Soil Dynamics
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Lecture 15
Multiple Degree of Freedom System (MDOF)

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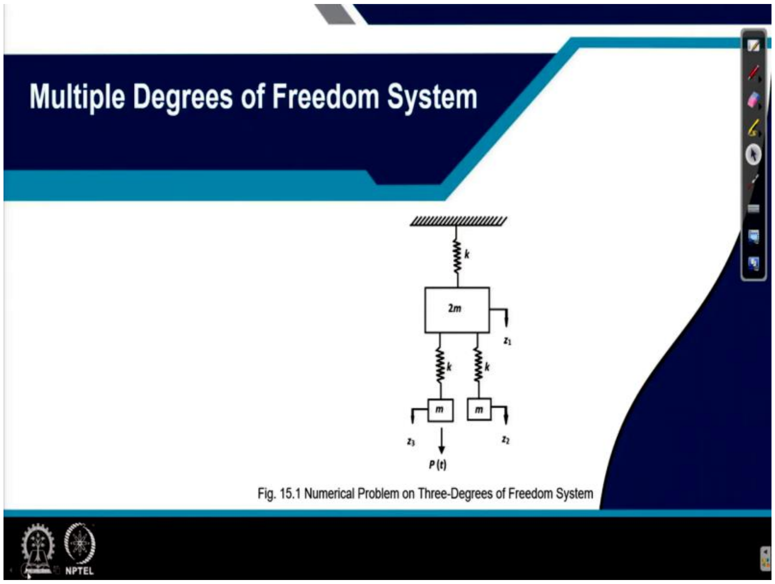

Hello friends, today we will continue our previous discussion on 3 degrees of freedom system under forced vibration. So, last class, we have formed the equation of motion for these kinds of three degrees of freedom system for 3 different masses to m m and m on the right hand side these three masses we have formed the equations of motion. So, let us continue that part.

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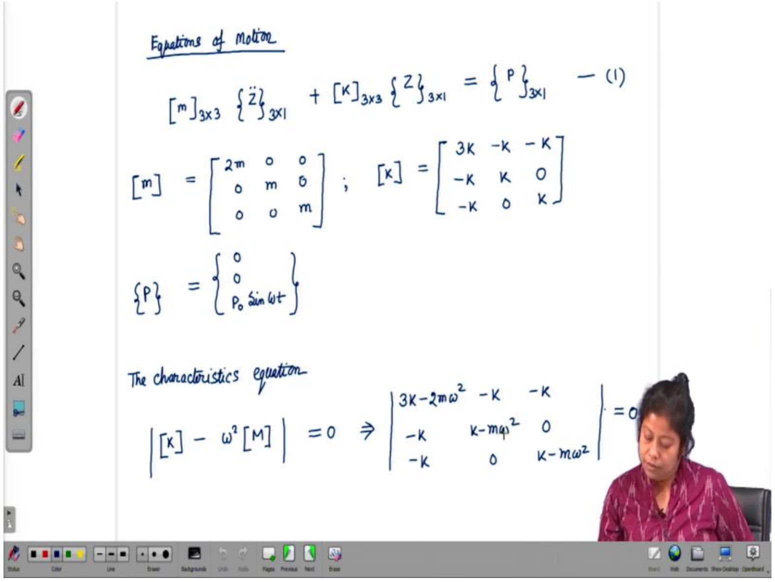

So, from equations of motion basically, we formed 3 equations of motion for 3 different masses if we will write it in a matrix form then what we can write here. So, they are easy and vast matrix in this case it will be 3 by 3 it is 3 by 3 then it it is multiplying with the acceleration vector which is 3 by 1 plus stiffness matrix 3 by 3 size times displacement vector 3 by 1 that is equal to force vector which is 3 by 1 now, if I will expand these matrices in k vector p then how it will look let us see that also.

So, m is a diagonal matrix and the diagonal terms are 2 m m and aim just like this one this is our A matrix now, we can see the stiffness matrix that these 3k minus k minus k then minus k see it will be k 0 and last one like this and p vector is like our 0 0 p 0 sin omega t. So, this is the different matrices and vector used in this expression one.

Now, we will write the characteristics equation. So, the characteristics equation determinant of K matrix minus omega squared times mass matrix is equal to 0. So, from these what we can write, we can write it as 3k minus 2 m omega squared minus k minus k then minus k k minus m omega square 0 and the last row is minus k 0 k minus m omega squared that should be equal to, it is in determining not matrix so, let me change the sign here itself.

So, this will be this is equal to 0. Now, we will expand it in the next page. So, what we can write from the previous equation. So, this minus this so, this is 0 already, 0 then minus k minus minus

K I can write here so, minus minus k then what I can do here is minus k times k minus m omega squared minus 0 times minus k.

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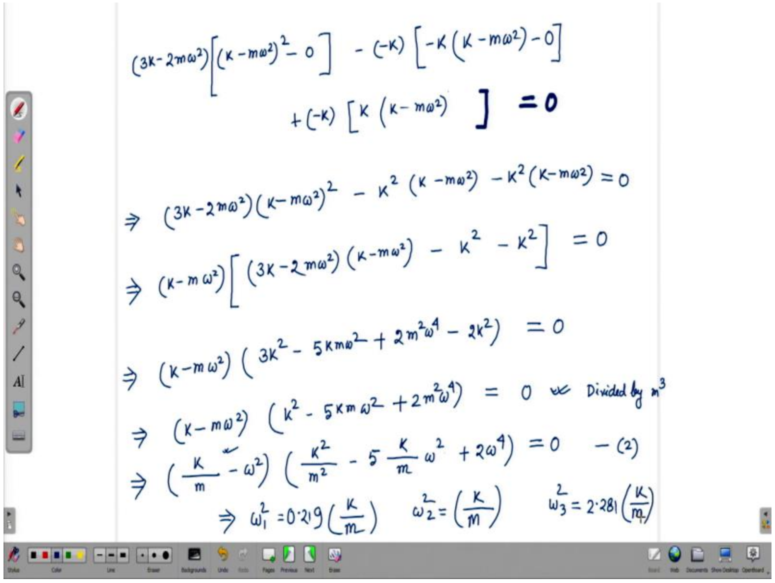

So, what I will get is minus I just change the color of this ink so, minus k times k minus m omega squared minus 0 for this and then my last minus k times once again we are getting k times k minus m omega square minus 0 obviously, is there this is equal to 0 just check it once again in the previous page whether I have written it correctly or not minus k then 0 minus.

So, here you can see there is a I think so, I have written it correctly 0 minus so, just it will be better if I will not write these here. So, from this we can set equation it is k minus m omega square whole square inside now, what we can see here that is interesting we can get k squared times k minus m omega square from the second term and from the third one we are getting another minus k squared times k minus m omega squared and this right hand side is 0 here.

So, now, we can see that k minus m omega squared is common for all the 3 terms on the left hand side. So, I can take it out and can then write rest of that term 3k minus 2 m omega squared times k minus m omega square this is the first term now, the second term is k minus k square third term is another minus k squared and that is equal to 0.

So, from these I can also write something like 3k square minus 5k m omega squared plus 2 m square omega squared minus 2k square that is equal to 0 or I can write it as k times m omega square this thing is multiplied with 2k, sorry 3k squared minus 2k squared that means only k squared is left here.

So, k squared minus 5k m omega squared plus to m square omega square, this entire thing is on the left hand side and the right hand side it is 0 or we can now divide the right hand side and the left hand side by k k is a non 0 quantity here. So, what I will get not k better I should try divide left hand side and right hand side by m square. If I will do that then I will get something like this k m squared or I can use mq that will be right choice.

So, what I have done these line is divided by n square these divided by m squared not m squared m q and that I can do because m is a non 0 quantity here. So, now, from these we can get the roots because there are how many roots for this equation I think somewhere there will be omega to the power 4 I made a mistake while writing the power of omega so, let me correct it in this case it is omega to the power 4 dot omega square let me check where I made this mistake on the this too.

So, these will be omega to the power 4 if it is omega to the power 4 then here also it should be omega to the power sorry not here here it will be omega to the power 4 so, now, you can see the highest order of this equation, I can give it our number let us take 2 so, the order of these omega in equation 2 is 6 that means, it has 6 roots there are 3 positive or there are 3 positive roots and remaining 3 roots are negative.

So, what are those 3 roots let me calculate. So, omega one is first root then second root is omega 2 and the third is another one. So, omega 1 omega 2 and omega 3 these 3 are the positive roots so so, the first root is omega 1 which is 0 point 219 times square root of k by m. The second root obviously is square root of k by m that we can get from this itself and the third root now we will see that is 2 point 281 times square root of k by m.

So, these are the 3 roots for this equation of for these characteristics equation 2. Now, next step is to find out the mode shape for these. So, please come to the next page. So, here we will find out the characteristics equation just 1 minute in previous page finally, I have calculated omega 1 omega 2 and omega 3 but this is not omega 1 actually omega 1 square omega 2 and then it is not square root of K by a it is just k by m all the 3. Sorry, so 219 k by m this is k by and this is k by m.

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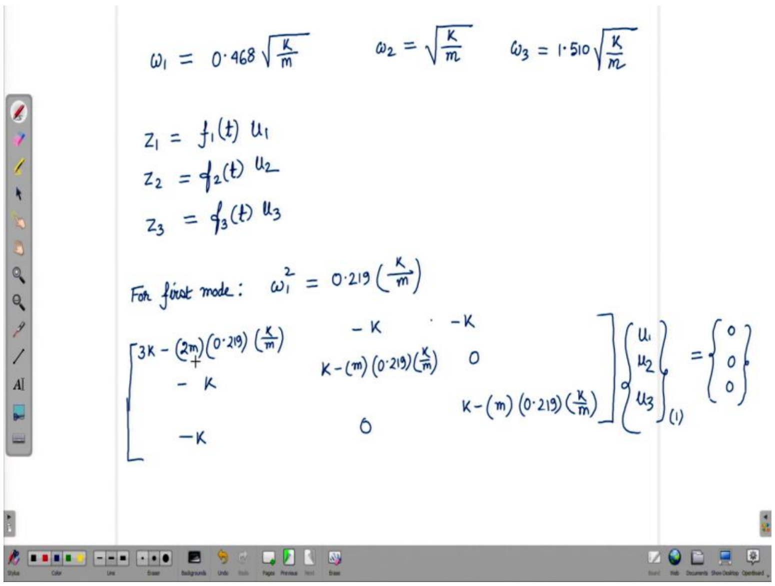

Then omega 1 is how much that we need to calculate. So, square root of 219 means it is 0 point 468 times square root of k by m this is the first root or first natural frequency then the second one is simply k by m and the third one will is and square root of 2 point 281 there is a square so, that is coming something 1 point 510, omega 3 is 1 point 510 times square root of k by m.

Now, what we can do now, we will assume that actually already we have assumed z 1 is equal to f 1 t times you want likewise z 2 is equal to f 2 t times u 2 and z 3 is equal to F 3 t times u 3 where we know f1 f2 f3 are time dependent function or time dependent coordinates.

Now, what we will do we will find out u1 u2 and u3 for three different modes of vibration for that what we need to do, for first mode I am writing here for first mode omega 1 or omega 1 squared I can write is equal to 0 point 219 times k by m. So, with this I can write once again the equation of motion that is 3k minus 2 m omega 1 squared.

So, 2 m omega 1 squared then minus k minus k now, we can write the second row second row is minus k k minus m omega square in this case it is omega 1 square 0 and the next row is minus k 0 k minus m times 0 point 219 times k by m. And that should be equal to 1 term I have missed here, that is u1 u2 u3 which is the elements of u vector for first mode of vibration, and that is equal to in this case is 0 this portion actually we have learned in last to last class.

So, the same thing now, I am repeating for 3 degrees of freedom system. So, from this if I will write the characteristics, if I will write the relationship between u1 and u2 what we will get that I am trying to write fast.

So, what we will get is something so, the first term is 3 minus 2 times point 219, which is 2 point 562 or I can write it as also some different way, but finally, what I am getting, I am just writing that expression that means the relationship between u1 and u2 from these what I am getting that I am writing here.

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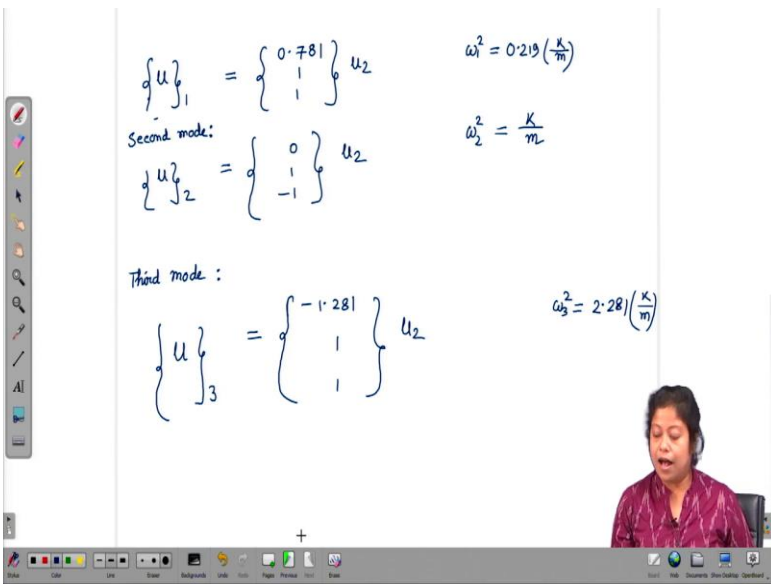

So, these relationship first I need to write, so that relationship is something you u1 I am not writing it only for the first mode, I am writing it for all the three modes or I can write it directly I am writing directly the so, u for the first mode I am writing that is coming 0 point 781 then 1 then 1 times u2 so, what is u2 just so, u2 is here, for the mass 2m sorry mass m on the right hand side, the displacement of that mass is defined by f 2 t times u2.

So, that is u2, which I am writing here. This is for the first mode. So, likewise the same thing I can write for the second mode also for the second mode, when I am writing for first mode, that time omega 1 squared is 0 point 219 times k by m for the second mode, I am writing for second mode. So, for the second boat, it is coming 01 minus 1 times u2 so, all the 3 modes. I am trying to write it in terms of u2 and these times these times omega 2 squared is equal to k by m.

Now, for the third mode u3 that is equal to for the third mode when I will write it omega 3 squared is 2 point 28 let me check previous 2 point 281 times k by m. And for these if I will write i 3 that will be actually I have already determined this value to reduce the time span. So, it is coming 1 point 281 1 and 1 this thing is multiplying with u2. So, in this way we can get actually the u vector and for three different modes of vibration.

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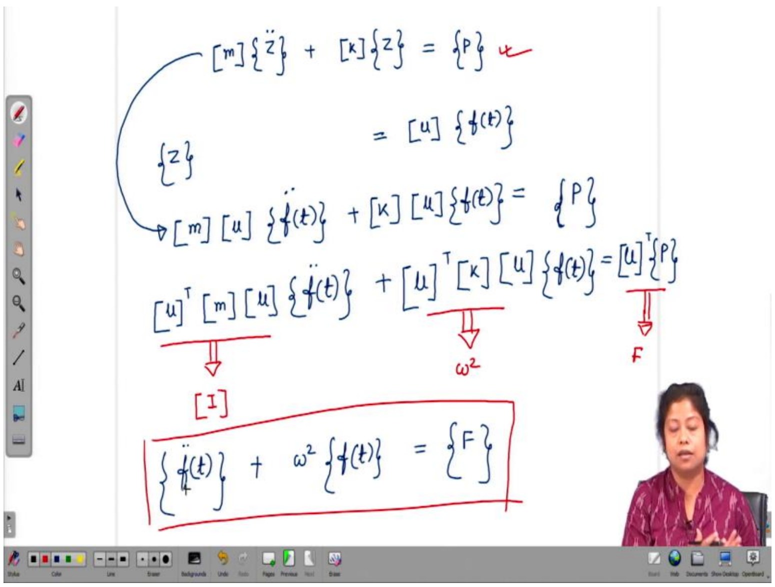

In the next step what we will do in the next step we need to just write I am writing one line we have the equation of motions that is mass matrix times z 2 dot which is acceleration vector plus k times displacement vector is equal to force that we have now, what we can do here.

So, what we have done, we have considered z vector is equal to f t times or I can write it as, I can write it this way times so, now, these equal in this equation I can write these values, so, I am writing I am trying to write that also. So, this is just a minute. So, here I will write this path.

So, what we can then write that I am trying to write somewhere, I can write it as actually, instead of these I can also write it as u times if ft also, if I will write it this way, then I am just erasing this one then what I can write here here I can write u times ft, plus k times u times ft and that is equal to on the right hand side it is p now, in the next day, so, these steps can be written like this.

Now, what I will do, I will pre multiply left and right hand side by u transpose. So, now I am multiplying multiplying these by u transpose. So, I am getting these expression there is a f2 dot plus and this is equal to u transpose times p. Now, here what we can write using the orthogonality condition this will be this gives us identity matrix I. Likewise this will give us if you see, these will give us omega square.

So, finally, and last term we can this one we can write, as, let us take capital F because we have already used a small f. So, I am considering it capital F. So, finally, what we are getting that I am trying to now, f2 dot t that is a vector plus omega squared ft vector omega squared times ft vector that is equal to on the right hand side capital F vector. So, now, what we have seen this equation is reduced to another form which is written here. So, if we solve this, then we can get 3 independent equations for ft and if we solve ft, then we will finally find out the response.

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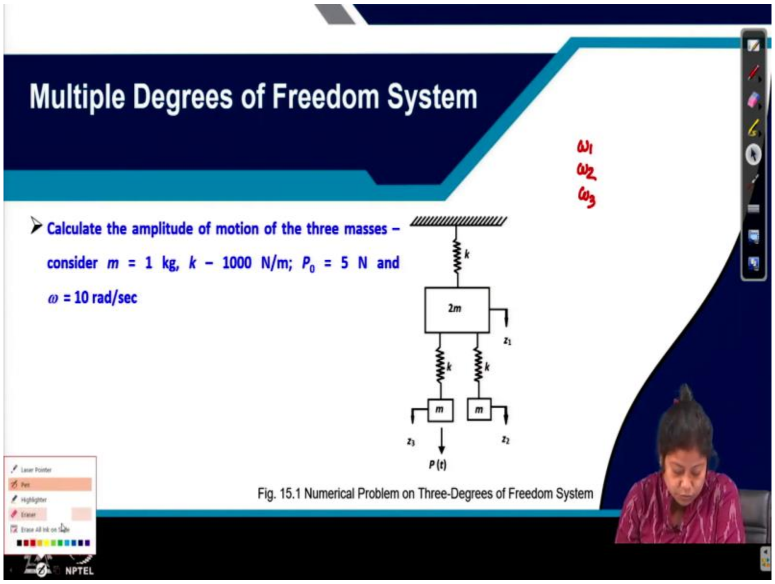

So, for the problem which we have taken here for this, so, far we have calculated natural frequencies omega 1 omega 2 and omega 3 and now we know how to proceed further. So, in next class we will take this value of m and k this should be equal using these value we will try to find out the final response for this system, I am stopping here. So, we will complete this exercise in the next class. Thank you.