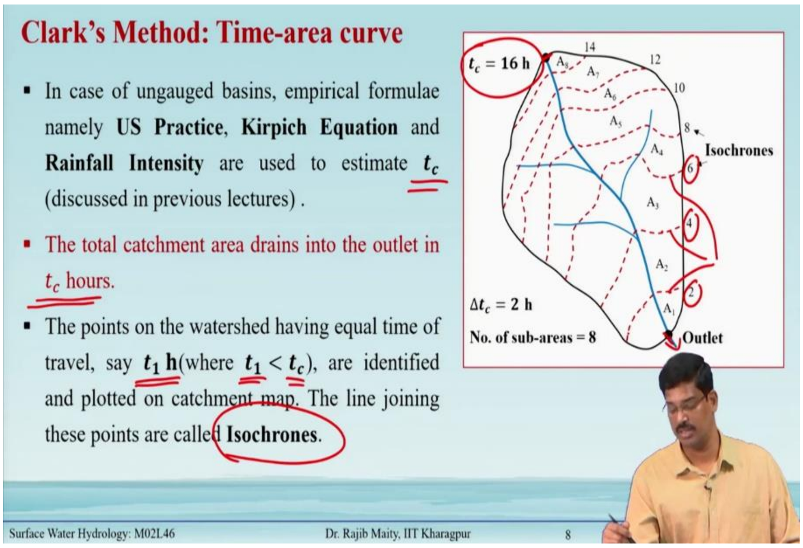

The inter-isochrone areas $\boldsymbol{A}_1$, $\boldsymbol{A}_2$, ..., $\boldsymbol{A}_N$ are used to construct a travel time-area histogram.

The travel time-area histogram represents a sequence in which the volume of runoff will reach the catchment outlet if a rainfall excess of 1 cm occurs instantly and uniformly over the catchment area.

In fig. 2 a sub-area $\boldsymbol{A}_r$ $\mathbf{km^2}$ represent a volume of $\boldsymbol{A}_r$ $\mathbf{km^2.cm} = \boldsymbol{A}_r \times \mathbf{10^4\ m^3}$ draining out in time $\Delta \boldsymbol{t}_c = \frac{t_c}{N}$ $\mathbf{h}$.

The outflow hydrograph is obtained using this figure, while properly accounting for the sequence of arrival of flows. However, it does not account for the storage of the catchment.

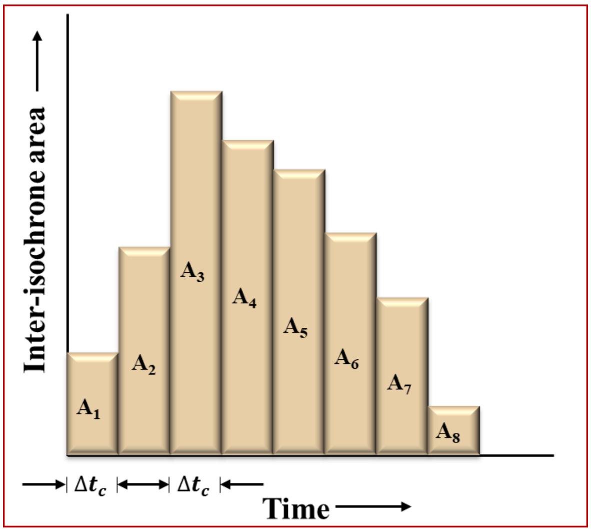

Figure 2 Time-area Histogram

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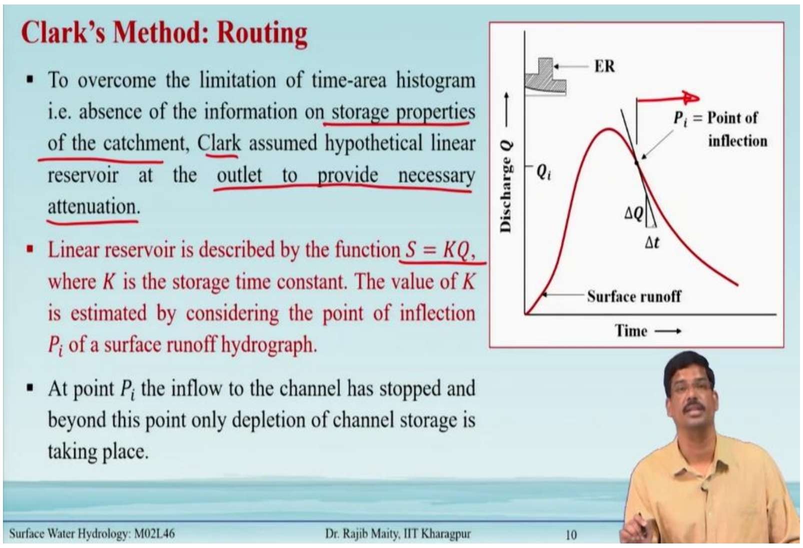

Clark's Method: Routing

To overcome the limitation of the time-area histogram i.e., the absence of the information on storage properties of the catchment, Clark assumed a hypothetical linear reservoir at the outlet to provide necessary attenuation.

A linear reservoir is described by the function $S = KQ$, where K is the storage time constant. The value of K is estimated by considering the point of inflection P_i of a surface runoff hydrograph as shown in fig.3. At point P_i the inflow to the channel has stopped and beyond this point, only depletion of channel storage is taking place.

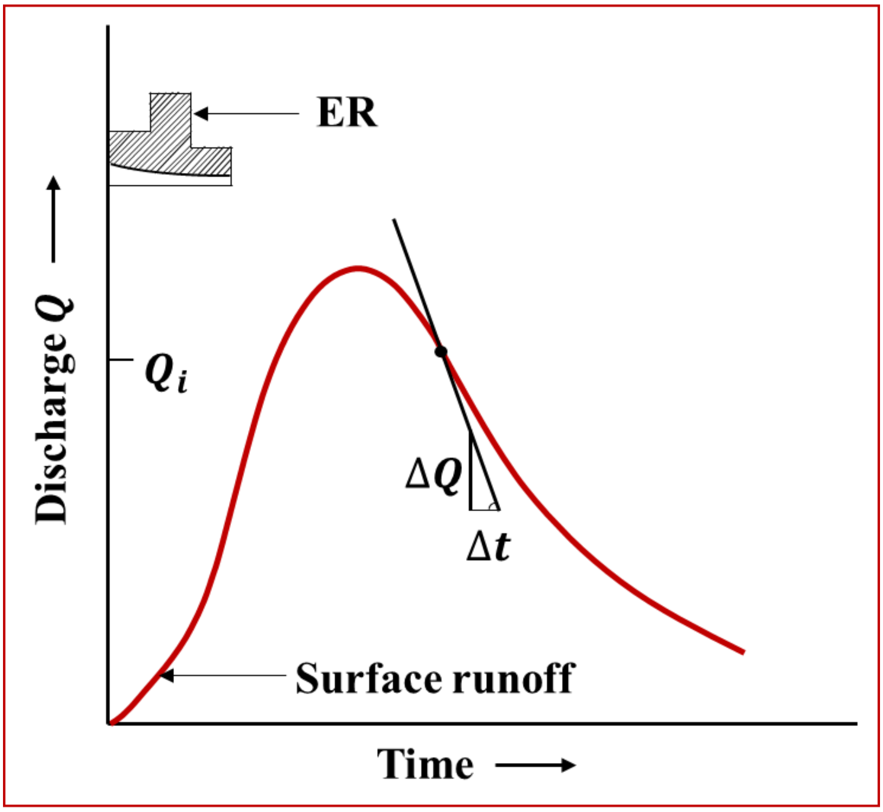

Figure 3: Surface Runoff of a Catchment

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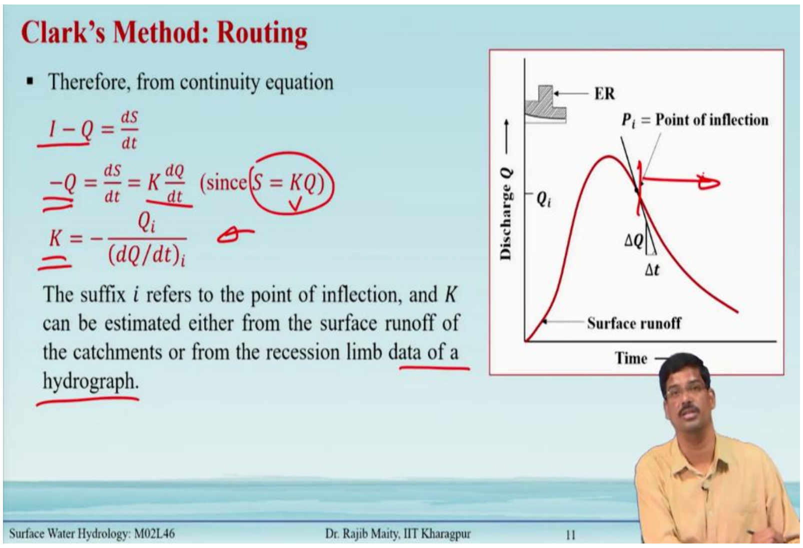

Therefore, from the continuity equation

$$I - Q = \frac{dS}{dt}$$

$$-Q = \frac{dS}{dt} = K\frac{dQ}{dt} \quad \text{(Since, } S = KQ\text{)}$$

$$K = -\frac{Q_i}{(dQ/dt)_i}$$

The suffix i refers to the point of inflection, and K can be estimated either from the surface runoff of the catchments or from the recession limb data of a hydrograph.

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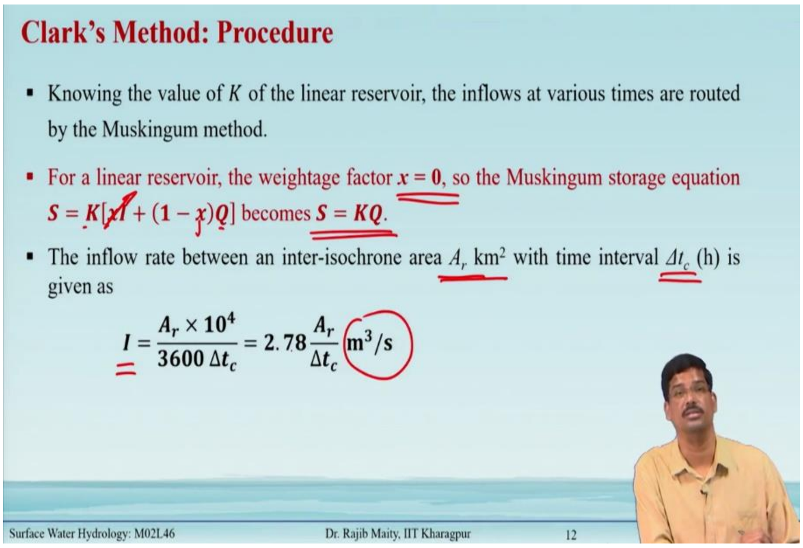

Clark's Method: Procedure

- Knowing the value of K of the linear reservoir, the inflows at various times are routed by the Muskingum method.

- For a linear reservoir, the weightage factor $\boldsymbol{x} = \mathbf{0}$, so the Muskingum storage equation $\boldsymbol{S = K[xI + (1 - x)Q]}$ becomes $\boldsymbol{S = KQ}$.

- The inflow rate between an inter-isochrone area A_r km^2 with time interval Δt_c (h) is given as

$$\boldsymbol{I = \frac{A_r \times 10^4}{3600\,\Delta t_c} = 2.78\frac{A_r}{\Delta t_c}\ \mathrm{m^3/s}}$$

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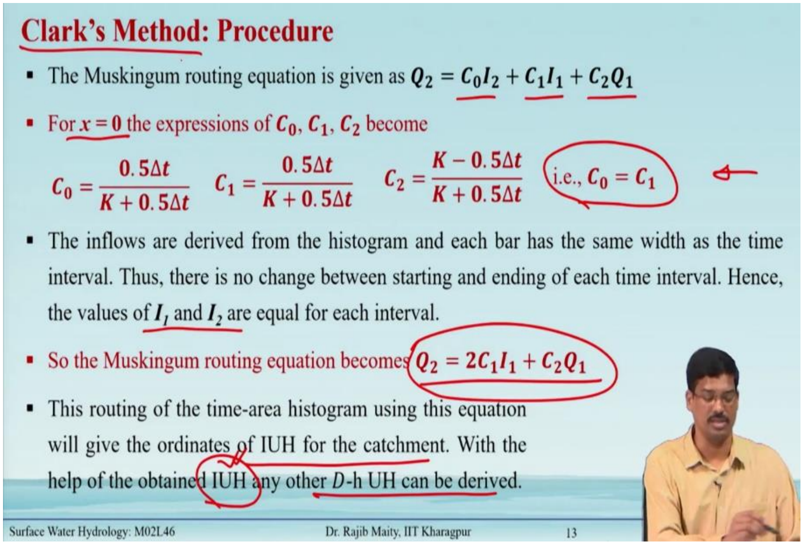

- The Muskingum routing equation is given as $\boldsymbol{Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1}$

- For $\boldsymbol{x = 0}$ the expressions of $\boldsymbol{C_0}$,$\boldsymbol{C_1}$, $\boldsymbol{C_2}$ become

$$\boldsymbol{C_0 = \frac{0.5\Delta t}{K+0.5\Delta t}} \quad \boldsymbol{C_1 = \frac{0.5\Delta t}{K+0.5\Delta t}} \quad \boldsymbol{C_2 = \frac{K-0.5\Delta t}{K+0.5\Delta t}} \qquad \text{i.e., } C_0 = C_1$$

- The inflows are derived from the histogram and each bar has the same width as the time interval. Thus, there is no change between starting and ending of each time interval. Hence, the values of $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$ are equal for each interval.

- So, the Muskingum routing equation becomes $\boldsymbol{Q_2 = 2C_1 I_1 + C_2 Q_1}$

- This routing of the time-area histogram using this equation will give the ordinates of IUH for the catchment. With the help of the obtained IUH any other D-h UH can be derived.

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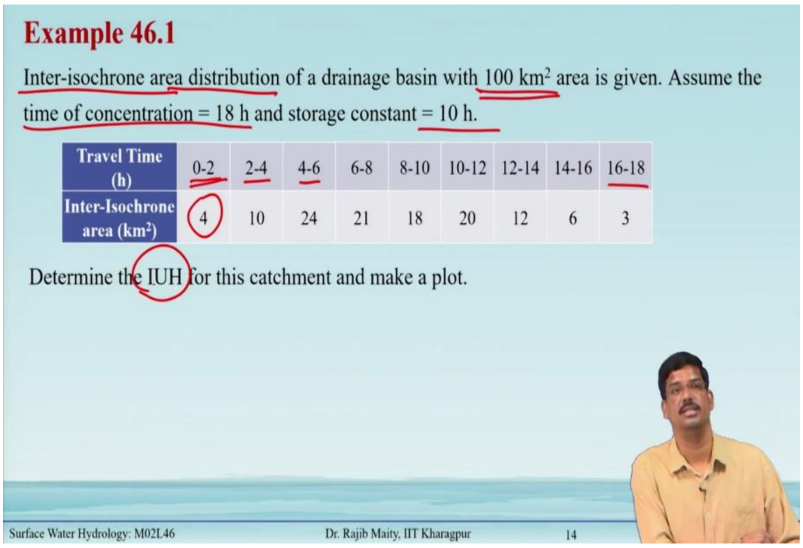

Example 46.1

Inter-isochrone area distribution of a drainage basin with a 100 km^2 area is given. Assume the time of concentration = 18 h and storage constant = 10 h.

Travel Time (h)	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
Inter-Isochrone area (km^2)	4	10	24	21	18	20	12	6	3

Determine the IUH for this catchment and make a plot.

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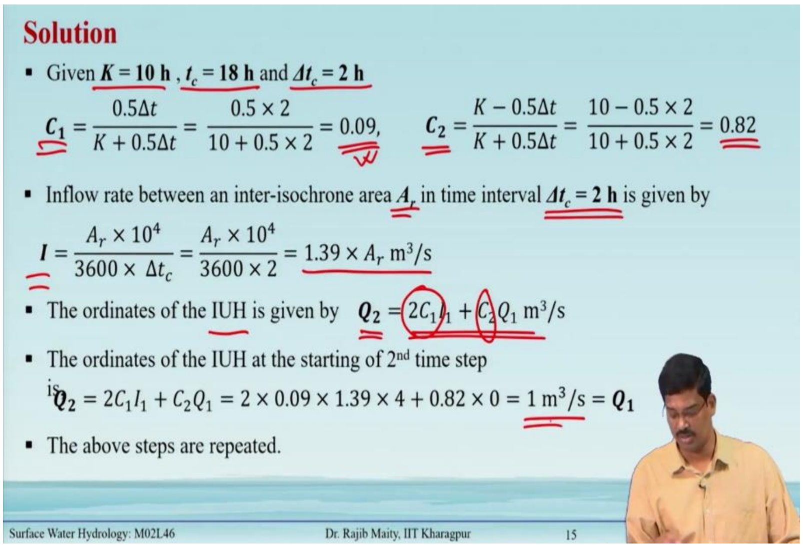

Solution

Given K = 10 h , tc = 18 h and $\Delta t_c = 2$ h

$$C_1 = \frac{0.5\Delta t}{K+0.5\Delta t} = \frac{0.5\times2}{10+0.5\times2} = 0.09, \qquad C_2 = \frac{K-0.5\Delta t}{K+0.5\Delta t} = \frac{10-0.5\times2}{10+0.5\times2} = 0.82$$

Inflow rate between an inter-isochrone area $\boldsymbol{A_r}$ in time interval $\boldsymbol{\Delta t_c}$ **= 2 h** is given by

$$\boldsymbol{I} = \frac{A_r \times 10^4}{3600\times\ \Delta t_c} = \frac{A_r \times 10^4}{3600\times 2} = 1.39 \times A_r \ \mathrm{m}^3/\mathrm{s}$$

The ordinates of the IUH are given by $\boldsymbol{Q_2} = 2C_1I_1 + C_2Q_1 \ \mathrm{m}^3/\mathrm{s}$

The ordinates of the IUH at the starting of the 2nd-time step are,

$$\boldsymbol{Q_2} = 2C_1I_1 + C_2Q_1 = 2\times0.09\times1.39\times4 + 0.82\times0 = 1\ \mathrm{m}^3/\mathrm{s} = \boldsymbol{Q_1}$$

The above steps are repeated.

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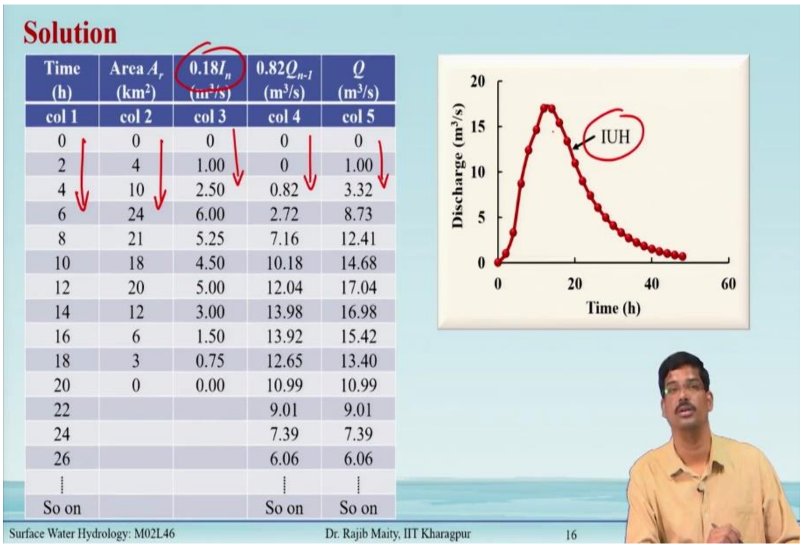

Time (h)	Area A_r (km²)	$0.18I_n$ (m³/s)	$0.82Q_{n-1}$ (m³/s)	Q (m³/s)
col 1	col 2	col 3	col 4	col 5
0	0	0	0	0
2	4	1.00	0	1.00
4	10	2.50	0.82	3.32
6	24	6.00	2.72	8.73
8	21	5.25	7.16	12.41
10	18	4.50	10.18	14.68
12	20	5.00	12.04	17.04
14	12	3.00	13.98	16.98
16	6	1.50	13.92	15.42
18	3	0.75	12.65	13.40
20	0	0.00	10.99	10.99
22			9.01	9.01
24			7.39	7.39
26			6.06	6.06
⋮			⋮	⋮
So on			So on	So on

Time (h)	Area A_r (km^2)	$0.18I_n$ (m^3/s)	$0.82Q_{n-1}$ (m^3/s)	Q (m^3/s)
col 1	col 2	col 3	col 4	col 5
0	0	0	0	0
2	4	1.00	0	1.00
4	10	2.50	0.82	3.32
6	24	6.00	2.72	8.73
8	21	5.25	7.16	12.41
10	18	4.50	10.18	14.68
12	20	5.00	12.04	17.04
14	12	3.00	13.98	16.98
16	6	1.50	13.92	15.42
18	3	0.75	12.65	13.40
20	0	0.00	10.99	10.99
22			9.01	9.01
24			7.39	7.39
26			6.06	6.06
⋮			⋮	⋮
So on			So on	So on

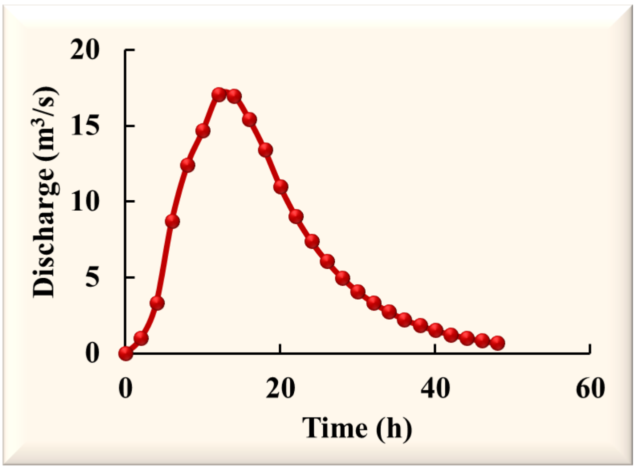

Figure 4: Instantaneous Unit Hydrograph of example 46.1

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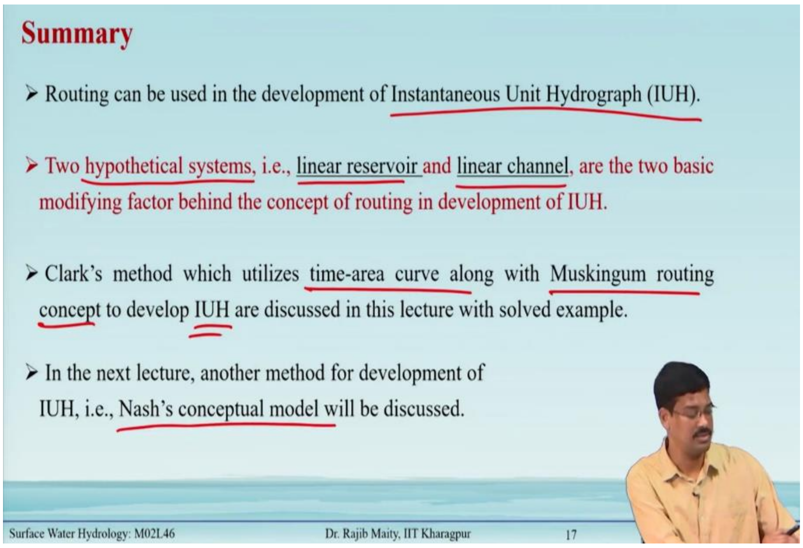

Summary

In summary, we learned the following points from this lecture:

- Routing can be used in the development of Instantaneous Unit Hydrograph (IUH).
- Two hypothetical systems, i.e., linear reservoir and linear channel, are the two basic modifying factors behind the concept of routing in the development of IUH.
- Clark's method which utilizes the time-area curve along with the Muskingum routing concept to develop IUH is discussed in this lecture with a solved example.
- In the next lecture, another method for the development of IUH, i.e., Nash's conceptual model will be discussed.