Surface Water Hydrology
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Lecture – 09
Areal Precipitation and Frequency Analysis

In week 2, we are overall discussing the hydrological analysis of precipitation. In this lecture, we will cover areal precipitation and its frequency analysis.

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The major two concepts will be covered. The First one is mean precipitation over an area and from there one important relationship that is depth area relationship will be discussed. And finally, we will see some initial introduction of frequency analysis.

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The outline of this lecture goes like this. First, we will give an introduction to why this areal precipitation is different from the point precipitation and how to estimate the areal precipitation from the point is way different from the rain gauge. Then we will proceed to the concept of depth area duration curves, and their interrelationship between depth and area and duration. So, next, we will take some procedures on how to develop these DAD curves. And finally, frequency analysis of point rainfall, and summary at the end.

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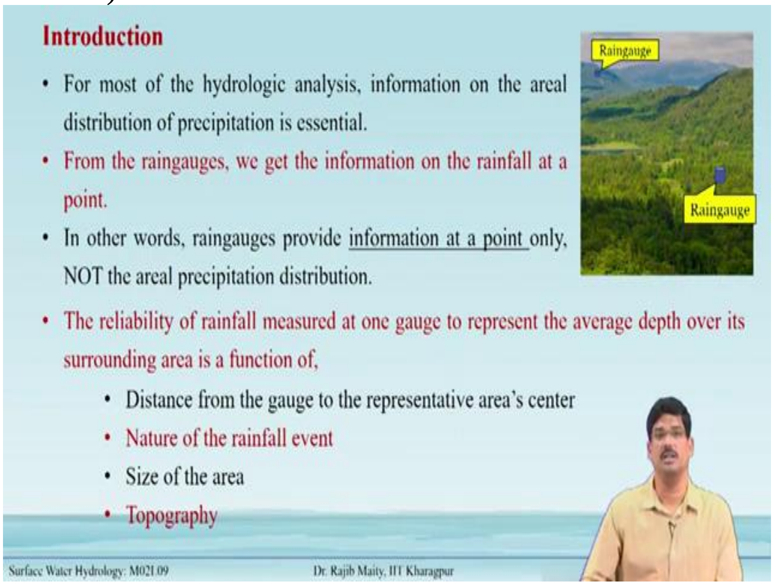

Introduction

Most of the hydrologic analysis, information on the areal distribution of the precipitation is essential. The rain gauge stations are give you the precipitation at that location.

But, the rain gauge provides information at a point only, not the areal precipitation distribution. And this reliability of the rainfall is measured as one gauge to represent and the average depth over its surroundings depends on various factors.

- Distance from the gauge to the representative area’s center
- Nature of the rainfall event
- Size of the area
- Topography

Depending on the nature of the storm or rainfall event its reliability of over a larger area depends on, thirdly the size of the area.. So, these are some of the considerations and generally, it should be considered when we are taking the point rainfalland using the point rainfall to estimate the areal distribution of the rainfall.

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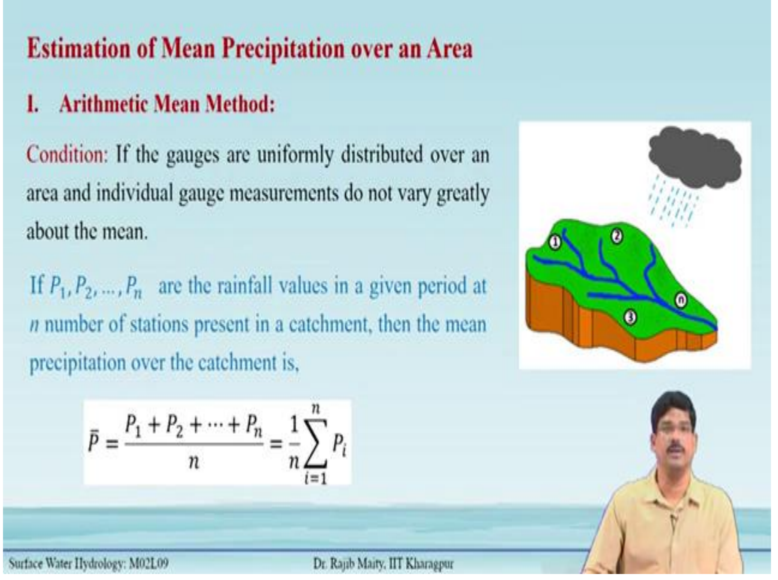

Estimation of Mean Precipitation over an Area

i. Arithmetic Mean Method:

Condition: If the gauges are uniformly distributed over an area and individual gauge measurements do not vary greatly about the mean. If P_1, P_2,…, P_n are the rainfall values in a given period at n number of stations present in a catchment, then the mean precipitation over the catchment is,

$$\bar{P} = \frac{P_1 + P_2 + \cdots + P_n}{n} = \frac{1}{n}\sum_{i=1}^{n} P_i$$

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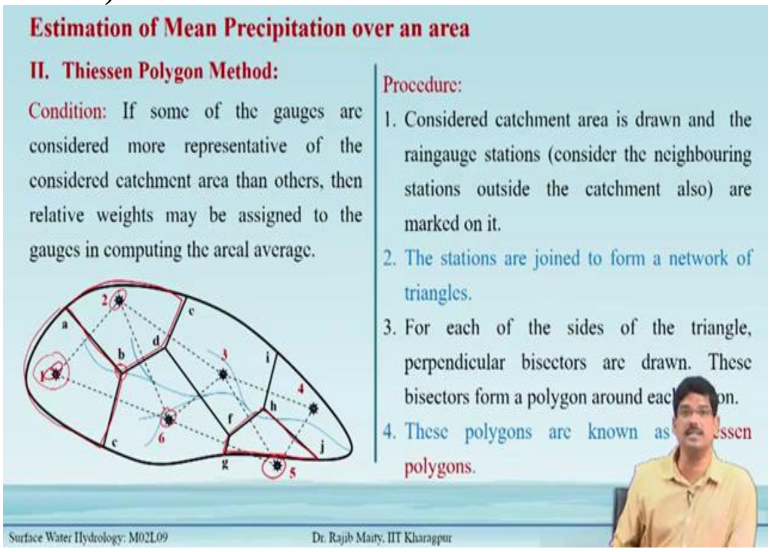

ii. Thiessen Polygon Method:

Condition: If some of the gauges are considered more representative of the considered catchment area than others, then relative weights may be assigned to the gauges in computing the areal average.

Procedure:

1. Considered catchment area is drawn and the rain gauge stations (consider the neighboring stations outside the catchment also) are marked on it.
2. The stations are joined to form a network of triangles.
3. For each of the sides of the triangle, perpendicular bisectors are drawn. These bisectors form a polygon around each station.
4. These polygons are known as Thiessen polygons.

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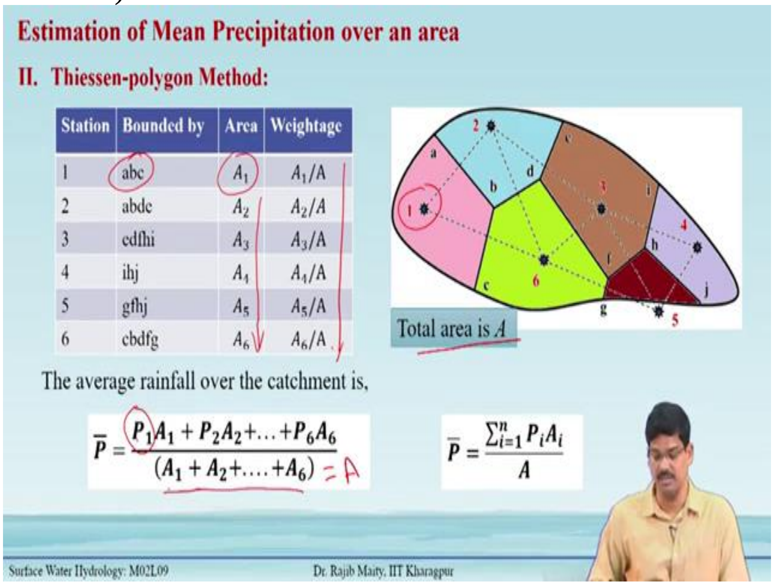

In Fig.1, you can see that these things are basically highlighted in different color and these are area replenishes, and for each of these engages rain gauge stations. So these areas are called the Thiessen polygon. Now, here if the total area entire catchment is having the area A. Then for each of these polygons, we estimate the areas.

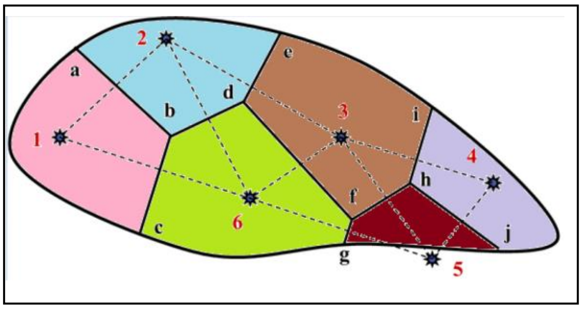

Fig.1 shows the Thiessen Polygons

Station	Bounded by	Area	Weightage
1	abc	A_1	A_1/A
2	abde	A_2	A_2/A
3	edfhi	A_3	A_3/A
4	ihj	A_4	A_4/A
5	gfhj	A_5	A_5/A
6	cbdfg	A_6	A_6/A

$$\overline{P} = \frac{\sum_{i=1}^{n} P_i A_i}{A}$$

The average rainfall over the catchment is,

$$\overline{P} = \frac{P_1 A_1 + P_2 A_2 + \ldots + P_6 A_6}{(A_1 + A_2 + \ldots . + A_6)}$$

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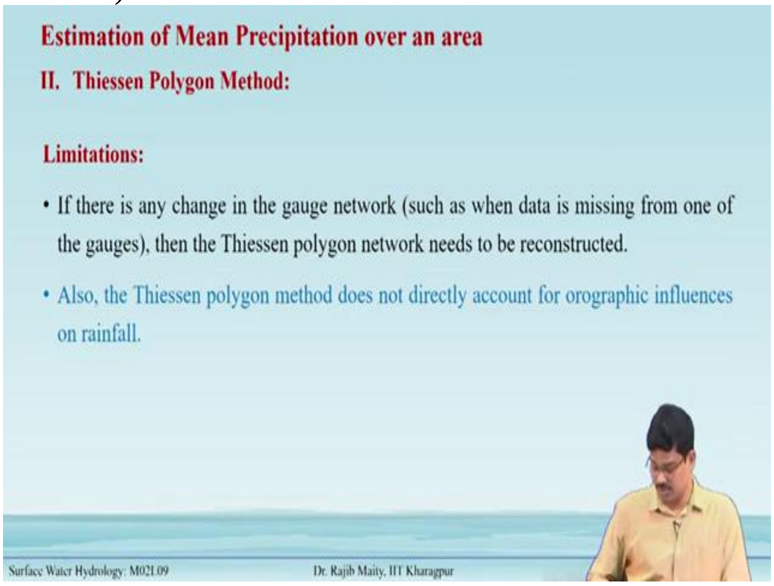

Limitations:

- If there is any change in the gauge network (such as when data is missing from one of the gauges), then the Thiessen polygon network needs to be reconstructed.
- Also, the Thiessen polygon method does not directly account for orographic influences on rainfall.

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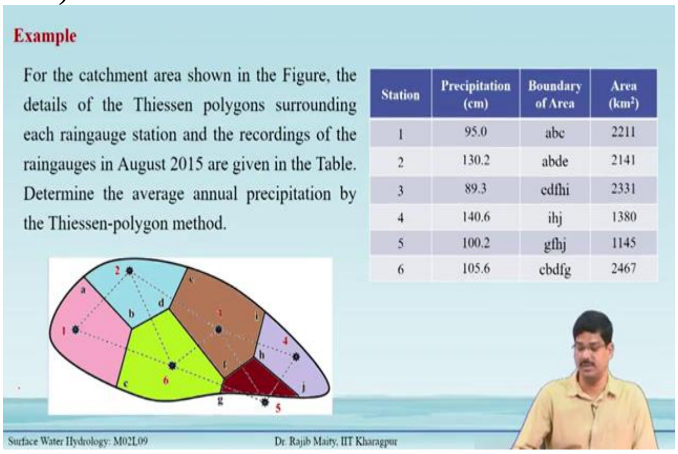

Example

For the catchment area shown in the Figure, the details of the Thiessen polygons surrounding each rain gauge station and the recordings of the rain gauges in August 2015 are given in the Table. Determine the average annual precipitation by the Thiessen-polygon method.

Station	Precipitation (cm)	Boundary of Area	Area (km^2)
1	95.0	abc	2211
2	130.2	abde	2141
3	89.3	edfhi	2331
4	140.6	ihj	1380
5	100.2	gfhj	1145
6	105.6	cbdfg	2467

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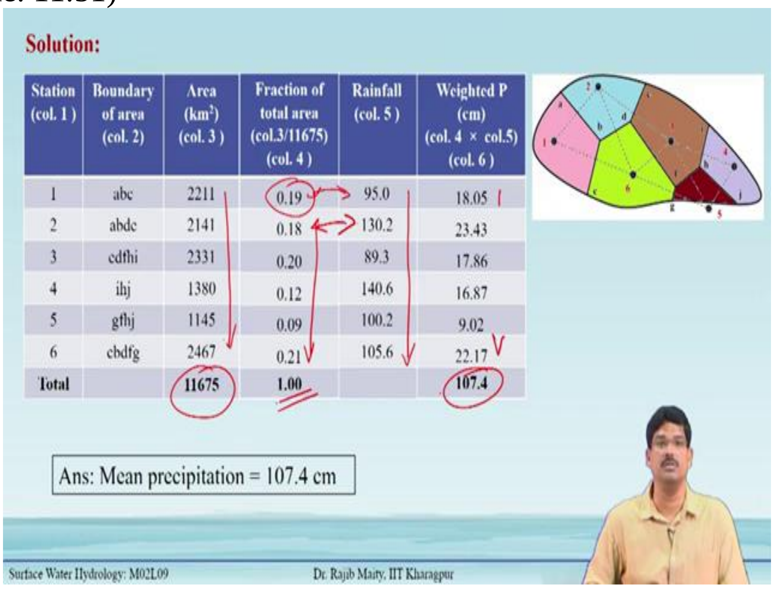
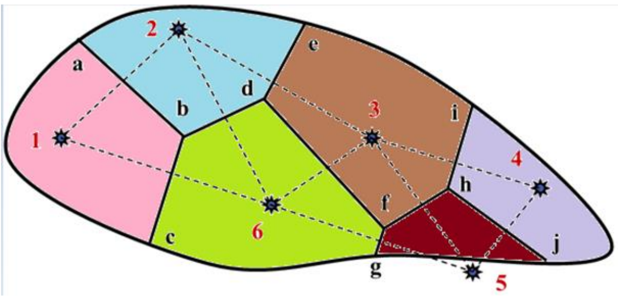


Solution



Station (col. 1)	Boundary of area (col. 2)	Area (km^2) (col. 3)	Fraction of total area (col.3/11675) (col. 4)	Rainfall (col. 5)	Weighted P (cm) (col. 4 × col.5) (col. 6)
1	abc	2211	0.19	95.0	18.05
2	abde	2141	0.18	130.2	23.43
3	edfhi	2331	0.20	89.3	17.86
4	ihj	1380	0.12	140.6	16.87
5	gfhj	1145	0.09	100.2	9.02
6	cbdfg	2467	0.21	105.6	22.17
Total		**11675**	**1.00**		**107.4**

Ans: Mean precipitation = 107.4 cm

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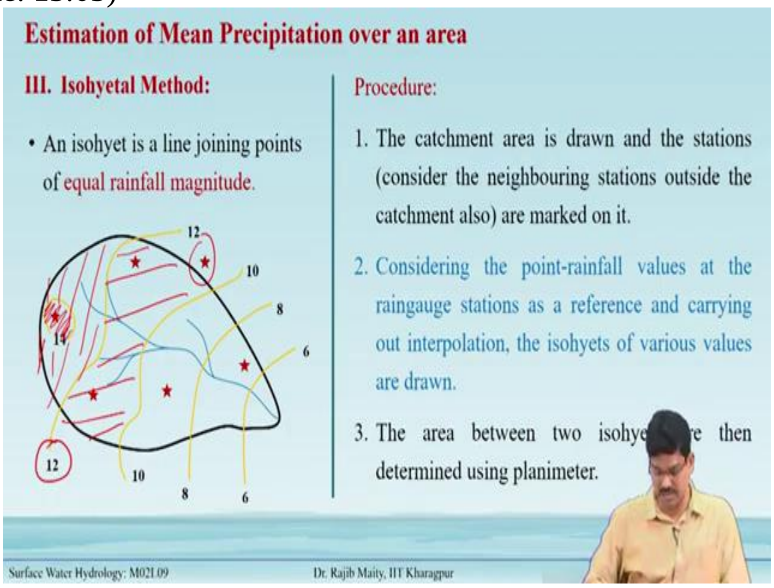

III. Isohyetal Method:

An isohyet is a line joining points of equal rainfall magnitude. So, it is a control map and those control is showing the rainfall magnitude over an area.

Procedure:

1. The catchment area is drawn and the stations (consider the neighboring stations outside the catchment also) are marked on it.
2. Considering the point-rainfall values at the rain gauge stations as a reference and carrying out interpolation, the isohyets of various values are drawn.
3. The area between two isohyets are then determined using the planimeter

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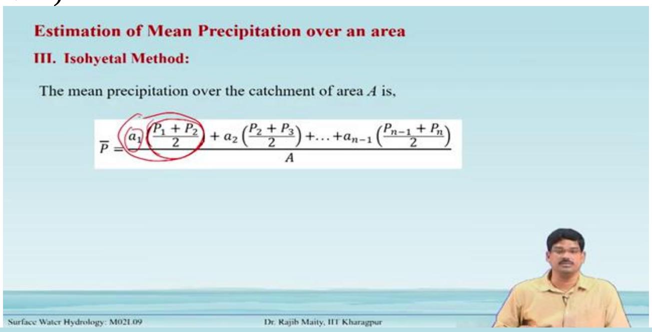

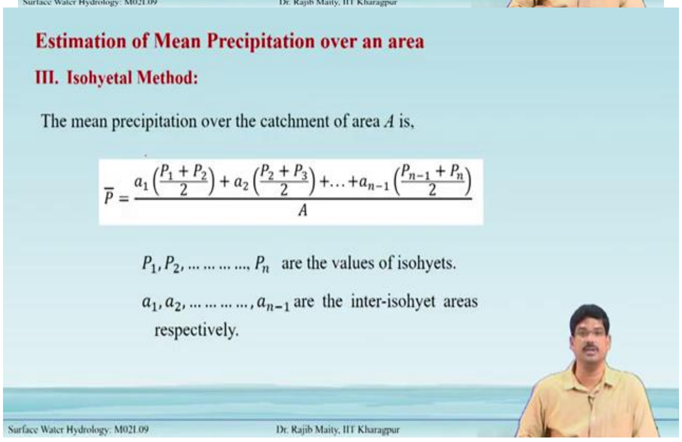

The mean precipitation over the catchment of area A is,

$$\overline{P} = \frac{a_1\left(\frac{P_1+P_2}{2}\right) + a_2\left(\frac{P_2+P_3}{2}\right) + \ldots + a_{n-1}\left(\frac{P_{n-1}+P_n}{2}\right)}{A}$$

$P_1, P_2 \ldots P_n$ are the values of isohyets.

$a_1, a_2 \ldots a_{n-1}$ are the inter-isohyet areas respectively.

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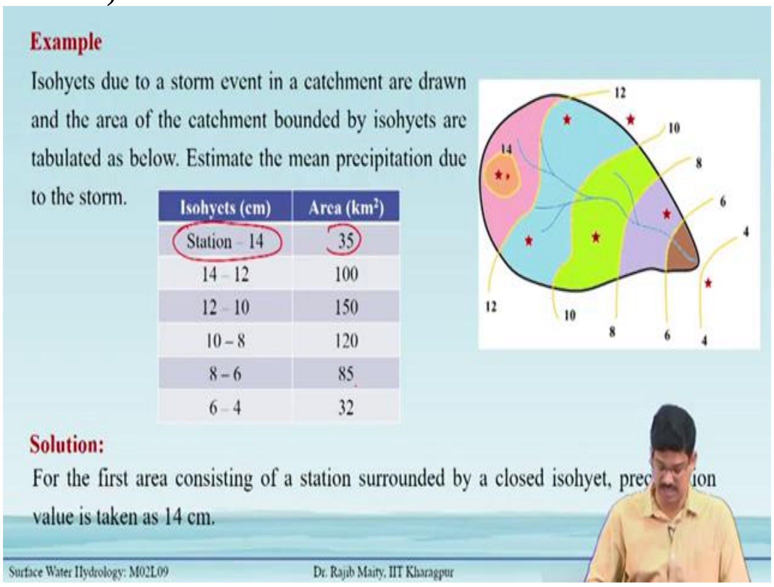
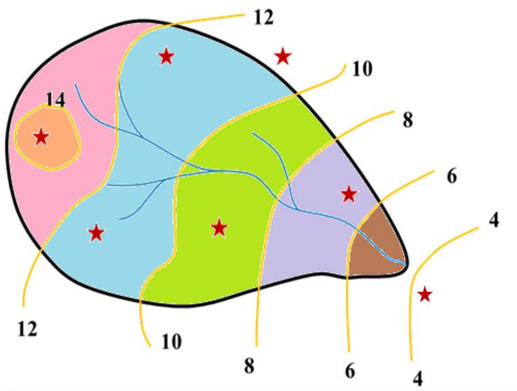


Example

Isohyets due to a storm event in a catchment are drawn and the area of the catchment bounded by isohyets are tabulated as below. Estimate the mean precipitation due to the storm.

Isohyets (cm)	Area (km^2)
Station – 14	35
14 – 12	100
12 – 10	150
10 – 8	120
8 – 6	85
6 – 4	32



Solution

For the first area consisting of a station surrounded by a closed isohyet, precipitation value is taken as 14 cm.

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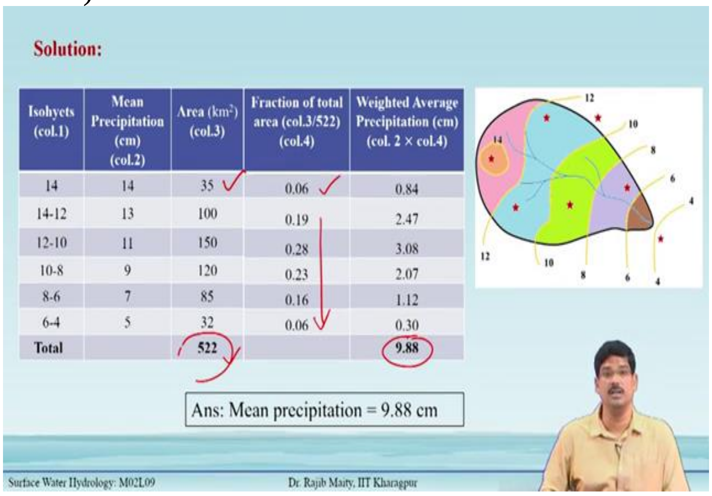

Isohyets (col.1)	Mean Precipitation (cm) (col.2)	Area (km^2) (col.3)	Fraction of total area (col.3/522) (col.4)	Weighted Average Precipitation (cm) (col. 2 × col.4)
14	14	35	0.06	0.84
14-12	13	100	0.19	2.47
12-10	11	150	0.28	3.08
10-8	9	120	0.23	2.07
8-6	7	85	0.16	1.12
6-4	5	32	0.06	0.30
TOTAL		**522**		**9.88**

Ans: Mean precipitation = 9.88 cm

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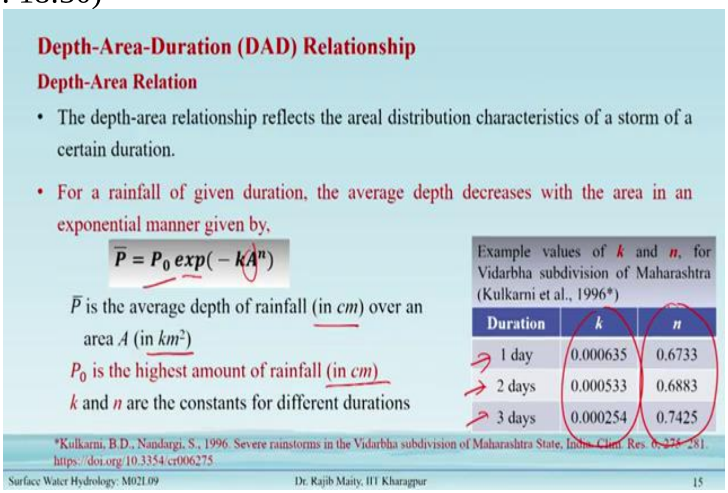

Depth-Area-Duration (DAD) Relationship

Depth-Area Relation

The depth-area relationship reflects the areal distribution characteristics of a storm of a certain duration. For a rainfall of given duration, the average depth decreases with the area in an exponential manner given by,

$$\overline{P} = P_0\, exp(-kA^n)$$

$\overline{P}$ is the average depth of rainfall (in cm) over an area A (in km^2)

P_0 is the highest amount of rainfall (in cm)

k and n are the constants for different durations

Example values of ***k*** and ***n***, for Vidarbha subdivision of Maharashtra (Kulkarni et al., 1996*)

Duration	*k*	*n*
1 day	0.000635	0.6733
2 days	0.000533	0.6883
3 days	0.000254	0.7425

*Kulkarni, B.D., Nandargi, S., 1996. Severe rainstorms in the Vidarbha subdivision of Maharashtra State, India. Clim. Res. 6, 275–281. https://doi.org/10.3354/cr006275

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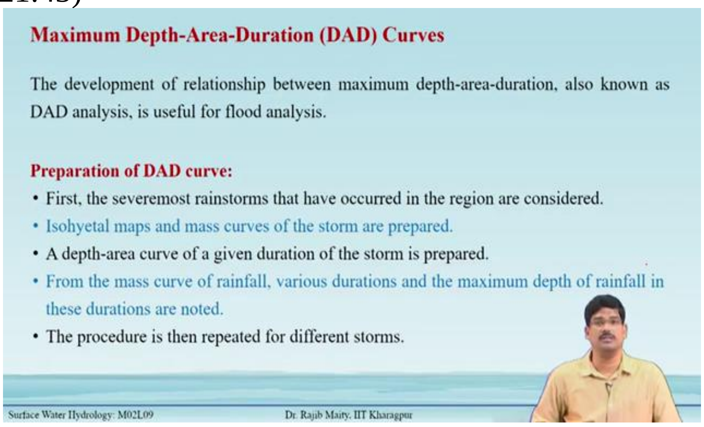

The development of the relationship between maximum depth-area-duration, also known as DAD analysis, is useful for flood analysis.

Preparation of DAD curve:

- First, the severe most rainstorms that have occurred in the region are considered.
- Isohyetal maps and mass curves of the storm are prepared.
- A depth-area curve of a given duration of the storm is prepared.
- From the mass curve of rainfall, various durations and the maximum depth of rainfall in these durations are noted.
- The procedure is then repeated for different storms.

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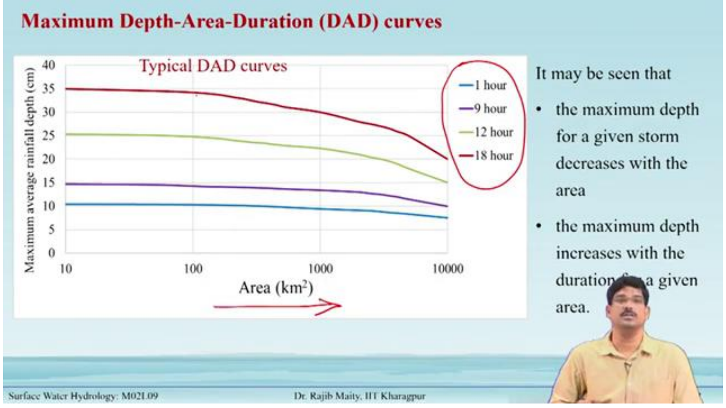

A typical DAD curve shown in fig.2, for different durations. And these durations are kept hold and for the different colour as we have shown. So, the lowermost one is basically for 1 hour, then 9 hours, 12 hours, and 18 hours, these four lines are shown here. So, one observation is that as the area. So, as the area increases the maximum average rainfall depth decreases. And this decrement is more for the higher durations. Another thing is that the maximum depth increases with the duration as it is, as it is given for a particular region.

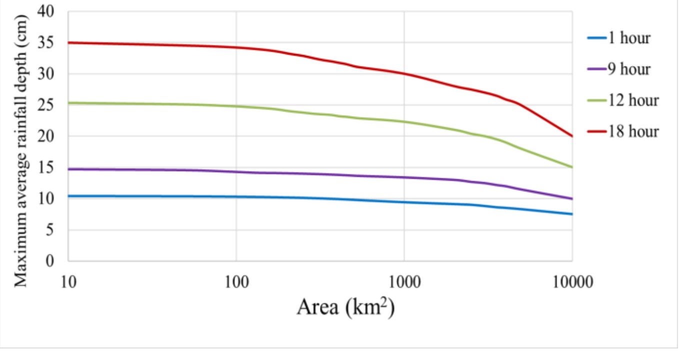

Fig.2 shows the DAD curve

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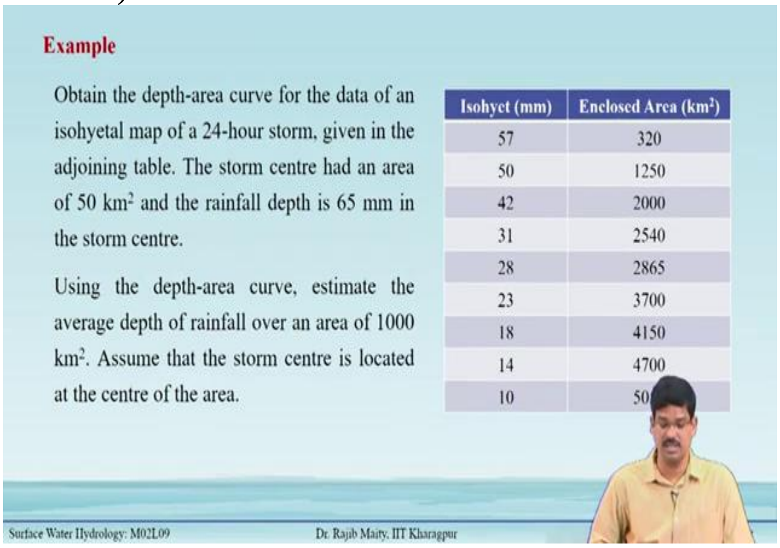

Example

Obtain the depth-area curve for the data of an Isohyetal map of a 24-hour storm, given in the adjoining table. The storm center had an area of 50 km^2 and the rainfall depth is 65 mm in the storm canter. Using the depth-area curve, estimate the average depth of rainfall over an area of 1000 km^2. Assume that the storm center is located at the centre of the area

Isohyet (mm)	Enclosed Area (km^2)
57	320
50	1250
42	2000
31	2540
28	2865
23	3700
18	4150
14	4700
10	5050

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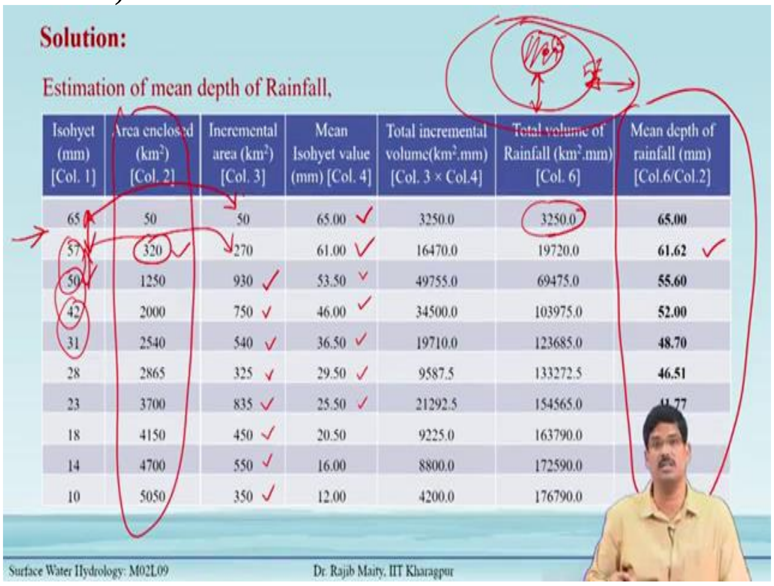

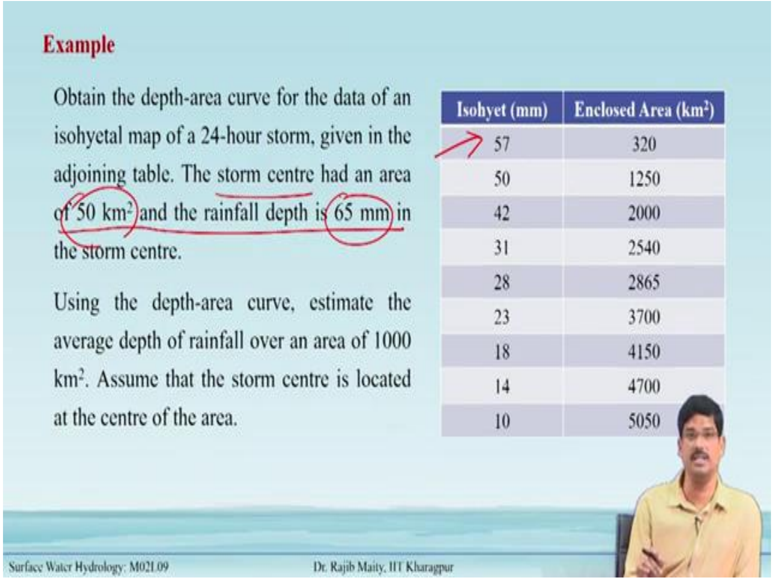

Solution

Estimation of mean depth of Rainfall,

Isohyet (mm) [Col. 1]	Area enclosed (km^2) [Col. 2]	Incremental area (km^2) [Col. 3]	Mean Isohyet value (mm) [Col. 4]	Total incremental volume(km^2.mm) [Col. 3 × Col.4]	Total volume of Rainfall (km^2.mm) [Col. 6]	Mean depth of rainfall (mm) [Col.6/Col.2]
65	50	50	65.00	3250.0	3250.0	**65.00**
57	320	270	61.00	16470.0	19720.0	**61.62**
50	1250	930	53.50	49755.0	69475.0	**55.60**
42	2000	750	46.00	34500.0	103975.0	**52.00**
31	2540	540	36.50	19710.0	123685.0	**48.70**
28	2865	325	29.50	9587.5	133272.5	**46.51**
23	3700	835	25.50	21292.5	154565.0	**41.77**
18	4150	450	20.50	9225.0	163790.0	**39.46**
14	4700	550	16.00	8800.0	172590.0	**36.72**
10	5050	350	12.00	4200.0	176790.0	**35.00**

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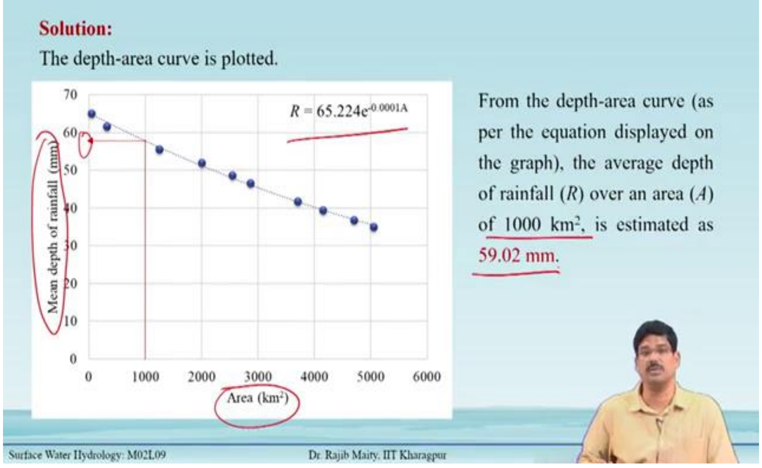

Solution

The depth-area curve is plotted.

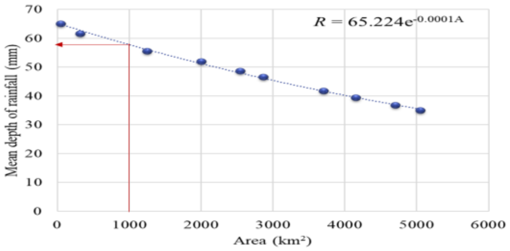

Fig.3 shows the depth-area curve

From the depth-area curve (as per the equation displayed on the graph), the average depth of rainfall (R) over an area (A) of 1000 km^2, is estimated as 59.02 mm.

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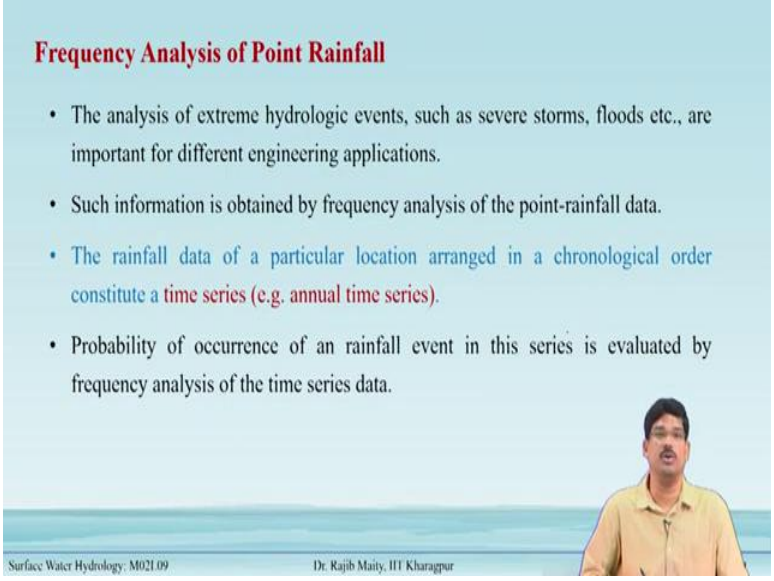

Frequency Analysis of Point Rainfall

- The analysis of extreme hydrologic events, such as severe storms, floods, etc., is important for different engineering applications.
- Such information is obtained by frequency analysis of the point-rainfall data.
- The rainfall data of a particular location arranged in a chronological order constitute a time series (e.g. annual time series).
- The probability of occurrence of a rainfall event in this series is evaluated by frequency analysis of the time series data.

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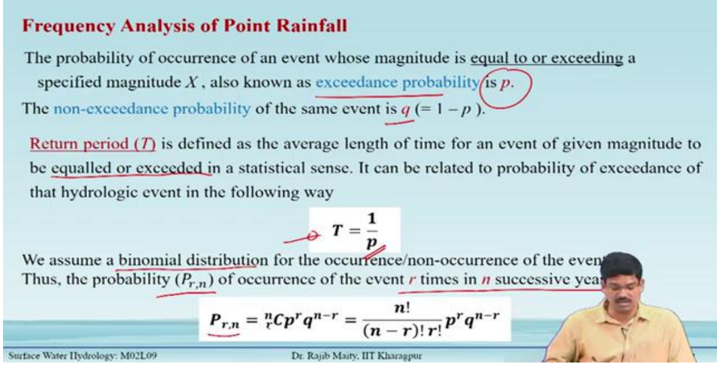

The probability of occurrence of an event whose magnitude is equal to or exceeding a specified magnitude *X*, also known as exceedance probability is *p*.

The non-exceedance probability of the same event is $q = (1 - p)$.

Return period (T) is defined as the average length of time for an event of a given magnitude to be equaled or exceeded in a statistical sense. It can be related to the probability of exceedance of that hydrologic event in the following way

T=1/P

We assume a binomial distribution for the occurrence/non-occurrence of the event.

Thus, the probability ($P_{r,n}$) of occurrence of the event r times in n successive years is:

$$P_{r,n} = {}^{n}_{r}Cp^{r}q^{n-r} = \frac{n!}{(n-r)!\,r!}p^{r}q^{n-r}$$

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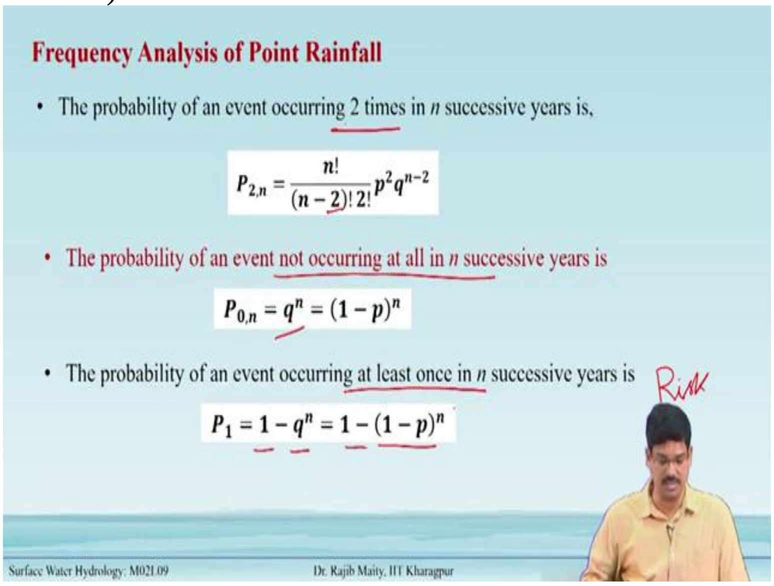

The probability of an event occurring 2 times in n successive years is,

$$P_{2,n} = \frac{n!}{(n-2)!\,2!}p^{2}q^{n-2}$$

The probability of an event not occurring at all in n successive years is

$$P_{0,n} = q^{n} = (1-p)^{n}$$

The probability of an event occurring at least once in n successive years is

$$\boldsymbol{P_1 = 1 - q^n = 1 - (1 - p)^n}$$

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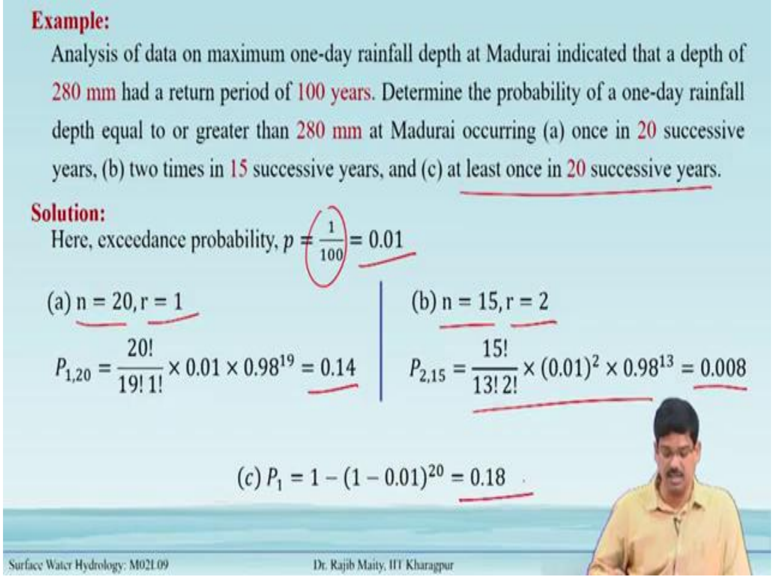

Example

Analysis of data on maximum one-day rainfall depth at Madurai indicated that a depth of 280 mm had a return period of 100 years. Determine the probability of a one-day rainfall depth equal to or greater than 280 mm at Madurai occurring (a) once in 20 successive years, (b) two times in 15 successive years, and (c) at least once in 20 successive years

Solution

Here, exceedance probability, p=1/100=0.01

(a) n=20, r=1

$$P_{1,20} = \frac{20!}{19!\,1!} \times 0.01 \times 0.98^{19} = 0.14$$

(b) n=15, r=2

$$P_{2,15} = \frac{15!}{13!\,2!} \times (0.01)^2 \times 0.98^{13} = 0.008$$

(c) $P_1=1-(1-0.01)^{20}=0.18$

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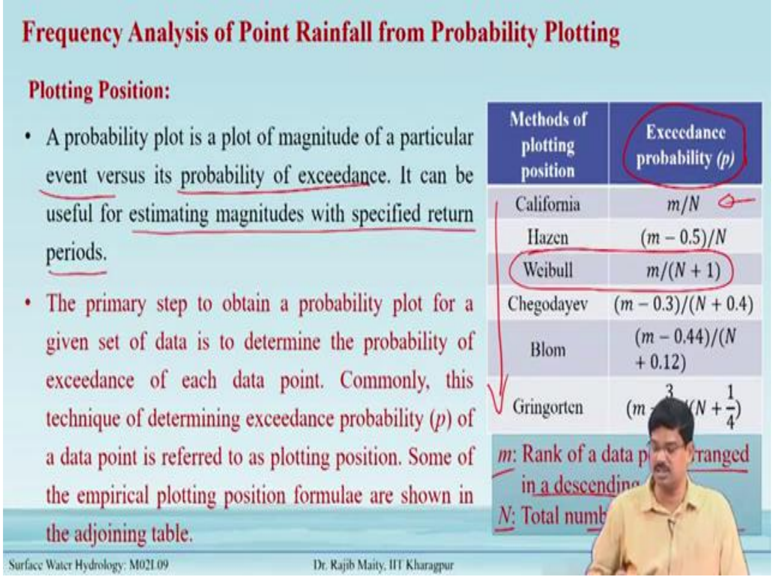

Frequency Analysis of Point Rainfall from Probability Plotting

Plotting Position:

A probability plot is a plot of magnitude of a particular event versus its probability of exceedance. It can be useful for estimating magnitudes with specified return periods. The primary step to obtain a probability plot for a given set of data is to determine the probability of exceedance of each data point. Commonly, this technique of determining exceedance probability (p) of a data point is referred to as plotting position. Some of the empirical plotting position formulae are shown in the adjoining table.

Methods of plotting position	Exceedance probability *(p)*
California	m/N
Hazen	$(m-0.5)/N$
Weibull	$m/(N+1)$
Chegodayev	$(m-0.3)/(N+0.4)$
Blom	$(m-0.44)/(N+0.12)$
Gringorten	$(m-\frac{3}{8})/(N+\frac{1}{4})$

where, Rank of a data point arranged in a descending order

N: Total number of data points

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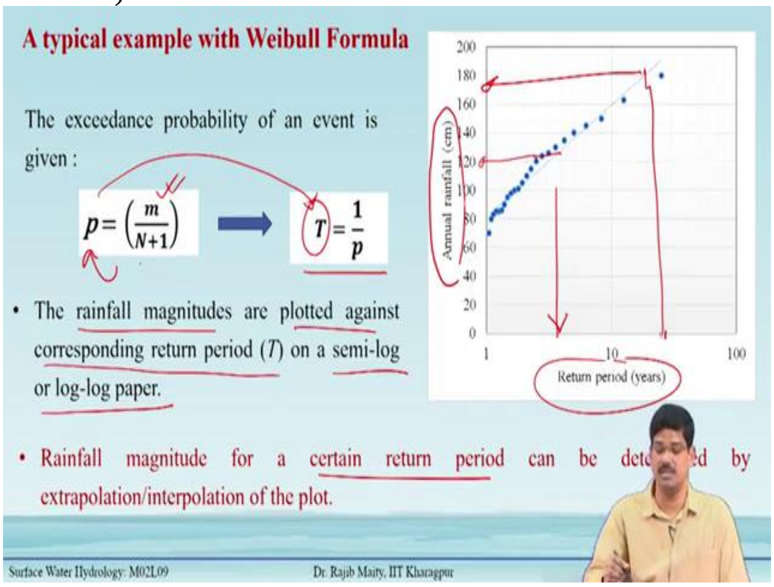

The exceedance probability of one particular event is

$$P=m/(n+1)$$

$$T=1/P$$

Now, this rainfall magnitude is plotted against the corresponding return period which is shown in fig.4. In this case, we are plotting in a semi-log or log paper. In the x-axis, it is the return period which is this There and y-axis it is the annual rainfall magnitude as the rainfall magnitude that we arrange it in the descending order, get the rank, from there we get the exceedance probability and then we get this return period.

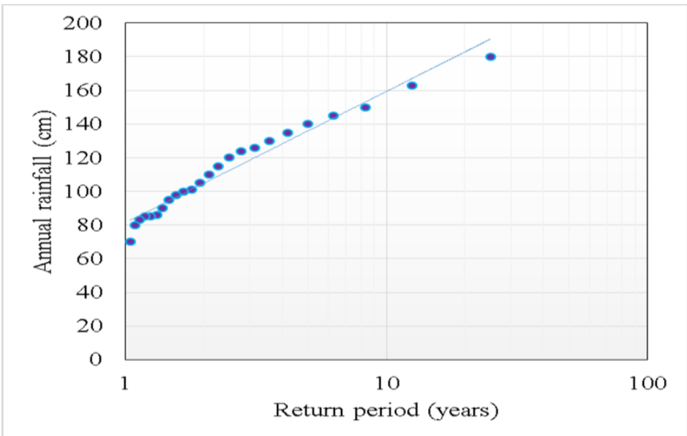

Fig.4 shows the plot between annual rainfall vs return period

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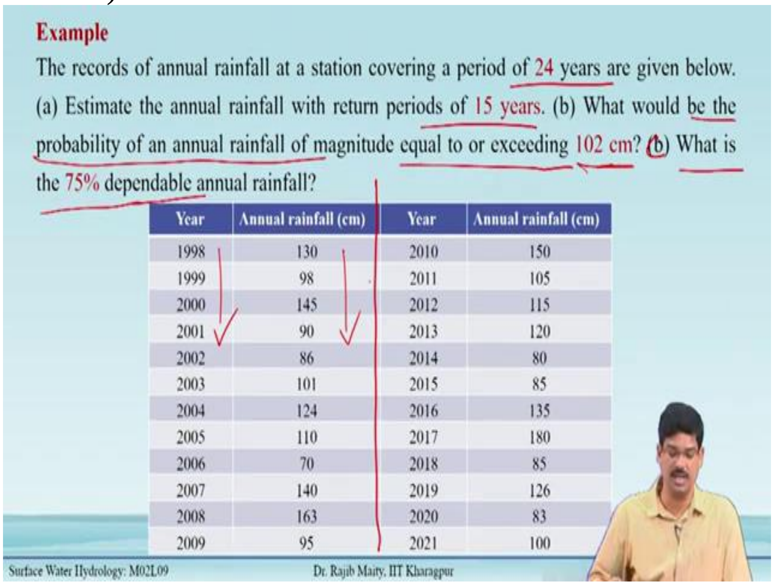

Example

The records of annual rainfall at a station covering a period of 24 years are given below. (a) Estimate the annual rainfall with return periods of 15 years. (b) What would be the probability of an annual rainfall of magnitude equal to or exceeding 102 cm? (b) What is the 75% dependable annual rainfall?

Year	Annual rainfall (cm)	Year	Annual rainfall (cm)
1998	130	2010	150
1999	98	2011	105
2000	145	2012	115
2001	90	2013	120
2002	86	2014	80
2003	101	2015	85
2004	124	2016	135
2005	110	2017	180
2006	70	2018	85
2007	140	2019	126
2008	163	2020	83
2009	95	2021	100

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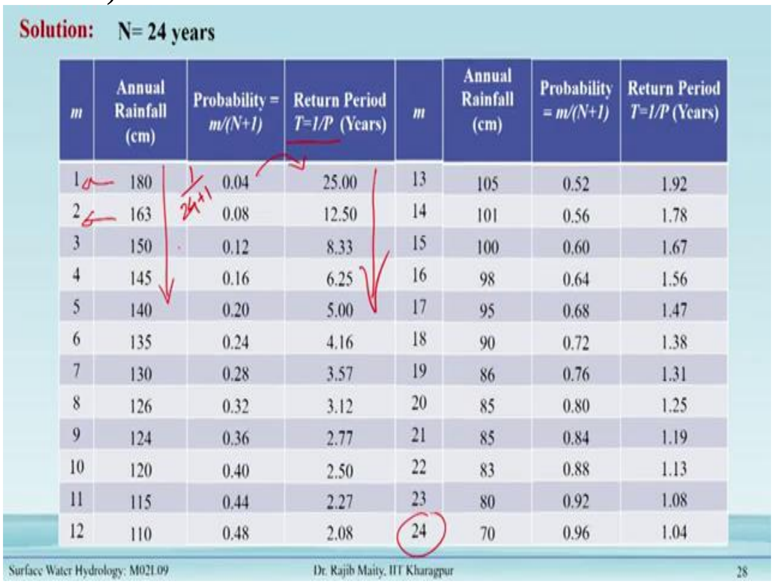

m	Annual Rainfall (cm)	Probability = *m/(N+1)*	Return Period *T=1/P* (Years)	*m*	Annual Rainfall (cm)	Probability = *m/(N+1)*	Return Period *T=1/P* (Years)
1	180	0.04	25.00	13	105	0.52	1.92
2	163	0.08	12.50	14	101	0.56	1.78
3	150	0.12	8.33	15	100	0.60	1.67
4	145	0.16	6.25	16	98	0.64	1.56
5	140	0.20	5.00	17	95	0.68	1.47
6	135	0.24	4.16	18	90	0.72	1.38
7	130	0.28	3.57	19	86	0.76	1.31
8	126	0.32	3.12	20	85	0.80	1.25
9	124	0.36	2.77	21	85	0.84	1.19
10	120	0.40	2.50	22	83	0.88	1.13
11	115	0.44	2.27	23	80	0.92	1.08
12	110	0.48	2.08	24	70	0.96	1.04

Solution

N=24 years

m	Annual Rainfall (cm)	Probability = *m/(N+1)*	Return Period *T=1/P* (Years)	*m*	Annual Rainfall (cm)	Probability = *m/(N+1)*	Return Period *T=1/P* (Years)
1	180	0.04	25.00	13	105	0.52	1.92
2	163	0.08	12.50	14	101	0.56	1.78
3	150	0.12	8.33	15	100	0.60	1.67
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8	126	0.32	3.12	20	85	0.80	1.25
9	124	0.36	2.77	21	85	0.84	1.19
10	120	0.40	2.50	22	83	0.88	1.13
11	115	0.44	2.27	23	80	0.92	1.08
12	110	0.48	2.08	24	70	0.96	1.04

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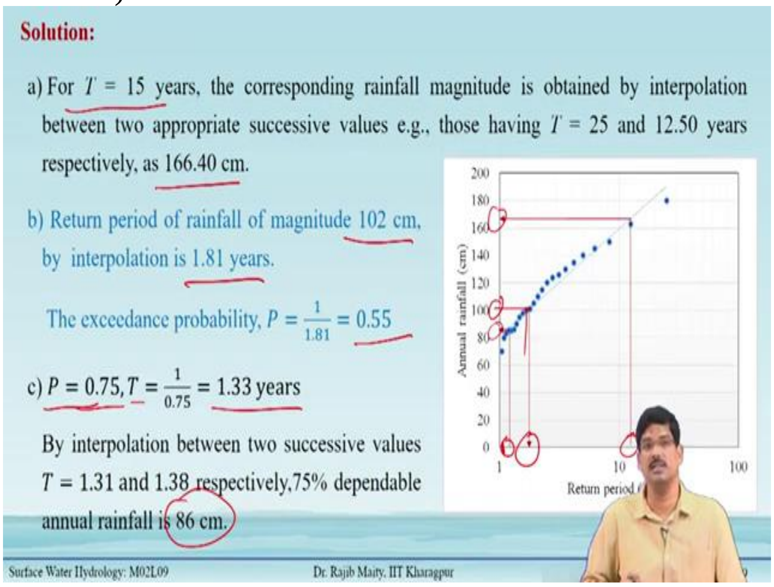

Solution

a) For T = 15 years, the corresponding rainfall magnitude is obtained by interpolation between two appropriate successive values e.g., those having T = 25 and 12.50 years respectively, as 166.40 cm.

b) Return period of rainfall of magnitude 102 cm, by interpolation is 1.81 years.

The exceedance probability, P=1/1.81=0.55

c) P=0.75, T=1/0.75=1.33 years

By interpolation between two successive values T=1.31 and 1.38 respectively,75% dependable annual rainfall is 86 cm.

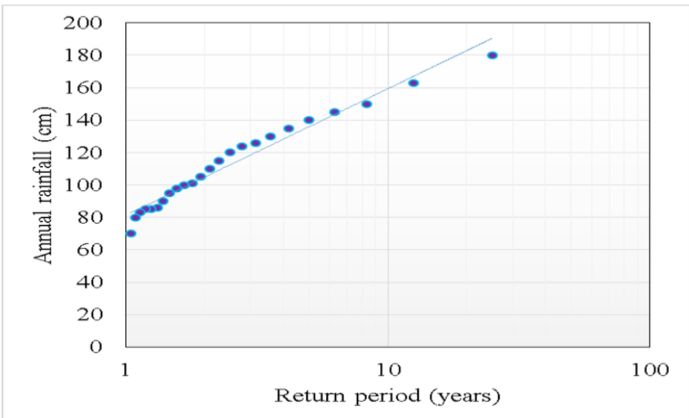

Fig.5 Return period of annual Rainfall at a Station

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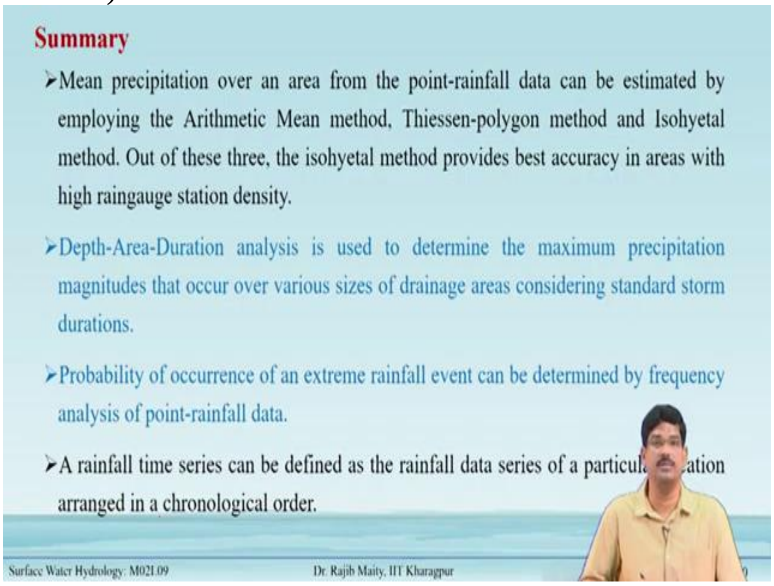

Summary

In summary, we learned the following points from this lecture:

- Mean precipitation over an area from the point-rainfall data can be estimated by employing the Arithmetic Mean method, Thiessen-polygon method, and Isohyetal method. Out of these three, the isohyetal method provides the best accuracy in areas with high rain gauge station density.

- Depth-Area-Duration analysis is used to determine the maximum precipitation magnitudes that occur over various sizes of drainage areas considering standard storm durations.

- The probability of occurrence of an extreme rainfall event can be determined by frequency analysis of point-rainfall data.

- A rainfall time series can be defined as the rainfall data series of a particular location arranged in chronological order.