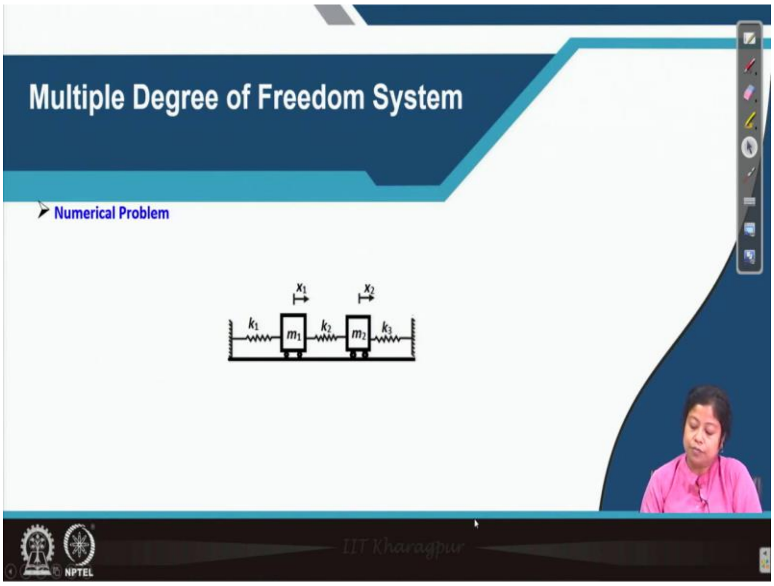
Soil Dynamics
Professor Paramita Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture – 13
Multiple Degree of Freedom System (MDOF) – Part 2

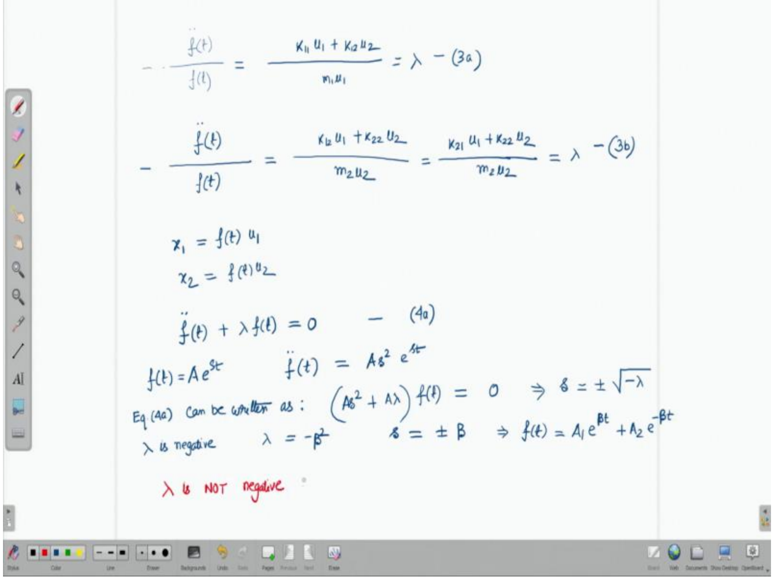
Hello friends, today we will continue the last class which is how to solve the Multiple Degree of Freedom System.

(Refer Slide Time: 0:39)



So, in the last class we have taken, there are 2 masses attached to springs like this, of course, in that example I have shown dashboard also, but when we are interested to find out the unnatural frequencies for undammed system we do not need to consider the effect of the damping Force that is a reason. Now I have just removed the dampers from this figure.

(Refer Slide Time: 1:25)



Now in this problem, so last class we have seen that minus of if double dot t divided by f t, this is equal to K 11 U 1 plus K 12 U 2 divided by mu 1 U 1 that is equal to some lambda. Similar I can give it as equation 3a, similarly I can write the second equation also which we have already seen in last class that minus of if 2 dot t that means second derivative of if with respect to t divided by f t is equal to K 12 times U 1 plus K 22 times U2 divided by mu 2 U2 or what we can write in place of K 12 we can write K 21 also.

So, we can write it as also K 21 times U 1 plus K 22 times U 2 divided by m 2 U 2 is equal to lambda, so this is our equation 3b. Now here what we have assumed, we assume that Z 1 is equal to f t times U 1 and Z 2 is equal to f t times U 2. So, in this case we are considering movement or displacement in X direction. So, in place of Z 1 and Z 2, I need to write x 1 and x 2. So, this is x 1 this is x 2.

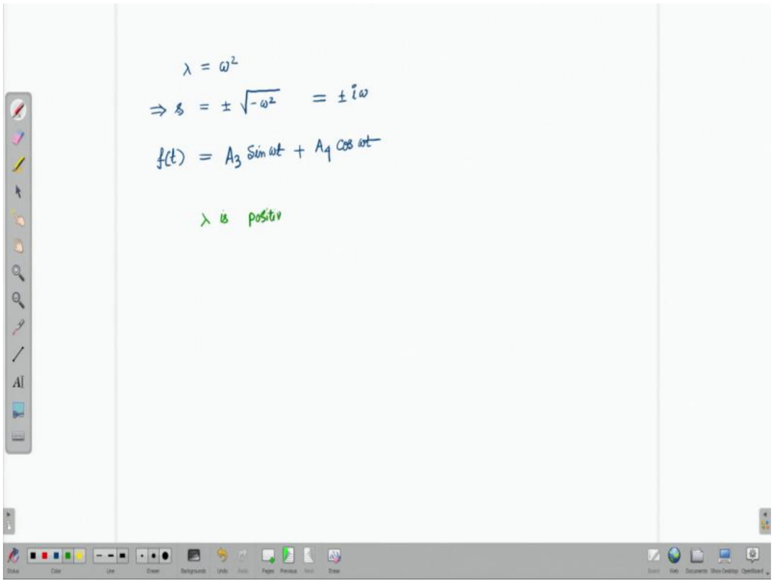
Now from equation 3a what we can write or equation 3b also gives the same thing, we can write f 2 dot t plus lambda times f t is equal to 0. Now I can give it a number 4a or now for equation 4a, we can assume a solution a to the power A times e to the power st as the solution for f t. Then what we can write for second derivative of f with respect to t, we can write it as A times A square times e to the power st.

Then equation 4a can be written here as, can be written here as, A square plus A times lambda A A square plus A times lambda times whole thing multiplied with f t and that is equal to 0 in the right hand side. So, from these we can write s is equal to plus minus square root of minus lambda. Now there are two possibilities either lambda is positive or lambda is negative, if lambda is for negative, first consider lambda is negative.

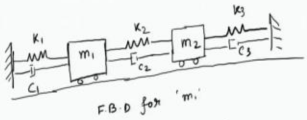
So, if lambda is negative, then what will be happening we can take lambda is equal to minus beta square whatever be beta positive or negative minus beta square is always positive. So, what we can get then s is equal to plus minus square root of beta square which is our beta. Now from these what we can get the general solution for f t from these we can write if T is equal to A 1 times e to the power beta t plus A 2 times e to the power minus beta t.

So, plus beta is one root and minus beta is another root for this s. Now from these what we can see f t is a function of t, but it changes exponentially. Now for an undamped system such kind of changes is not possible. So, whatever we have assumed that Alpha, sorry, that lambda is negative that is not possible so, lambda is not negative.

(Refer Slide Time: 7:47)



For λ is +ve $\lambda = \omega^2$

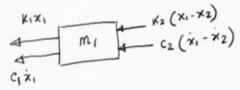


Equation of Motion

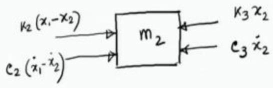
$$m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 + (c_1+c_2)\dot{x}_1 - c_2\dot{x}_2 = 0$$

Damping forces

F.B.D for 'm_1'



F.B.D. for 'm_2'



$$m_2\ddot{x}_2 - k_2x_1 + (k_2+k_3)x_2 - c_2\dot{x}_1 + (c_2+c_3)\dot{x}_2 = 0$$

Damping forces

For λ is +ve $\lambda = \omega^2$ $\forall$ all non-zero values of 'ω'.

$$\frac{k_{11}u_1 + k_{12}u_2}{m_1u_1} = -\frac{\ddot{f}(t)}{f(t)} = \omega^2 = \text{positive.} \quad -(4a)$$

$k_{11} = (k_1+k_2) = (k+k) = 2k$

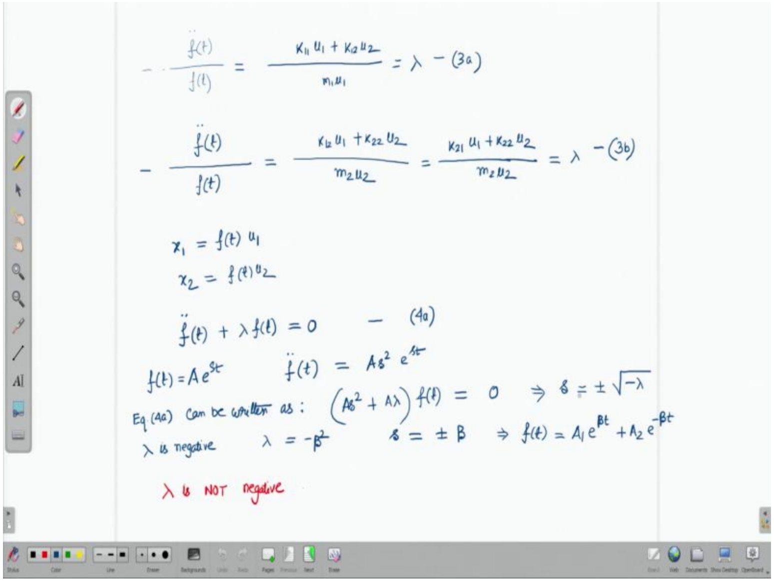
$k_{12} = -k_2 = -k = k_{21}$

$k_{22} = (k_2+k_3) = 2k$

Equation (4a) as :

$$\frac{(2k)u_1 + (-k)u_2}{(m)u_1} = \omega^2 \Rightarrow 2ku_1 - ku_2 - m\omega^2u_1 = 0$$

$-(5a)$



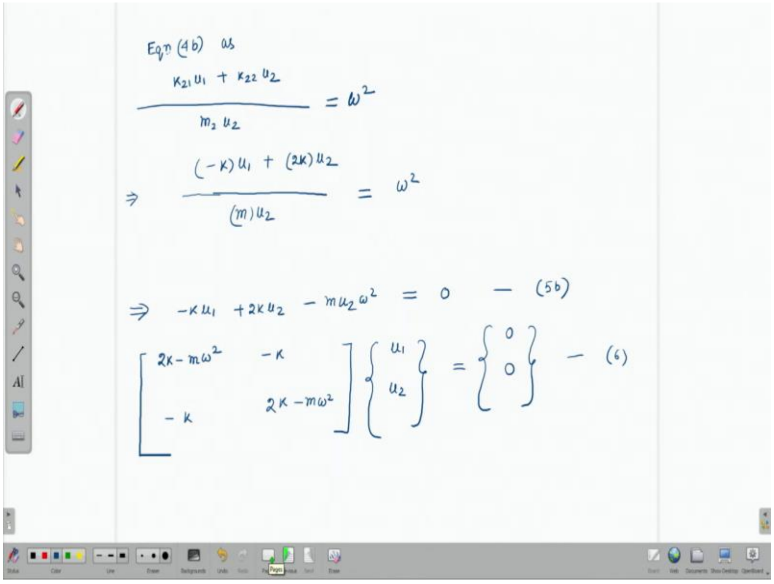
Now there is second possibility. So, let me write that, what is the second possibility? Second possibility is lambda is positive if lambda is positive, then we can write lambda is equal to omega square, omega may be positive or negative but omega square is always positive. So, lambda is positive in this case. Now if lambda is positive, then what is s, then you can see what I can write, I can write then s is equal to plus minus square root of minus omega square.

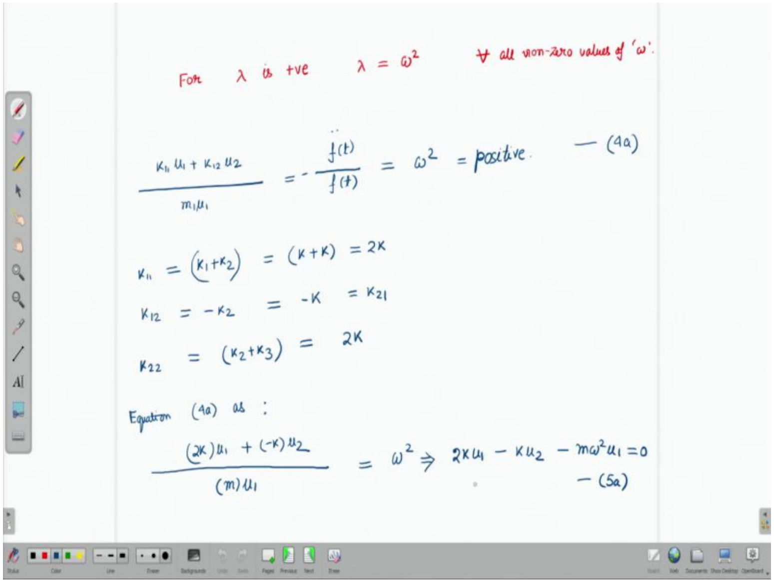
So, it means plus minus I times omega, then what we can write for f t, f t is equal to A 1 sin omega t plus, we can use A 1 A 2 or we can use here A3 A 4 I am writing A 3 A 4. So, A 3 sin omega t plus A 4 cosine omega t, that means f t is addition of two simple harmonic motion and it is possible. So, the Assumption lambda is positive is correct here, I can write then lambda is positive.

I am taking lambda is equal to omega square, omega may be positive or negative it does not matter but omega square is always positive, so lambda is always positive for all non zero values of omega, that we have seen in the previous class. So, K 1 plus K 2 means in this case now, K plus K which is 2 K, likewise K 12 means minus K 2 that means it is K with negative sign, same should be K 21 also, what about K 22, K 22 is K2 plus K3 which are K in this example problem, so here it will be 2 K.

Then we can write this is this was our equation 4a. So, now we can then write equation 4a as 2K U 1 plus minus K times U 2 divided by m 1 times U 1 m 1 means m times U 1 that is equal to omega square. Now then we can write here also 2K U 1 minus K U 2 minus m omega square U 1 is equal to 0, this is another form of equation 4a. I am giving it a name 5a.

(Refer Slide Time: 12:20)



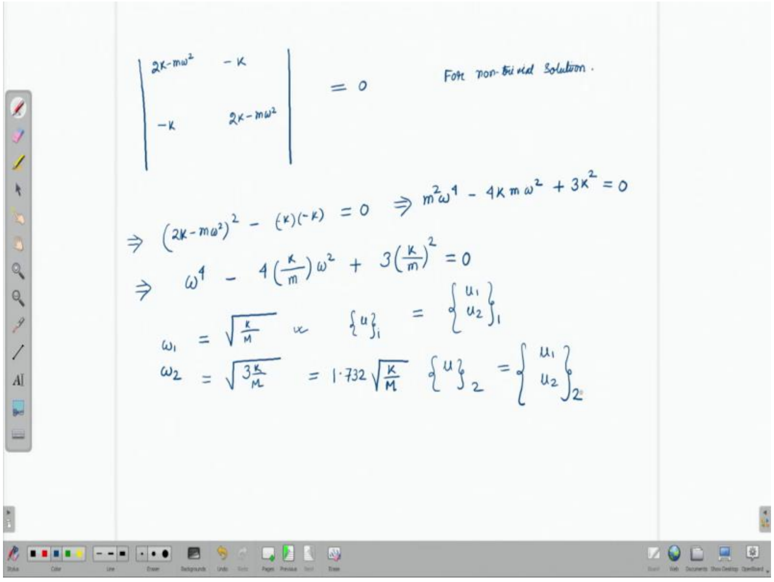


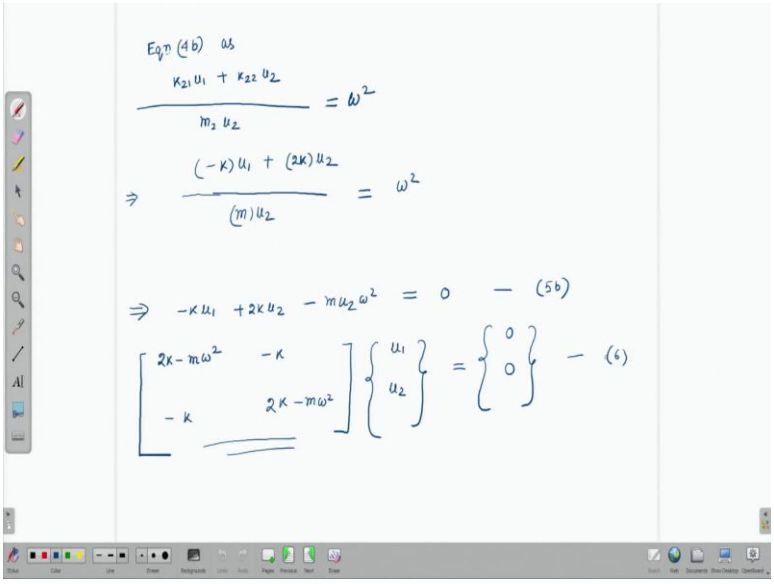
Now let us write, the next equation 4b as, what is the equation 4b? Equation 4b says K 21 times U 1 plus K 22 times U 2 divided by m 2 U 2 is equal to omega square. Now we will put the, in place of K 21 K 22 and m 2 whatever we have assumed that we will put, then K 21 will become minus K 21 will become minus K times U 1 K t 22 will be 2K times U 2 divided by m times U 2 that is equal to omega square.

So, let us give it a number 5b or I 5b here I will not write, in the next line I will write it as 5b. So, finally I can write it as K U 1 minus K U 1 plus 2 K U 2 minus m U 2 omega square is equal to 0, this is 5b, then together 5a and 5b can be written as... So, the form of 5a was this and the form of 5b is this. So, if I will write these two equations together, what I can write, I can write it as 2K minus m omega square minus K minus K here again 2K minus m omega square this will be multiplied with U 1 U 2 which is equal to U or null vector.

Because we are considering free vibration. So, hand side is a null vector, I can give it a name 6. Now solving this we will find out omega first of all we need to think whether U 1 and U 2, 0 is possible, no, we are con we will consider only the non-trivial solution for equation 6 or expression 6.

(Refer Slide Time: 15:47)





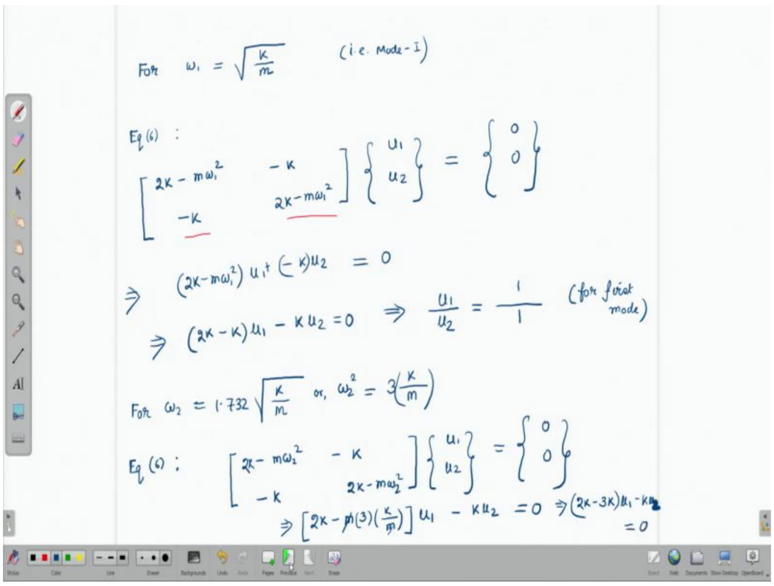
For non-trivial solution what is the procedure? The determinant of the coefficient matrix will be 0 then so coefficient matrix is this one. So, let me write then 2K minus m omega square minus K here minus K and again here 2K minus m omega square, so this can be 0 for non-trivial solution. Then we can write it as 2K minus m omega square whole square minus, if I will expand it, it will be or I can write it as, let me check whether I have written it correctly here it is not m square but it is only m.

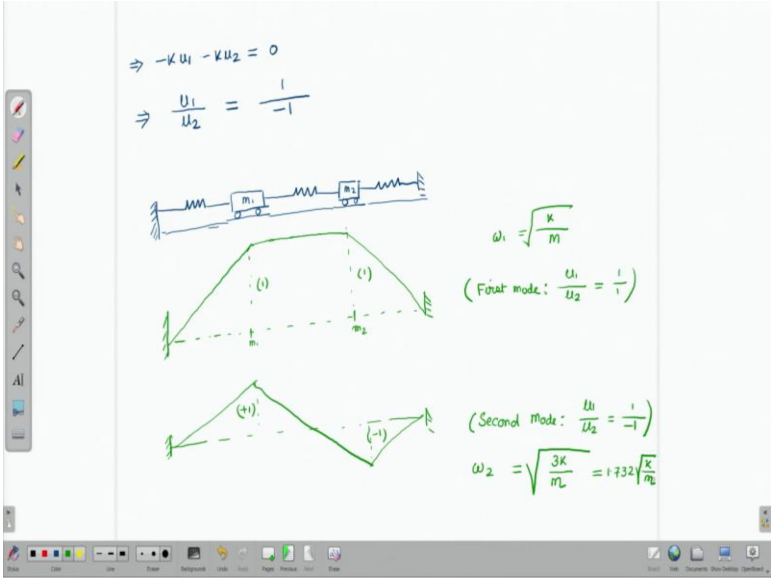
So, now we can find out the solution for omega there are four roots because it is a fourth order equation among four roots, two roots are negative which we will not consider because frequency cannot be negative. So, the 2 positive roots for omega is the 2 positive roots are omega 1 and omega 2 which are if we solve it, it will be square root of K by m and square root of 3K by m. So, square root of 3K by m means how much let me check.

So, omega 2 is 1.732 times square root of K by m, first one is K by m. So, among these two, first one is the fundamental natural frequency and second one is the higher natural frequency. So, now we have omega 1 and omega 2. And we get it from expression 6. Now we need to know what is U 1 and U 2, what physically it means U 1 and U 2 are called the mode shapes for this 2 degree of freedom system.

So, we will get two different values for this U vector, one when omega is equal to square root of K by m that means for this case we will get one U vector and for the second case we will get the another U vector. So, first one we can write it as U 1 U 2 for first mode, for second one we can write it as U 1 U 2 for second mode. Now let us see what are these two mode shapes.

(Refer Slide Time: 20:40)





First one is for omega 1 is equal to square root of K by n, what equation 6 says I am just rewriting equation 6 in terms of omega 1 first. So, it is coming 2K minus m omega 1 square minus K minus K again 2K minus in omega 1 square. This is multiplying by U vector right hand side is a null vector. So, from these now we can get a relationship between U 1 and U 2. So, what is that relationship?

This one or here I can now write in place of omega 1 square, I can write K by m for mode 1 or first mode. So, I am writing then it is foreign or we can write it as U 1 divided by U 2 is equal to 1 by 1. So, this is for first mode. You can see here if you will use the second relationship that means this one, then also you will get U 1 divided by U 2 for the first mode is equal to 1 by 1.

So, we can get an idea about the boat shape but we cannot get the exact value for U 1 and U 2. The same way for second mode now, we can find out, for second boat omega 2 is 1.738, sorry, 2 square root of K by m or we can write omega 2 square is equal to 3 times K by m this also, then equation 6 will become 2K minus m times omega 2 square minus K, second row minus K and 2K minus m omega 2 square, this coefficient matrix will be multiplied by the mode shape vector, that is U vector for the second mode right hand side is a null vector.

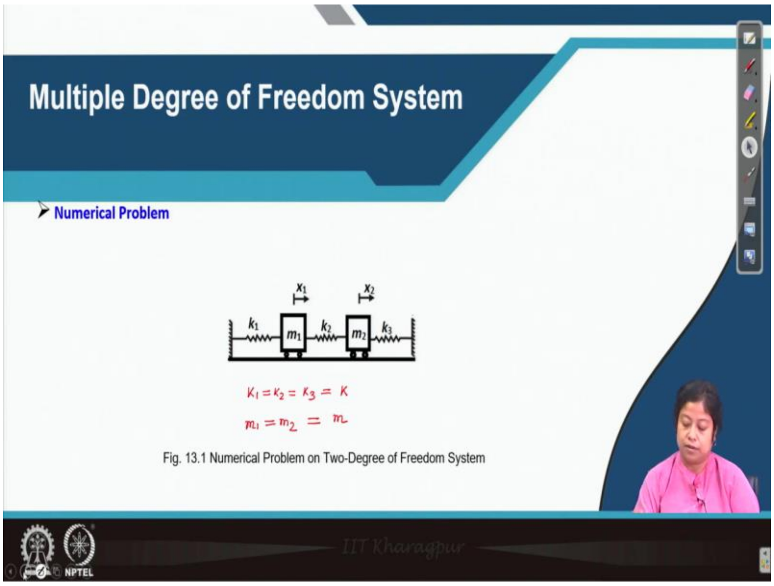
So, from here what we can write, we can write K minus m in place of omega 2 square I can write 3 times K by m also, it is multiplying with U 1 minus K times U2 is equal to 0. So, here finally what we are getting is 2K minus 3K times U 1 minus K times U 2 is equal to 0. Go to the next page minus K times U 1 minus K times U 2 is equal to 0 from which we can conclude U 1 and U 2 is equal to 1 minus 1.

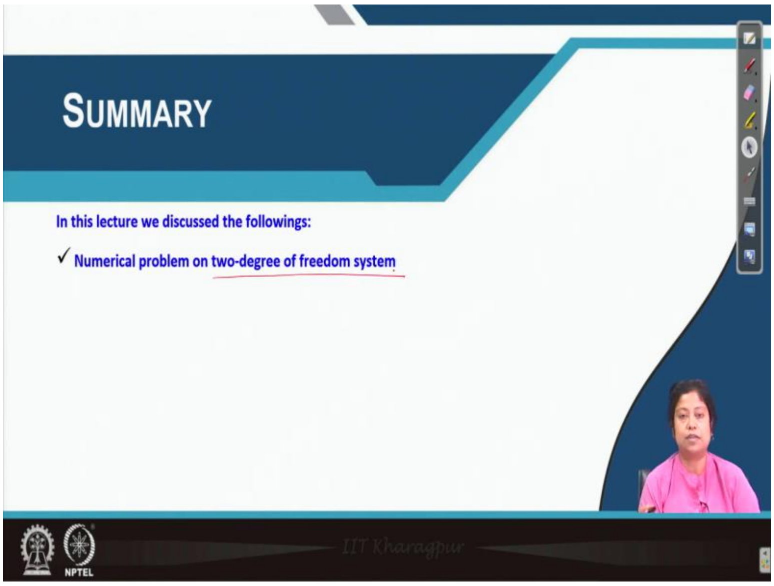
So, in this way we can express, we can express U 1 and U 2 as a ratio. Now if we know the ratio of U 1 and U 2, then we can give some idea about the mode shape of the system under consideration, I am just writing, just drawing first the original system, there is a mass here m1 another mass m 2 you can see, this is the problem which we have taken or which we have solved I can say.

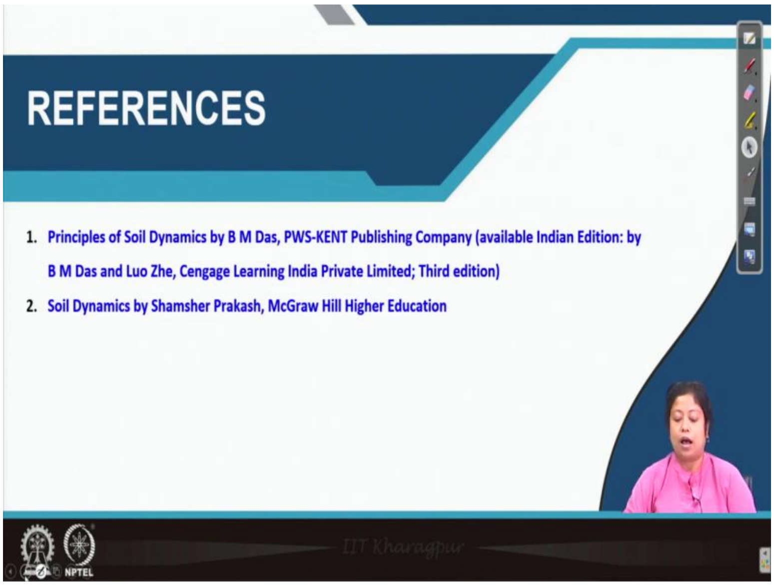
Now the idea of mode shape which we finally obtained from these class work from that, if someone will ask us to draw the boat shape, then how we will draw it, for the first case, this is equilibrium position, this is the for mass m 1 this is m 2. Then how the shape will be, it will be some for first mood I can. So, I can connect this, this something like this similarly this. So, I should write here first mode, this is first mode.

For the second mode how it will look. So, for first mode the ratio of U 1 and U 2 is 1 by 1. For the second mode if U 1 is 1 then U 2 is minus 1, then accordingly how the mode shape will look, then it will look like. So, this is minus 1 this is plus 1. This is some approximate idea about the mode shape. And for first mode omega 1 is square root of K by m, for the second mode natural frequency omega 2 is 3K by square root of 3K by m or we can write it as 1.732 times square root of K by m.

(Refer Slide Time: 29:51)







So, in this way we can find out the two mode shapes of the problem given here. So, we have discussed how to solve the (numeri), how to solve a problem of 2 degree of freedom system in today's class. So, for this lecture I have used, prefer these two books. So, this is all about the thing 2 degree of freedom system under undamped condition and subjected to no external force, with this I conclude today's class. Next class we will see what is the difference, if we consider an external force in the 2 degree of freedom system. Thank you.