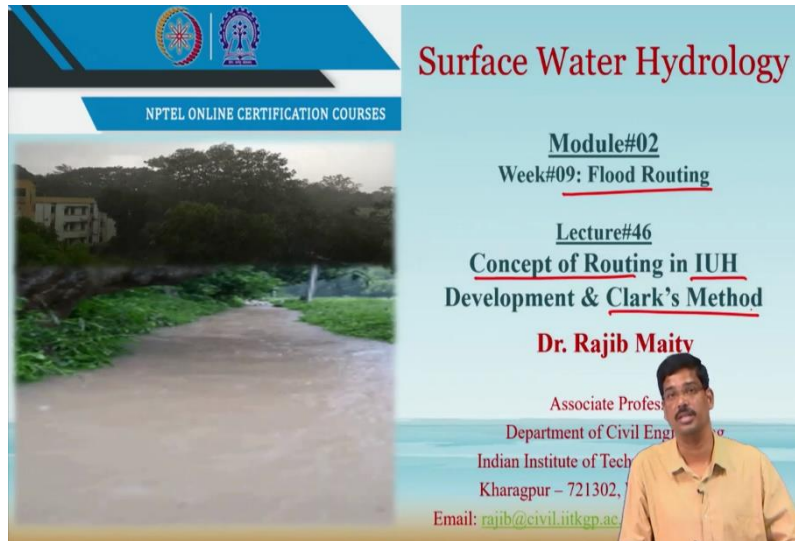


**Surface Water Hydrology**  
**Professor Rajib Maity**  
**Department of Civil Engineering**  
**Indian Institute of Technology Kharagpur**  
**Lecture 46**  
**Concept of Routing in IUH Development & Clark's Method**

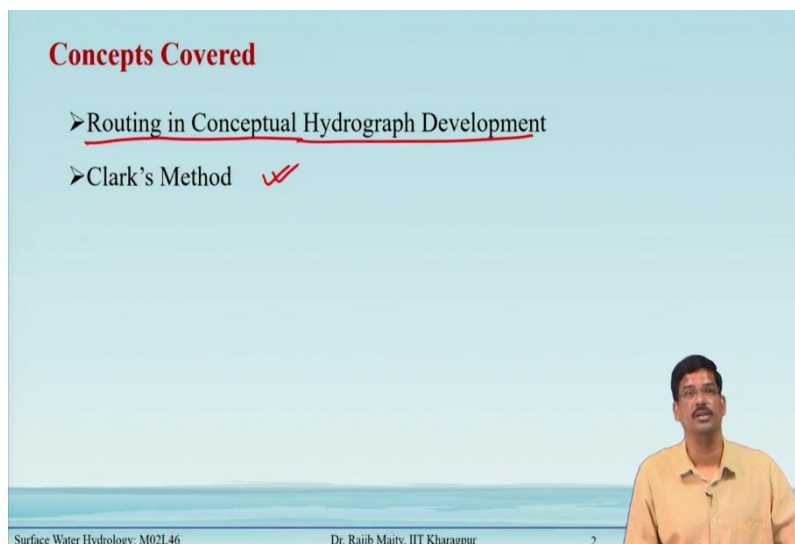
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The slide features a blue header with the NPTEL logo and the text "NPTEL ONLINE CERTIFICATION COURSES". Below the header is a photograph of a flooded road. To the right of the photo, the text reads: "Surface Water Hydrology", "Module#02", "Week#09: Flood Routing", "Lecture#46", "Concept of Routing in IUH Development & Clark's Method", and "Dr. Rajib Maity". Below the name, it lists his title as "Associate Professor", his department as "Department of Civil Engineering", his affiliation as "Indian Institute of Technology Kharagpur - 721302, India", and his email as "rajib@civil.iitkgp.ac.in". A small portrait of Dr. Rajib Maity is visible in the bottom right corner of the slide.

In this particular lecture, we will be discussing the concept of routing in the instantaneous unit hydrograph development and the Clark method.

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The slide has a light blue background. At the top left, the text "Concepts Covered" is written in red. Below it, there are two bullet points: "➤ Routing in Conceptual Hydrograph Development" and "➤ Clark's Method ✓". In the bottom right corner, there is a small portrait of Dr. Rajib Maity. At the very bottom of the slide, there is a footer with the text "Surface Water Hydrology: M02L46", "Dr. Rajib Maity, IIT Kharagpur", and the number "2".

The concept covered in this lecture is the routing in conceptual hydrograph development and there are two methods there will be discussed. The first one is Clark's method and the second one is the Nash model.

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**Outline**

- Application of Routing in Conceptual Hydrograph Development
- Clark's Method ✓
- Time-Area Curve
- Routing
- Solved Example
- Summary

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The hydrograph analysis and the unit hydrograph concepts can be utilized to develop the conceptual unit, conceptual hydrograph. So, this Clark's method is one of them and that we will discuss in detail today.

(Refer Slide Time: 1:38)

**Application of Routing in Conceptual Hydrograph Development**

- Concept of routing is also used in the conceptual studies of hydrographs.
- Attenuation and translation of an input hydrograph during routing are two basic modifying operators in this study.
- Based on these modifying operators, two hypothetical systems are developed which are the base for development of synthetic hydrographs through conceptual models i.e.,
  - ✓ **Linear reservoir**
  - ✓ **Linear channel**

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## Application of Routing in Conceptual Hydrograph Development

The concept of routing is also used in the conceptual studies of hydrographs. The attenuation and translation of an input hydrograph during the routing are the two basic modifying operators in this study. So, based on these modifying operators that are attenuation and translation, two hypothetical systems are developed which are the base for the development of these synthetic hydrographs through the conceptual approach, two conceptual models that we will consider. One is the linear reservoir; the other one is a linear channel.

(Refer Slide Time: 2:58)

**Application of Routing in Conceptual Hydrograph Development**

**Linear reservoir:**  
A reservoir in which the storage ( $S$ ) is directly proportional to the discharge ( $Q$ ) i.e.,  $S = KQ$ .

**Linear channel:**

- A hypothetical channel in which the time required to translate a discharge  $Q$  through a given reach is constant.
- An inflow hydrograph traverses such a channel with only translation but no attenuation.

In this lecture two conceptual models for development of Instantaneous Unit Hydrograph (IUH) are discussed i.e.,

Clark's Method      Nash's Conceptual Model

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## Application of Routing in Conceptual Hydrograph Development

### Linear reservoir:

A reservoir in which the storage ( $S$ ) is directly proportional to the discharge ( $Q$ ) i.e.,  $S = KQ$ .

### Linear channel:

A hypothetical channel in which the time required to translate a discharge  $Q$  through a given reach is constant. An inflow hydrograph traverses such a channel with only translation but no attenuation.

In this lecture two conceptual models for the development of Instantaneous Unit Hydrograph (IUH) are discussed i.e., i) Clark's Method; ii) Nash's Conceptual Model

(Refer Slide Time: 4:28)

**Clark's Method**

- Clark's method is also known as Time-Area histogram method.
- It aims at creating an IUH for a catchment as a result of an instantaneous rainfall excess, which is assumed to undergo pure translation (channel movement) first and then attenuation (reduced peak and elongated time base).
- The translation is achieved by a travel time-area histogram (by ignoring the storage of the catchment) at the catchment outlet.
- Then the attenuation is computed by routing the results of the above through a hypothetical linear reservoir at the catchment outlet.

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## Clark's Method

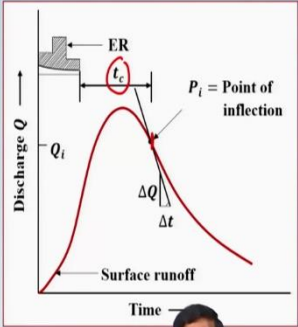
Clark's method is also known as another name is called the Time-Area histogram method and its target is to create an Instantaneous Unit Hydrograph for a catchment as a result of an instantaneous rainfall excess, which is assumed to undergo the pure translation first, that is the channel moment and then the attenuation the reduced peak and elongated time base.

The translation is achieved by a travel time-area histogram. And when it is computing the translation at that time the storage in the catchment is completely ignored. And then when we come to this attenuation, this attenuation is computed by routing the result of the above through the hypothetical linear reservoir at the catchment outlet.

(Refer Slide Time: 5:43)

**Clark's Method: Time-area curve**

- Here, time refers to the time of concentration ( $t_c$ ).
- It represents the maximum time of translation of the surface runoff of the catchment.
- In case of gauged basins, the time of concentration can be computed with the help of rainfall-runoff data.
- The rainfall-runoff data allows the computation of the elapsed time between the end of rainfall excess and the point of inflection on the recession limb of the resulting DRH/surface runoff hydrograph, which is the  $t_c$ .



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### Clark's Method: time-area curve

Here the time is referred to as the time of concentration ( $t_c$ ). Once the excess rainfall stops and from there up to the inflection point on the recession limb, the limb from this hydrograph is called the time of concentration. In other words, it is the time that takes for water to reach from the farthest point of the watershed to the outlet.

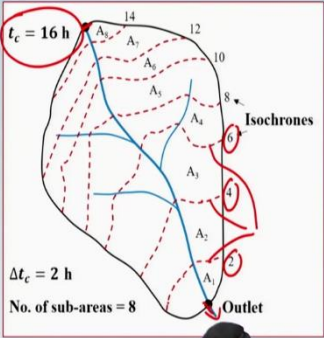
After the rainfall starts and thus once the time of concentration is reached and then the entire catchment is contributing to the outlet. So, it represents the maximum time of the translation of the surface runoff of this catchment. And in the case of a gauged basin, the time of concentration can be computed with the help of the rainfall-runoff data.

But when the rainfall-runoff data allows the computation of this elapsed time between the end of the rainfall excess and the point of inflection on the recession limb of the resulting direct runoff hydrograph or surface runoff hydrograph which is the  $t_c$ .

(Refer Slide Time: 7:13)

### Clark's Method: Time-area curve

- In case of ungauged basins, empirical formulae namely **US Practice**, **Kirpich Equation** and **Rainfall Intensity** are used to estimate  $t_c$  (discussed in previous lectures).
- The total catchment area drains into the outlet in  $t_c$  hours.
- The points on the watershed having equal time of travel, say  $t_1$  h (where  $t_1 < t_c$ ), are identified and plotted on catchment map. The line joining these points are called **Isochrones**.



The diagram shows a catchment area divided into 8 sub-areas (A<sub>1</sub> to A<sub>8</sub>) and an outlet. Isochrones are shown as dashed lines representing equal travel times (2, 4, 6, 8, 10, 12, 14, 16 hours). The total catchment area drains into the outlet in  $t_c = 16$  hours. The time interval between isochrones is  $\Delta t_c = 2$  h. The number of sub-areas is 8.

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In the case of an ungauged basin, the empirical formulae: the US practice, the Kirpich Equation, and the Rainfall Intensity are used to estimate the time of concentration if it is the ungauged basin.

The total catchment area drains into the outlet in  $t_c$  hours. That means when the  $t_c$  hours are elapsed from the furthest point it comes to this outlet. So, the entire catchment starts contributing. There is one example is given in Fig.1. For example, in this watershed in fig.1 that  $t_c$  is 16 hours. So, from this furthest point, it will take that 16 hours' time to reach the outlet.

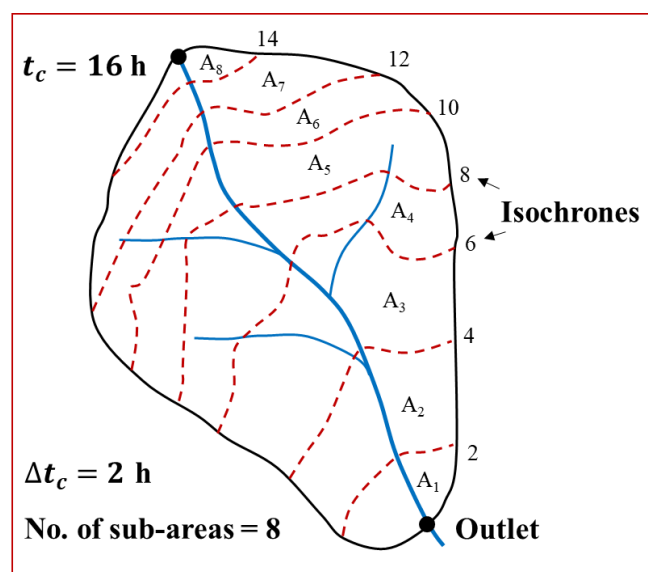


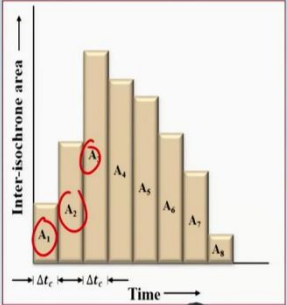
Figure 1: Isochrones in a Catchment

The points on the watershed having an equal time of travel, say  $t_1$  hour where  $t_1$  is less than  $t_c$  are identified and plotted on the catchment map and this line joins these points, points of the equal time of travel it is known as isochrones.

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### Clark's Method: Time-area curve

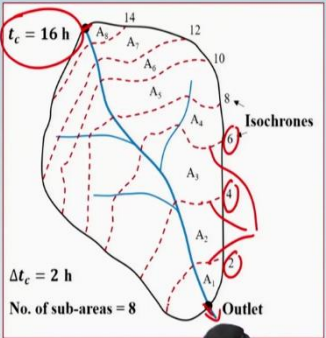
- The inter-isochrone areas  $A_1, A_2, \dots, A_N$  are used to construct a travel time-area histogram.
- The travel time-area histogram represents a sequence in which the volume of runoff will reach the catchment outlet if a rainfall excess of 1 cm occurs instantly and uniformly over the catchment area.
- In the figure a sub-area  $A_r \text{ km}^2$  represent a volume of  $A_r \text{ km}^2 \cdot \text{cm} = A_r \times 10^4 \text{ m}^3$  draining out in time  $\Delta t_c = \frac{t_c}{N} \text{ h}$ .
- The outflow hydrograph is obtained using this figure, while properly accounting for the sequence of arrival of flows. However, it does not account for the storage of the catchment.



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### Clark's Method: Time-area curve

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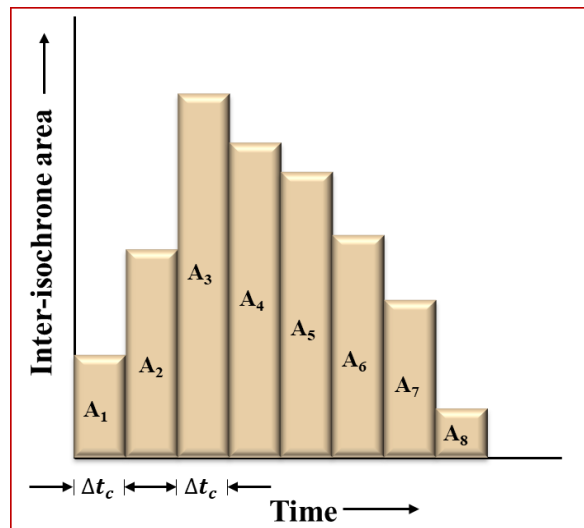
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The inter-isochrone areas  $A_1, A_2, \dots, A_N$  are used to construct a travel time-area histogram.

The travel time-area histogram represents a sequence in which the volume of runoff will reach the catchment outlet if a rainfall excess of 1 cm occurs instantly and uniformly over the catchment area.

In fig. 2 a sub-area  $A_r \text{ km}^2$  represent a volume of  $A_r \text{ km}^2 \cdot \text{cm} = A_r \times 10^4 \text{ m}^3$  draining out in time  $\Delta t_c = \frac{t_c}{N} \text{ h}$ .

The outflow hydrograph is obtained using this figure, while properly accounting for the sequence of arrival of flows. However, it does not account for the storage of the catchment.



**Figure 2 Time-area Histogram**

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**Clark's Method: Routing**

- To overcome the limitation of time-area histogram i.e. absence of the information on storage properties of the catchment, Clark assumed hypothetical linear reservoir at the outlet to provide necessary attenuation.
- Linear reservoir is described by the function  $S = KQ$ , where  $K$  is the storage time constant. The value of  $K$  is estimated by considering the point of inflection  $P_i$  of a surface runoff hydrograph.
- At point  $P_i$  the inflow to the channel has stopped and beyond this point only depletion of channel storage is taking place.

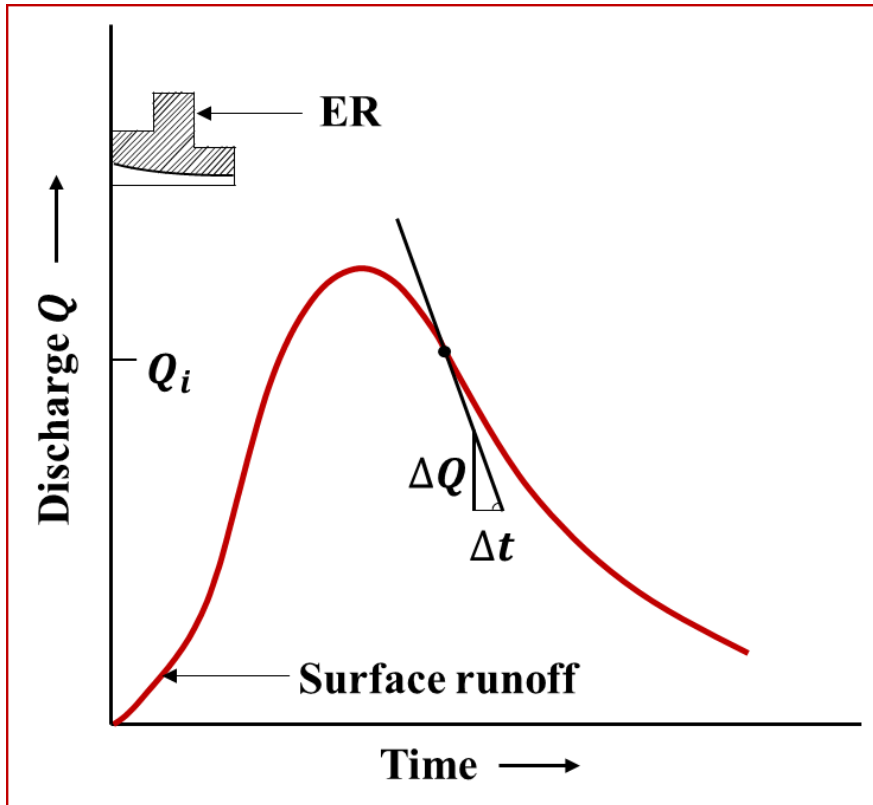
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### Clark's Method: Routing

To overcome the limitation of the time-area histogram i.e., the absence of the information on storage properties of the catchment, Clark assumed a hypothetical linear reservoir at the outlet to provide necessary attenuation.



A linear reservoir is described by the function  $S=KQ$ , where  $K$  is the storage time constant. The value of  $K$  is estimated by considering the point of inflection  $P_i$  of a surface runoff hydrograph as shown in fig.3. At point  $P_i$  the inflow to the channel has stopped and beyond this point, only depletion of channel storage is taking place.



**Figure 3: Surface Runoff of a Catchment**

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**Clark's Method: Routing**

- Therefore, from continuity equation

$$I - Q = \frac{dS}{dt}$$

$$\underline{-Q = \frac{dS}{dt} = K \frac{dQ}{dt}} \quad (\text{since } S = KQ)$$

$$\underline{K = -\frac{Q_i}{(dQ/dt)_i}}$$

The suffix  $i$  refers to the point of inflection, and  $K$  can be estimated either from the surface runoff of the catchments or from the recession limb data of a hydrograph.

Therefore, from the continuity equation

$$I - Q = \frac{dS}{dt}$$

$$-Q = \frac{dS}{dt} = K \frac{dQ}{dt} \quad (\text{Since, } S = KQ)$$


$$K = - \frac{Q_i}{(dQ/dt)_i}$$

The suffix  $i$  refers to the point of inflection, and  $K$  can be estimated either from the surface runoff of the catchments or from the recession limb data of a hydrograph.

(Refer Slide Time: 13:58)

**Clark's Method: Procedure**

- Knowing the value of  $K$  of the linear reservoir, the inflows at various times are routed by the Muskingum method.
- For a linear reservoir, the weightage factor  $x = 0$ , so the Muskingum storage equation  $S = K[xI + (1-x)Q]$  becomes  $S = KQ$ .
- The inflow rate between an inter-isochrone area  $A_r$ , km<sup>2</sup> with time interval  $\Delta t_c$  (h) is given as

$$I = \frac{A_r \times 10^4}{3600 \Delta t_c} = 2.78 \frac{A_r}{\Delta t_c} \text{ m}^3/\text{s}$$


### Clark's Method: Procedure

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$$I = \frac{A_r \times 10^4}{3600 \Delta t_c} = 2.78 \frac{A_r}{\Delta t_c} \text{ m}^3/\text{s}$$

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**Clark's Method: Procedure**

- The Muskingum routing equation is given as  $Q_2 = C_0I_2 + C_1I_1 + C_2Q_1$
- For  $x = 0$  the expressions of  $C_0, C_1, C_2$  become
$$C_0 = \frac{0.5\Delta t}{K + 0.5\Delta t} \quad C_1 = \frac{0.5\Delta t}{K + 0.5\Delta t} \quad C_2 = \frac{K - 0.5\Delta t}{K + 0.5\Delta t} \quad \text{i.e., } C_0 = C_1$$
- The inflows are derived from the histogram and each bar has the same width as the time interval. Thus, there is no change between starting and ending of each time interval. Hence, the values of  $I_1$  and  $I_2$  are equal for each interval.
- So the Muskingum routing equation becomes  $Q_2 = 2C_1I_1 + C_2Q_1$
- This routing of the time-area histogram using this equation will give the ordinates of IUH for the catchment. With the help of the obtained IUH any other  $D$ -h UH can be derived.

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➤ The Muskingum routing equation is given as  $Q_2 = C_0I_2 + C_1I_1 + C_2Q_1$

➤ For  $x = 0$  the expressions of  $C_0, C_1, C_2$  become

$$C_0 = \frac{0.5\Delta t}{K + 0.5\Delta t} \quad C_1 = \frac{0.5\Delta t}{K + 0.5\Delta t} \quad C_2 = \frac{K - 0.5\Delta t}{K + 0.5\Delta t} \quad \text{i.e., } C_0 = C_1$$

➤ The inflows are derived from the histogram and each bar has the same width as the time interval. Thus, there is no change between starting and ending of each time interval. Hence, the values of  $I_1$  and  $I_2$  are equal for each interval.

➤ So, the Muskingum routing equation becomes  $Q_2 = 2C_1I_1 + C_2Q_1$

➤ This routing of the time-area histogram using this equation will give the ordinates of IUH for the catchment. With the help of the obtained IUH any other  $D$ -h UH can be derived.


(Refer Slide Time: 17:26)

**Example 46.1**

Inter-isochrone area distribution of a drainage basin with 100 km<sup>2</sup> area is given. Assume the time of concentration = 18 h and storage constant = 10 h.

Travel Time (h)	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
Inter-Isochrone area (km <sup>2</sup> )	4	10	24	21	18	20	12	6	3

Determine the IUH for this catchment and make a plot.



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**Example 46.1**

Inter-isochrone area distribution of a drainage basin with a 100 km<sup>2</sup> area is given. Assume the time of concentration = 18 h and storage constant = 10 h.

Travel Time (h)	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
Inter-Isochrone area (km <sup>2</sup> )	4	10	24	21	18	20	12	6	3

Determine the IUH for this catchment and make a plot.

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**Solution**

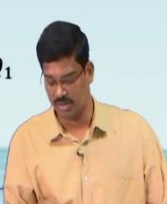
- Given  $K = 10 \text{ h}$ ,  $t_c = 18 \text{ h}$  and  $\Delta t_c = 2 \text{ h}$

$$\underline{C_1} = \frac{0.5\Delta t}{K + 0.5\Delta t} = \frac{0.5 \times 2}{10 + 0.5 \times 2} = 0.09, \quad \underline{C_2} = \frac{K - 0.5\Delta t}{K + 0.5\Delta t} = \frac{10 - 0.5 \times 2}{10 + 0.5 \times 2} = 0.82$$

- Inflow rate between an inter-isochrone area  $A_r$  in time interval  $\Delta t_c = 2 \text{ h}$  is given by

$$\underline{I} = \frac{A_r \times 10^4}{3600 \times \Delta t_c} = \frac{A_r \times 10^4}{3600 \times 2} = 1.39 \times A_r \text{ m}^3/\text{s}$$

- The ordinates of the IUH is given by  $Q_2 = 2C_1I_1 + C_2Q_1 \text{ m}^3/\text{s}$
- The ordinates of the IUH at the starting of 2<sup>nd</sup> time step is  $Q_2 = 2C_1I_1 + C_2Q_1 = 2 \times 0.09 \times 1.39 \times 4 + 0.82 \times 0 = 1 \text{ m}^3/\text{s} = Q_1$
- The above steps are repeated.



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## Solution

Given  $K = 10 \text{ h}$ ,  $t_c = 18 \text{ h}$  and  $\Delta t_c = 2 \text{ h}$

$$C_1 = \frac{0.5\Delta t}{K+0.5\Delta t} = \frac{0.5 \times 2}{10+0.5 \times 2} = 0.09, \quad C_2 = \frac{K-0.5\Delta t}{K+0.5\Delta t} = \frac{10-0.5 \times 2}{10+0.5 \times 2} = 0.82$$

Inflow rate between an inter-isochrone area  $A_r$  in time interval  $\Delta t_c = 2 \text{ h}$  is given by

$$I = \frac{A_r \times 10^4}{3600 \times \Delta t_c} = \frac{A_r \times 10^4}{3600 \times 2} = 1.39 \times A_r \text{ m}^3/\text{s}$$

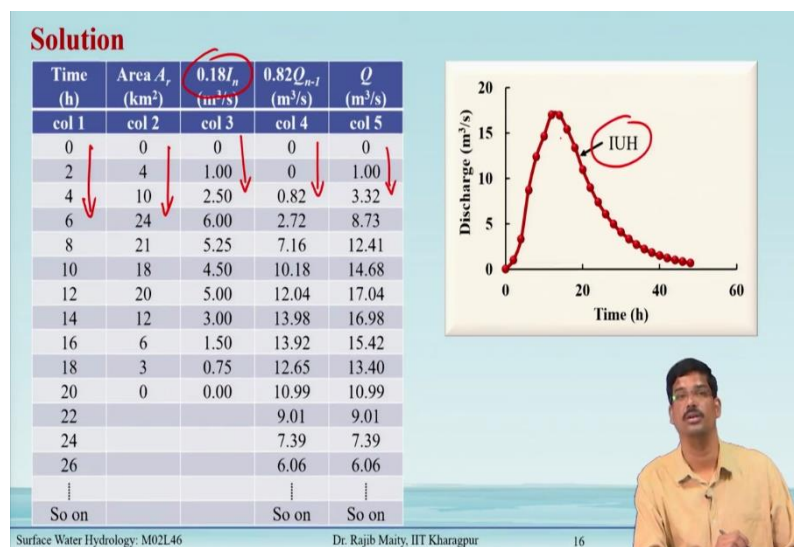
The ordinates of the IUH are given by  $Q_2 = 2C_1I_1 + C_2Q_1 \text{ m}^3/\text{s}$

The ordinates of the IUH at the starting of the 2<sup>nd</sup>-time step are,

$$Q_2 = 2C_1I_1 + C_2Q_1 = 2 \times 0.09 \times 1.39 \times 4 + 0.82 \times 0 = 1 \text{ m}^3/\text{s} = Q_1$$

The above steps are repeated.

(Refer Slide Time: 19:54)



Time (h)	Area $A_r$ (km <sup>2</sup> )	$0.18I_n$ (m <sup>3</sup> /s)	$0.82Q_{n-1}$ (m <sup>3</sup> /s)	$Q$ (m <sup>3</sup> /s)
col 1	col 2	col 3	col 4	col 5
0	0	0	0	0
2	4	1.00	0	1.00
4	10	2.50	0.82	3.32
6	24	6.00	2.72	8.73
8	21	5.25	7.16	12.41
10	18	4.50	10.18	14.68
12	20	5.00	12.04	17.04
14	12	3.00	13.98	16.98
16	6	1.50	13.92	15.42
18	3	0.75	12.65	13.40
20	0	0.00	10.99	10.99
22			9.01	9.01
24			7.39	7.39
26			6.06	6.06
⋮			⋮	⋮
So on			So on	So on

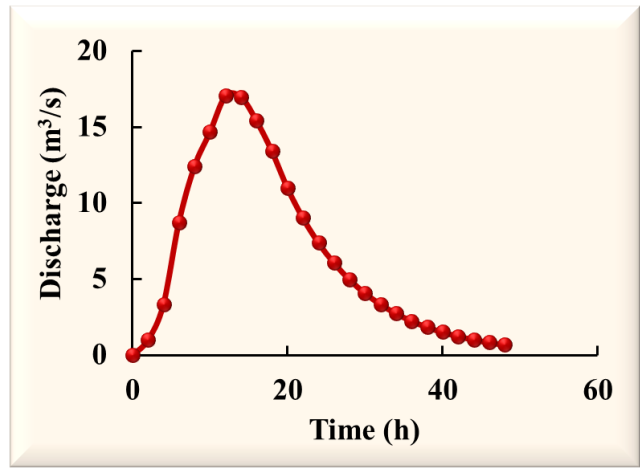



Figure 4: Instantaneous Unit Hydrograph of example 46.1

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**Summary**

- Routing can be used in the development of Instantaneous Unit Hydrograph (IUH).
- Two hypothetical systems, i.e., linear reservoir and linear channel, are the two basic modifying factor behind the concept of routing in development of IUH.
- Clark's method which utilizes time-area curve along with Muskingum routing concept to develop IUH are discussed in this lecture with solved example.
- In the next lecture, another method for development of IUH, i.e., Nash's conceptual model will be discussed.



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## Summary

In summary, we learned the following points from this lecture:

- Routing can be used in the development of Instantaneous Unit Hydrograph (IUH).
- Two hypothetical systems, i.e., linear reservoir and linear channel, are the two basic modifying factors behind the concept of routing in the development of IUH.
- Clark's method which utilizes the time-area curve along with the Muskingum routing concept to develop IUH is discussed in this lecture with a solved example.
- In the next lecture, another method for the development of IUH, i.e., Nash's conceptual model will be discussed.