Availability and Management of Groundwater Resources
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Lecture - 28
Law of Groundwater Movement, Darcy's Law and Application (Continued)

Welcome you all in the part 4 of the module 6, law of groundwater movement, Darcy's law and application. So, the Darcy's law in general states about the groundwater movement underneath the earth's surface. A law has been given for the porous media because we know that the groundwater remains in some porous media, the thermal formation should be the saturated one. So, through the porous media how the groundwater moves this has been described in the Darcy law. We have learned about this in the previous lectures.

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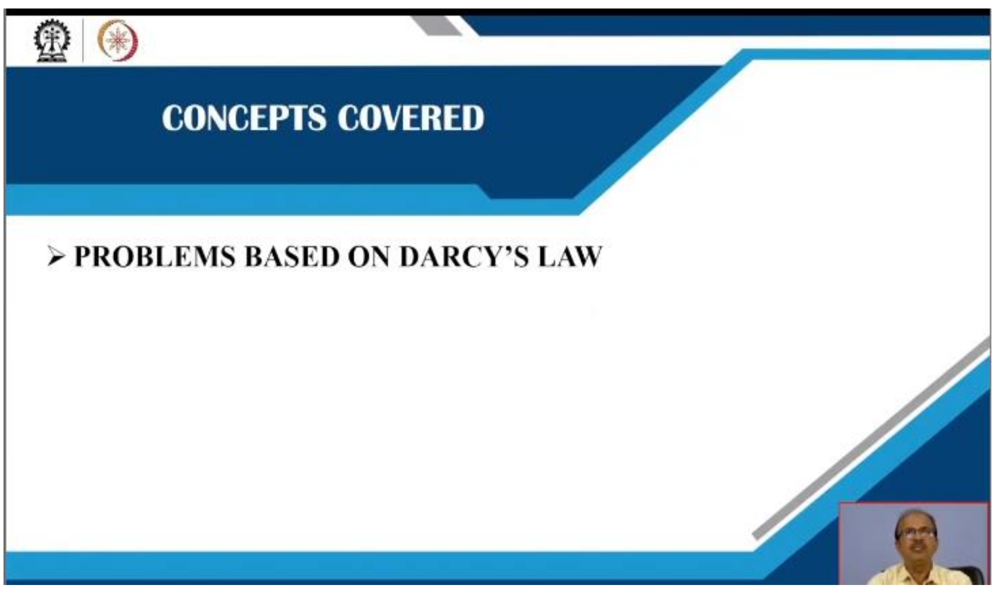

So, now in this lecture, in this part we will try to cover some of the problems which are very well related with the Darcy law.

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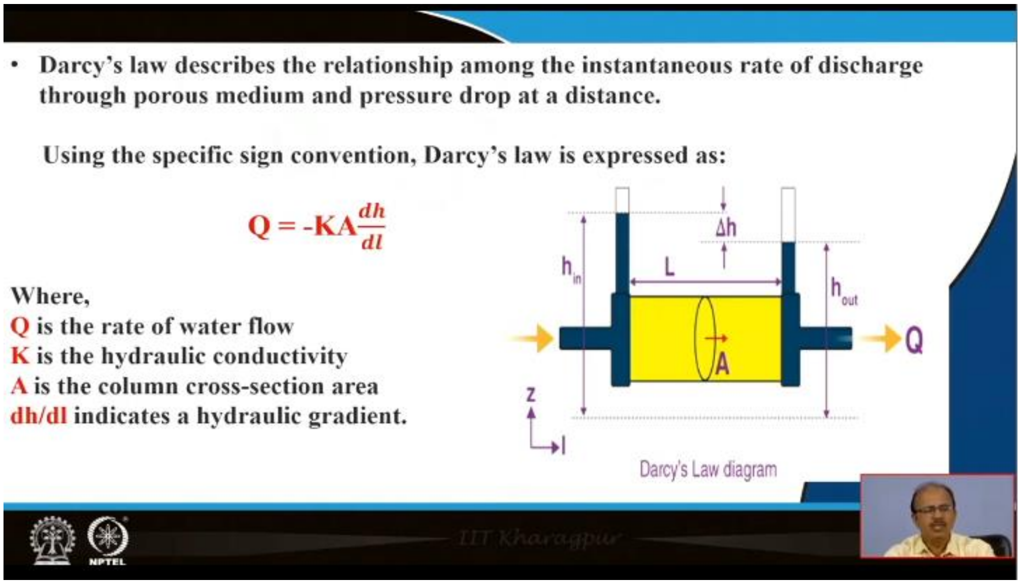

So, we will start from the, just we will recall the Darcy law, once again that Darcy law describes the relationships among the instantaneous rate of discharge through the porous medium and pressure drop at a distance. So, if you will recall we have learnt about the

$$Q = -KA\frac{dh}{dl}$$

where Q is the rate of water flow. Somewhere you may get in small letter q or in capital letter Q. K is the hydraulic conductivity of the porous media.

A is the cross-sectional area of the porous media and

$\frac{dh}{dl}$ indicates the hydraulic gradient. You can see that the distance between the two column is generally taken as the, your hydraulic gradient. So, we also know that water needs energy for its movement and it generally flow from higher energy to lower energy from higher hydraulic head to lower hydraulic head. So, this thing already we have covered in greater detail in the previous three lectures.

In this lecture just on the basis of the formula of the Darcy law $Q = -KA\frac{dh}{dl}$.

We will solve some of the numerical some of the problems which we usually face for finding the availability of groundwater resources in the earth's surface.

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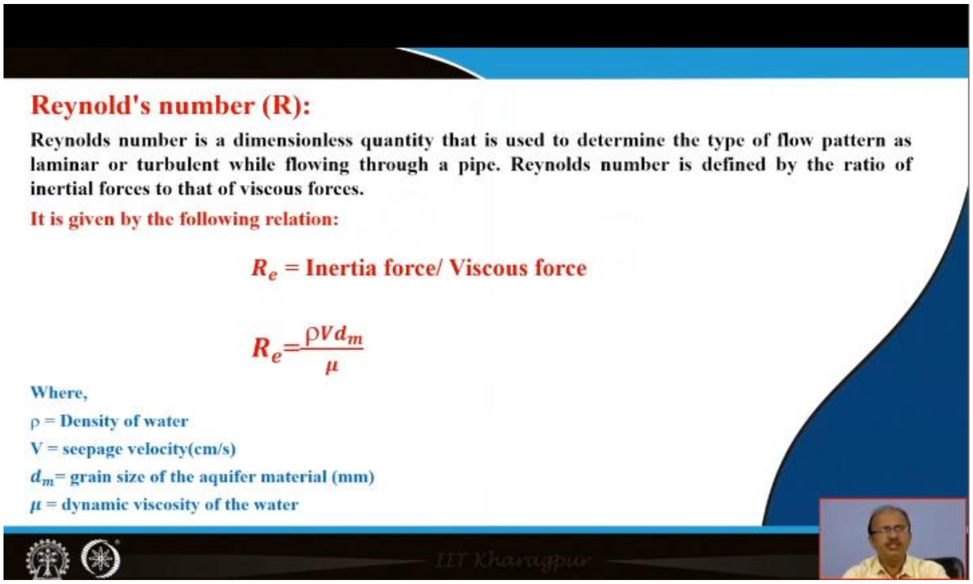

So, now the first one more thing we will just I will discuss here Reynold number. Reynold number is a dimensionless quantity which is used to determine the type of flow pattern. Which type of flow pattern is there, laminar or turbulent while flowing through a pipe. So, it is defined by the ratio of inertial forces to that of the viscous forces. Because it will the water will flow through certain pipe.

So, the two different units that is the; inertial forces as well as the viscous forces. The ratio of these two is generally called as the Reynold number which is usually given by the

$$R_e = \frac{\rho V d_m}{\mu}$$

Where, ρ is the density of water,

V is the seepage velocity in cm/s,

dm is the grain size of the aquifer material that is in mm and

μ is the dynamic viscosity of water.

So, these two concepts we have just recalled once again. One is the Darcy's law and second is the Reynold number. So, Reynold number is very important it is a dimensionless quantity and usually it is measured by inertial force divided by viscous force which can be rewritten as

$$R_e = \frac{\rho V d_m}{\mu}$$

So, in with this background we will try to solve some of the numerical which are very popular with your Darcy law.

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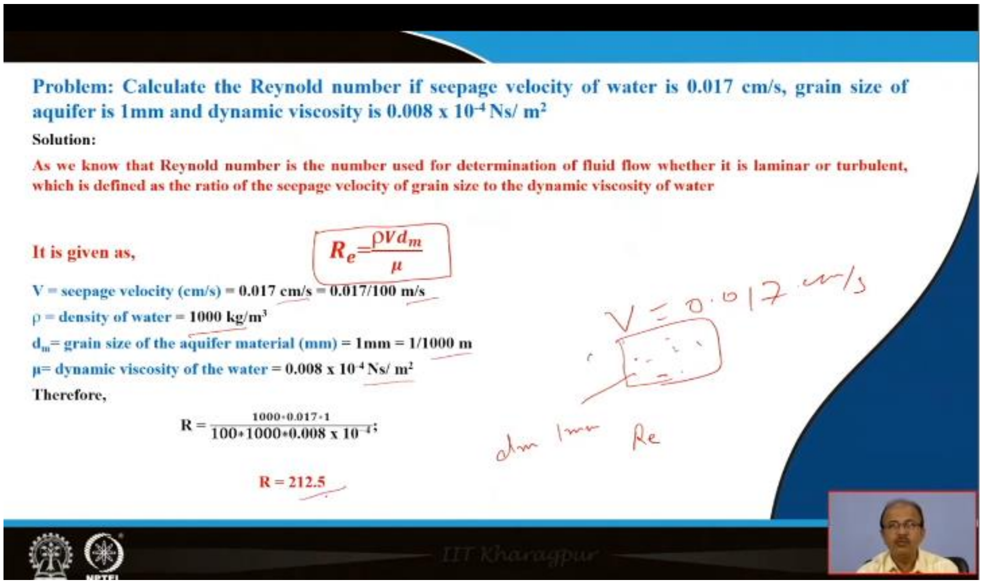

So, based on the Darcy law, what we have discussed and the Reynold number. Now we will solve some of the problem. The first problem is

Problem: Calculate the Reynold number if seepage velocity of water is 0.017 cm/s, grain size of aquifer is 1mm and dynamic viscosity is 0.008×10^{-4} Ns/ m^2

Solution:

As we know that Reynold number is the number used for determination of fluid flow whether it is laminar or turbulent, which is defined as the ratio of the seepage velocity of grain size to the dynamic viscosity of water

It is given as, $R_e = \frac{\rho V d_m}{\mu}$

V = seepage velocity (cm/s) = 0.017 cm/s = 0.017/100 m/s

ρ = density of water = 1000 kg/m^3

d_m= grain size of the aquifer material (mm) = 1mm = 1/1000 m

μ= dynamic viscosity of the water = 0.008×10^{-4} Ns/ m^2

Therefore,

Put the value in the above equation, we get

$$R = \frac{1000*0.017*1}{100*1000*0.008 \text{ x } 10\text{-}4};$$

$$R = 212.5$$

Now next we will see some another problem.

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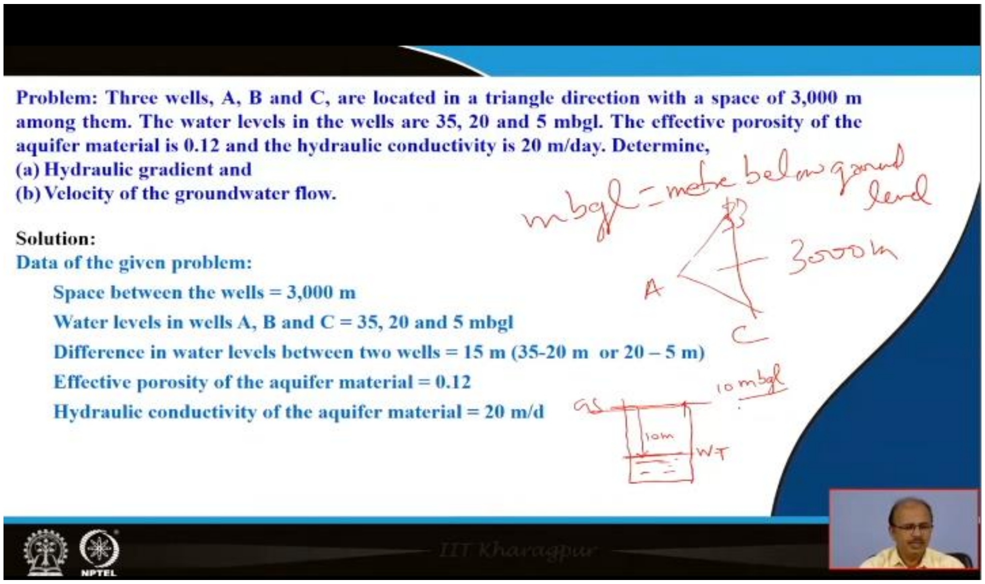

So, this is another type of Problem, three wells, A, B and C, are located in a triangle direction with a space of 3,000 m among them. The water levels in the wells are 35, 20 and 5 mbgl. The effective porosity of the aquifer material is 0.12 and the hydraulic conductivity is 20 m/day. Determine,

(a) Hydraulic gradient and
(b) Velocity of the groundwater flow.

Solution:

Data of the given problem:

Space between the wells = 3,000 m

Water levels in wells A, B and C = 35, 20 and 5 mbgl

Difference in water levels between two wells = 15 m (35-20 m or 20 – 5 m)

Effective porosity of the aquifer material = 0.12

Hydraulic conductivity of the aquifer material = 20 m/d

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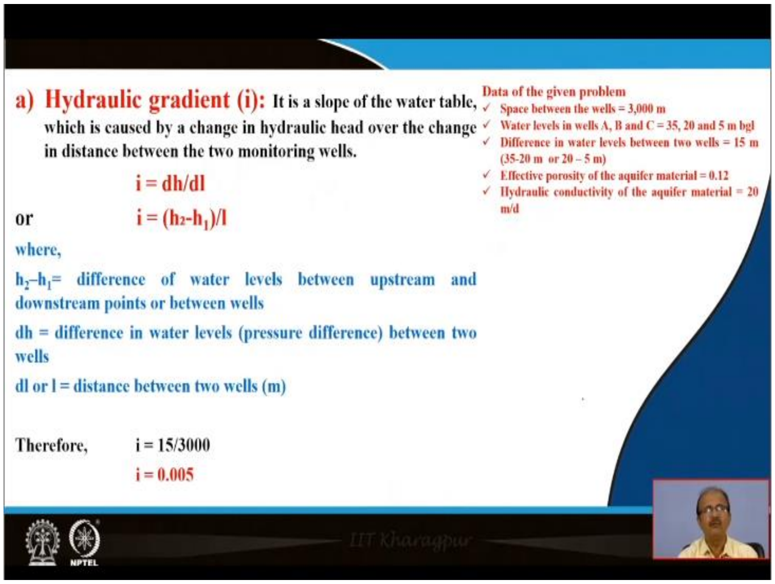

And then first question where they have to find out the hydraulic gradient of the aquifer material. So, we know we have discussed earlier also we know that hydraulic gradient is a slope of the water table which is caused by a change in the hydraulic head over the change in distance between the two monitoring wells.

a) Hydraulic gradient (i): It is a slope of the water table, which is caused by a change in hydraulic head over the change in distance between the two monitoring wells.

$$i = dh/dl$$

or $$i = (h_2-h_1)/l$$

where,

h_2–h_1= difference of water levels between upstream and downstream points or between wells

dh = difference in water levels (pressure difference) between two wells

dl or l = distance between two wells (m)

Therefore, $i = 15/3000$

$i = 0.005$

Now second was the about the velocity, find out the velocity of the groundwater flow.

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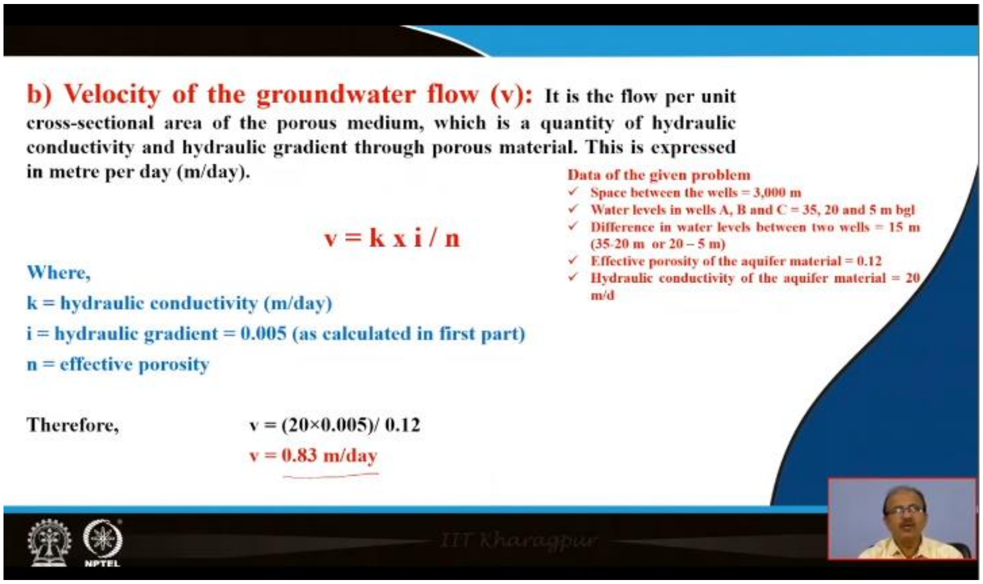

So, now we will find out the flow, velocity of the groundwater flow.

b) Velocity of the groundwater flow (v): It is the flow per unit cross-sectional area of the porous medium, which is a quantity of hydraulic conductivity and hydraulic gradient through porous material. This is expressed in metre per day (m/day).

$$v = k * i / n$$

Where,

k = hydraulic conductivity (m/day)

i = hydraulic gradient = 0.005 (as calculated in first part)

n = effective porosity

Therefore, $v = (20 \times 0.005)/ 0.12$

$v = 0.83$ m/day

So, now this is about this problem the two different aspects we have seen in this problem first is the how to find out the Renold number and second how to find out the hydraulic gradient and second how to find out the velocity of the groundwater flow.

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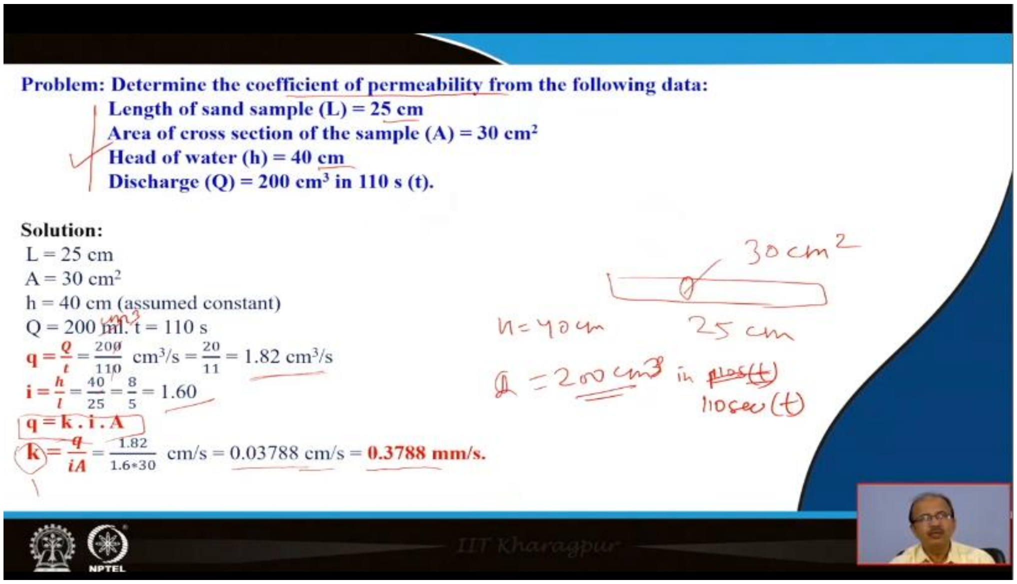

So, now think over this problem this is another problem and, in this problem the question is Determine the coefficient of permeability from the following data:

Length of sand sample (L) = 25 cm

Area of cross section of the sample (A) = 30 cm^2

Head of water (h) = 40 cm

Discharge (Q) = 200 cm^3 in 110 s (t).

Solution:

L = 25 cm

A = 30 cm^2

h = 40 cm (assumed constant)

Q = 200 ml. t = 110 s

$q = \frac{Q}{t} = \frac{200}{110}$ $cm^3/s = \frac{20}{11} = 1.82$ cm^3/s

$i = \frac{h}{l} = \frac{40}{25} = \frac{8}{5} = 1.60$

q = k . i . A

$k = \frac{q}{iA} = \frac{1.82}{1.6*30}$ cm/s = 0.03788 cm/s = 0.3788 mm/s.

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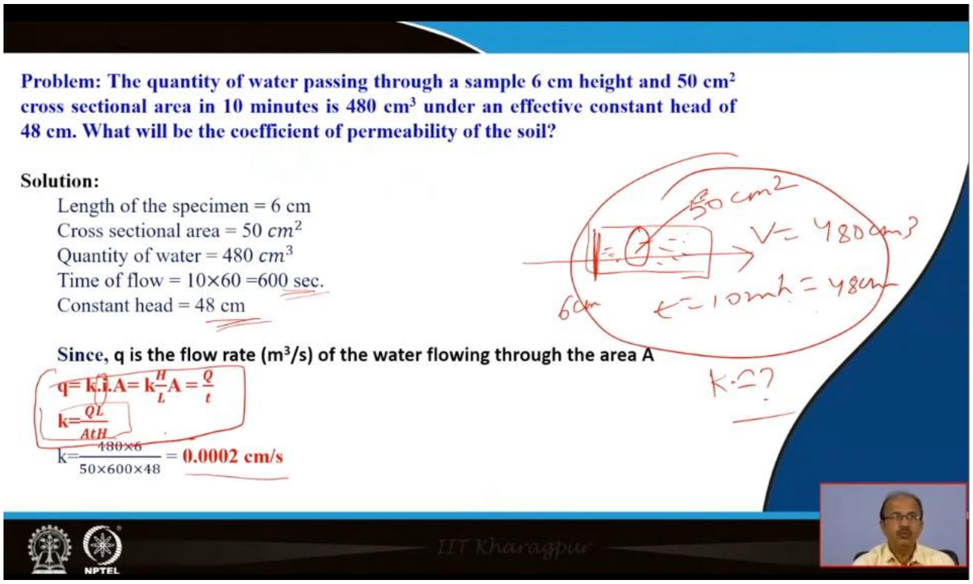

So, now we will discuss another type of problem.

Problem: The quantity of water passing through a sample 6 cm height and 50 cm^2 cross sectional area in 10 minutes is 480 cm^3 under an effective constant head of 48 cm. What will be the coefficient of permeability of the soil?

Solution:

Length of the specimen = 6 cm

Cross sectional area = 50 cm^2

Quantity of water = 480 cm^3

Time of flow = 10×60 =600 sec.

Constant head = 48 cm

Since, q is the flow rate (m^3/s) of the water flowing through the area A

q= k.i.A= k$\frac{H}{L}$A = $\frac{Q}{t}$

k=$\frac{QL}{AtH}$

k=$\frac{480\times6}{50\times600\times48}$ = 0.0002 cm/s

So, on the basis of this we can find out the different parameters related to the flow of the groundwater inside the earth surface.

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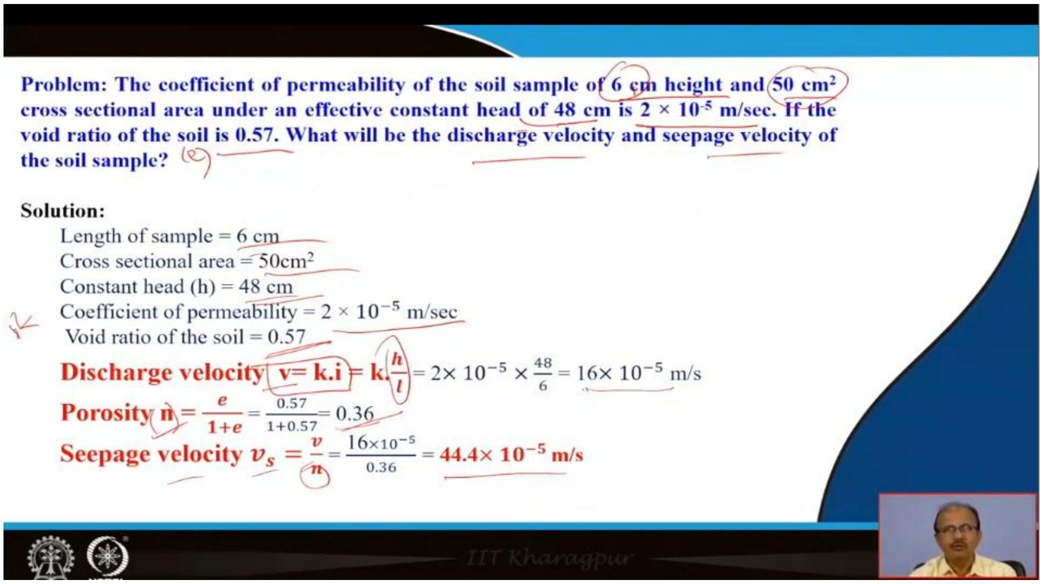

Now next another problem just

Problem: The coefficient of permeability of the soil sample of 6 cm height and 50 cm^2 cross sectional area under an effective constant head of 48 cm is 2 × 10^{-5} m/sec. If the void ratio of the soil is 0.57. What will be the discharge velocity and seepage velocity of the soil sample?

Solution:

Length of sample = 6 cm

Cross sectional area = 50cm^2

Constant head (h) = 48 cm

Coefficient of permeability = 2 × 10^{-5} m/sec

Void ratio of the soil = 0.57

Discharge velocity v= k.i = k.$\frac{h}{l}$ = 2× 10^{-5} × $\frac{48}{6}$ = 16× 10^{-5} m/s

Porosity n = $\frac{e}{1+e}$ = $\frac{0.57}{1+0.57}$= 0.36

Seepage velocity $v_s = \frac{v}{n}$ = $\frac{16\times10^{-5}}{0.36}$ = 44.4× 10^{-5} m/s

So, in this way we can solve many more problems related with your coefficient of permeability related to the discharge velocity related to the seepage velocity.

Only thing we have to just summarize or we have to keep all the different basic concepts in our mind related to the Darcy law. So, in this way we can find we can solve several problems if the different values were given. Now let us see next problem.

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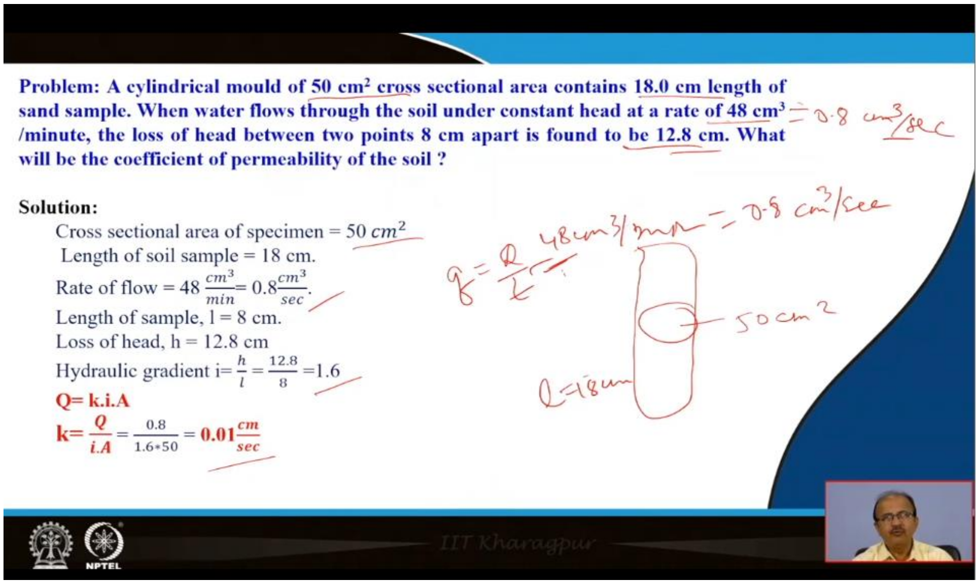

So, this is another problem a

A cylindrical mould of 50 cm^2 cross sectional area contains 18.0 cm length of sand sample. When water flows through the soil under constant head at a rate of 48 cm^3 /minute, the loss of head between two points 8 cm apart is found to be 12.8 cm. What will be the coefficient of permeability of the soil?

Solution:

Cross sectional area of specimen = 50 cm^2

Length of soil sample = 18 cm.

Rate of flow = 48 $\frac{cm^3}{min}$= 0.8$\frac{cm^3}{sec}$.

Length of sample, l = 8 cm.

Loss of head, h = 12.8 cm

Hydraulic gradient i= $\frac{h}{l}$ = $\frac{12.8}{8}$ =1.6

Q= k.i.A

k= $\frac{Q}{i.A}$ = $\frac{0.8}{1.6*50}$ = 0.01 cm/s

So, this whole problem can be solved if we are having the concept of the Darcy laws in greater detail. So, based on this for the Darcy law formula we can solve the different types of problems related with the hydraulic conductivity, hydraulic head then rate of flow, so Reynold number. We can solve different types of problems which are very well related with the groundwater movement in the earth's surface because we have to find out the availability of the groundwater resources. So, for this we should know how the groundwater flows inside the earth through the porous formation. So, with this; thank you to you all.