Solid Dynamics
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Lecture - 19
Wave Propagation in an Infinite Elastic Medium

Hello friends, today we will discuss Wave Propagation in An Infinite Elastic Medium.

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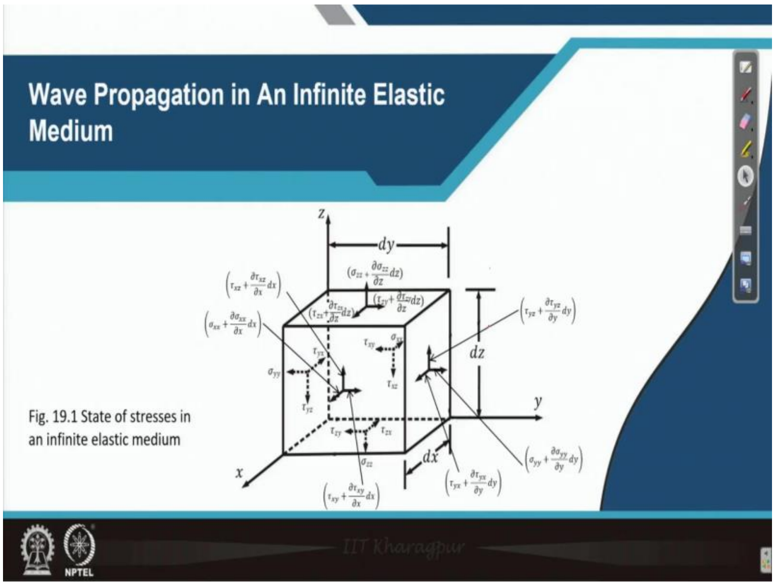

So, let us take and soil element collected from the infinite soil medium. Here we can see the dimensions of the soil elements, so dx dy dz is the dimension of the soil element that means its volume is dx times dy times dz. Now, when longitudinal wave is propagating through the elastic medium, what will be happened? It will pass through the element dx dy dz then the stresses at two opposite phases if we see will be of different magnitude.

For an example, you can take the two planes this one and this one. So, the left one is we can call it as negative y plane, whereas, this one we can call it as positive y plane. Why I am giving this type of name? Because, if you see this plane, if I will draw a normal to this plane, what will be the direction of the normal?

The direction of the normal will be positive y direction. Likewise, if I will draw a normal to the negative y plane or I can say the left hand side plane, the direction of the normal will be in the negative y axis, that is the reason I am giving this kind of name. Now, how I will give that name of the stress components?

We can see the I first time writing the stress matrix that is sigma xx tau xy tau xz then tau yx sigma yy tau yz and then tau zx tau zy sigma zz, this is our stress matrix or we can call it stress tensor. Now, here how these XY somewhere somewhere I am writing exit how these things are coming. So, let us see first the stress component this one I am just writing here 1. So, this component is acting on which plane?

This component is acting on y plane. And what is the direction of this component? The direction of the stress component is positive z direction. So, I can give the name of this component one as tau in subscript first later will indicate the direction of the normal to that plane that means, y then the direction of the stress or the direction of the line of action of the stress component which is z here, so, it is tau yz.

But, interestingly, if you see the another stress component acting on the negative y plane, what will be its name? Its name is already tau minus y minus z, so, there are two minus, so, I can write it this as tau yz and at the beginning itself I told there is an increase in the magnitude of the stress component.

So, basically this stress component which you can see here is having magnitude tau yz plus del tau yz by del y times dy, because this component is already tau yz. I hope in this way we can understand how the names are coming. So, now, you can see that soil element of dimension dx dy dz is subjected to longitudinal wave and we can see the stress developed at the six different phases.

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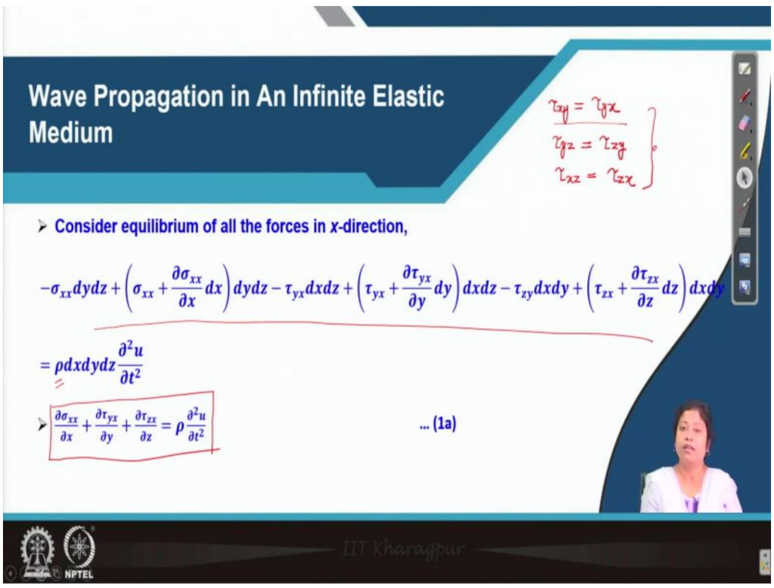

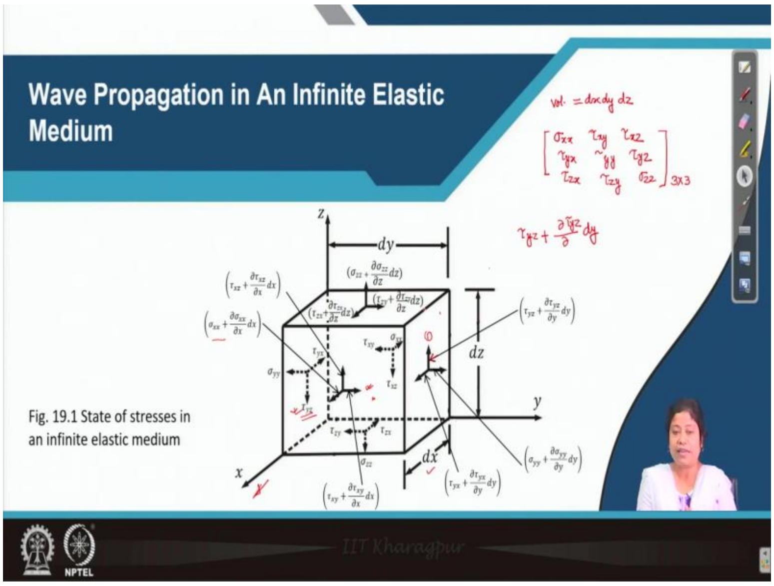

Now, in equilibrium condition if we consider the force equilibrium in x direction, what we can write here? We can write minus sigma x better I just refer to the diagram once again. So, in xx direction means, we are interested right now, for this direction. So, in this plane if I consider what is the normal force? Normal force is sigma xx plus del sigma xx by del x times dx that is the stress that should be multiplied by the area which is dy dz for this plane or I can call it as positive x plane, area is dy dz.

So, I will multiply the area to get the force in x direction acting on positive x plane. From the under normal stress component acting on negative x direction I will get another force which is acting on negative x direction and the magnitude of that force is sigma xx times dx dy dz. Now, from the shear components also, we will get force in x direction and that we need to consider.

So, if we consider all the stress components which are contributing to the force in x direction, then we can write this equation or expression on the left hand side. And what will be the right hand side? The right hand side is mass times acceleration which is inertia force. So, if rho is the density of these material the mass will be rho times dx dy dz and acceleration is del 2 u by del t 2. So, in this way we get the inertia force in x direction.

So, after simplifying we can write these that del sigma xx by del x plus del tau x del tau better I should write here xy del tau xy by del y plus del tau here also I can write xz by del z is equal to rho times del 2 to u by del t 2. Now, if we take the moment equilibrium condition about the point within that soil element then we will see another interesting relation. Actually, we will see three relationships that those three are tau xy is equal to tau yx.

Second one is tau yz is equal to tau zy and the last one is tau xz is equal to tau zx. So, if I use these three relationships coming from the masssorry moment equilibrium equation, then I do not need to change this convention because already I have written here that tau xy is equal to tau yx. So, tau yx is correct here. Let us give it a number 1a.

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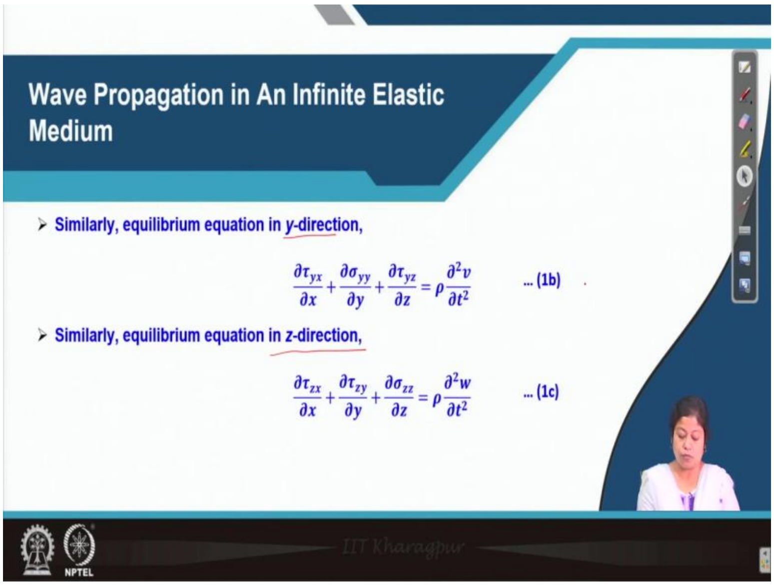

The same way, I can write the equilibrium equation in one direction. So, the equilibrium equation in y direction is tau yx by del tau x plus del tau yx by del x plus del sigma y by del y plus del tau yz by del z is equal to rho times del 2 v by del t 2, v is the displacement in y direction of the soil element under consideration.

For the z direction, we can write the equilibrium equation as del tau zx of del x plus del tau zy of del y plus del sigma zz of del z is equal to rho times del 2 w by del t 2. Here w is the displacement of the soil element in z direction. Let us give the number for these two equations as 1 b and 1 c.

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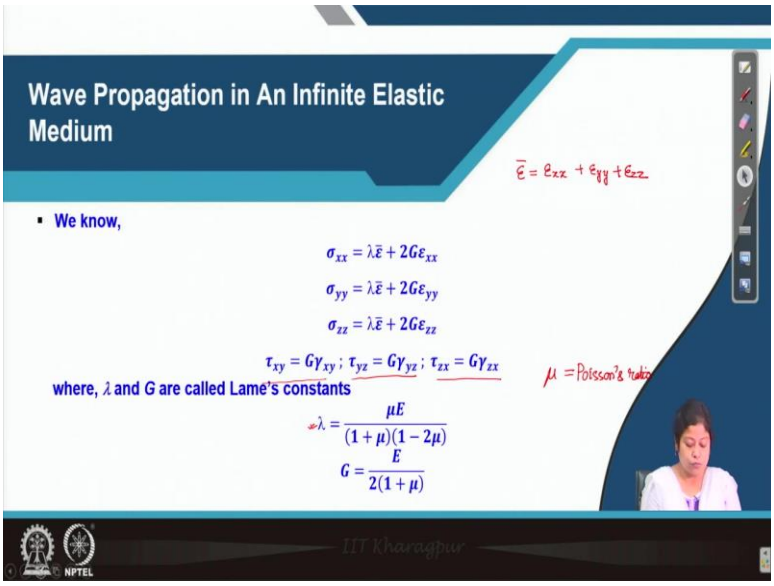

Now, we know from the course solid mechanics, which we have gone through probably in the first or second year of our undergraduate studies that sigma xx that means, the stress normal stress components in x direction is equal to lambda times epsilon bar plus 2 G times epsilon xx. I will come to the point what is lambda, what is epsilon bar and what is G, before that let me write this expression for other two normal stress components sigma yy and sigma zz. So, here what is lambda?

Lambda and G are called lame's constant and epsilon bar is volumetric strain which or we can call it as some of the three actual strain components epsilon xx plus epsilon yy plus epsilon zz. G is also called as shear modulus of the material. Similarly, for the shear stresses, we can write down xy is equal to g times gamma xy, where gamma xy is the shear strain, tau yz is equal to G times gamma yz and tau zx is equal to G times gamma zx.

Now, the lame's constant E and G can be represented by the elastic modulus E and the Poisson's ratio. First lame's constant lambda can be written as mu times capital E divided by 1 plus mu times 1 minus 2 mu, where mu is portions ratio, E is elastic modulus. Similarly, shear modulus G is equal to E divided by 2 times of 1 plus mu.

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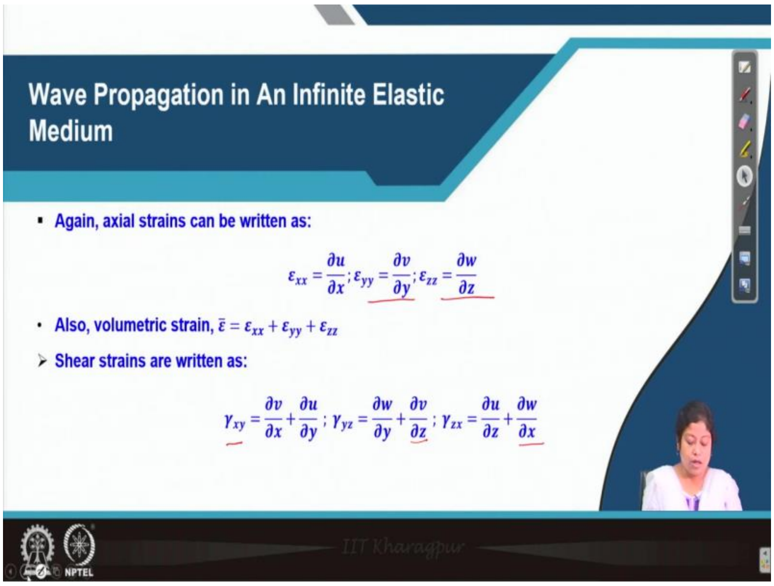

So, now, we are familiar with the three actual string components which are epsilon xx, epsilon yy and epsilon zz, how we can write epsilon xx in differential form? Epsilon xx is equal to del u of del x. Similarly, epsilon y is equal to del v by del y and epsilon zz is equal to del w of del z. Likewise, we can write the shear strain components also.

What are the shear strain components? Gamma xy, gamma yz, gamma xz or zx, So, gamma xy is del v of del x plus del u of del y. Similarly, gamma yz is equal to del v of del z plus del w of del y and gamma zx is equal to del u of del z plus del w of del x.

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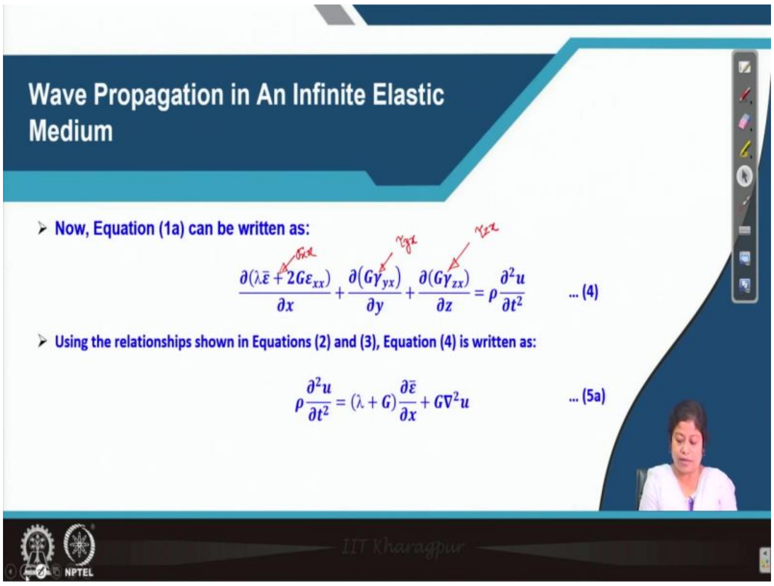

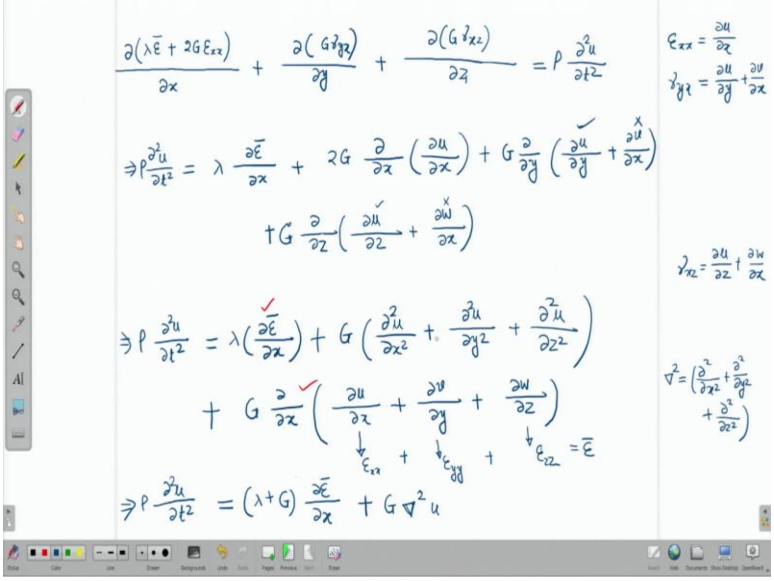

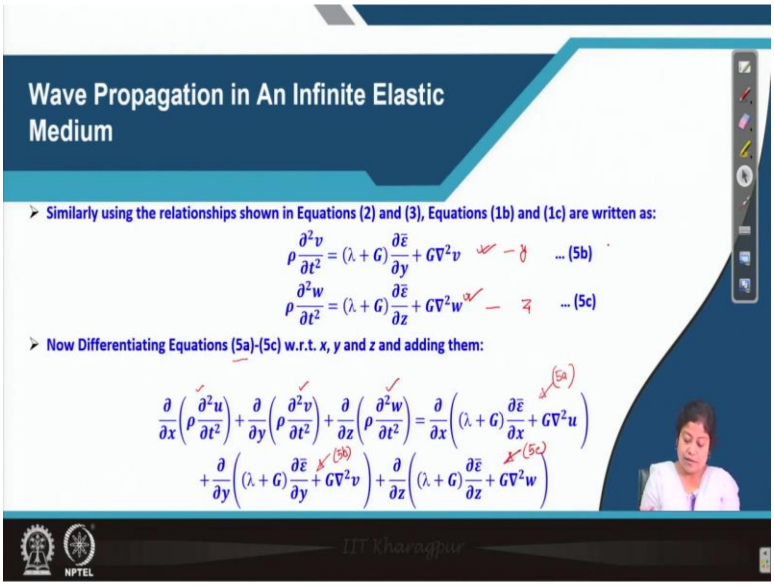

Now, with this what we can do? We have our first equation 1 a, so, there instead of sigma xx first I will write here instead of sigma xx I have written lambda times epsilon bar plus 2 g times of epsilon xx. Here it was tau yx. So, in place of tau yx I have written it as G times of gamma yx. Similarly, in the third term in place of tau zx, I have written G times of gamma zx. Now, what I can write here?

Now, I will use the relationship in I will expand it in whiteboard. So, let me write it here what I have first I am writing that part so, I get del of lambda times epsilon bar plus 2 g times epsilon xx. In place of del sigma xx I am writing this expression plus in place of tau yx I am writing this expression plus there is no two terms here only G.

Here I can write xz or zx both are same. I think in the expressions which I have shown there it was gamma zx. So, that is equals to rho times del 2 u by del t 2. I can take now, I can just interchange left and right hand sides, I am just writing here rho times del 2 u of del t 2. Now, what I can write here? I am just expanding the first term once again.

So, del epsilon bar of del x plus 2 G times, now, epsilon xx means, we have already seen in place of epsilon xx I can write del u of del x. So, that I will now do. So, then this expression will be del of del x times del u by del x. Similarly, in place of gamma yx I can write del u of del y plus del v of del x. So, I am writing it that way del u of del y plus del v of del x.

Similarly, for the third term, third term is gamma xz which I can write as del u del z plus del w del x. So, I am writing then the same way. Now, here the I am, there is no change on the left hand side. What I can write on the right hand side? Now, first term, I will not make any change. I will keep it as it is.

For the remaining terms, now, I can write if you see, if you note the expression very carefully, you can see, I can write it as G times del 2 u of del x 2 plus first term I am taking from this del 2 u of del y 2. Here I am taking, I made this small typo here it is the last term is del z. So, here it will be del z.

So, from the last term now I can take this and can write it as del 2 u and del z 2, so, something I have already taken and written here. Remaining term now, I am writing, so, remaining term means del 2 u by del x 2 plus I can write here. In the remaining term, what we can see? Del of del x is common. So, instead of writing it this way, how I can write?

I can write it as G times del of del x which is common for all the three remaining terms now del u del x plus del v del y plus del w del z. So, what is del u del x? It is epsilon xx. What is del v del y? It is epsilon yy and del w del z is epsilon zz. So, now in place of these we can write epsilon xx plus epsilon yy plus epsilon zz which is nothing but epsilon bar that we have already seen.

So, in the next line left hand side will remain as it is like this and it is equal to lambda times del epsilon del x. This is the first component. Then from the third term, I can write G times del epsilon bar by del x. So, for that I need to erase just this line once. So, first I am writing first and last term which I can write as lambda plus G times del epsilon bar by del x. So, the two terms which I have taken is this one and this one.

Now, the second term is left to right. So, second term can be written this way. We can introduce the operator grad 2 here so, grad 2 u, what is grad 2 operator that is some of del 2 of del x 2 del 2 of del y 2 plus del 2 del z 2, so, in this way I have written this one. I think let me check once whether it is written correctly or not. I think I have written it correctly. So, just let me check it once again. It is u grad 2 u. It is written correctly.

So, let us go back to the slide, what I have written for the x direction. The same thing is written here also. Now for the y and z directions we can do the same exercises and get this equation this is for y direction and this is for z direction. Now, next step what I have done, I have differentiating, just give two numbers for these two equations 5 b and 5 c.

So, now, what we can do? We can differentiate if equation 5 a with respect to x, 5 b with respect to y and 5 c with respect to z, and then sum up these three expressions. So, you can see this is for the left hand side of 5 a, this is for the left hand side of 5 b, this is for the left hand side of 5 c. So, for 5 a left hand side we have differentiated it by x, for 5 b differentiated with respect to y and for 5 c differentiated with respect to z.

And the expression which we get on the right hand side is del del x of the right hand side of equation 5 a. This is coming from 5 a, this is coming from 5 b and this is coming from 5 c you can check. So, after that what we are getting let us see.

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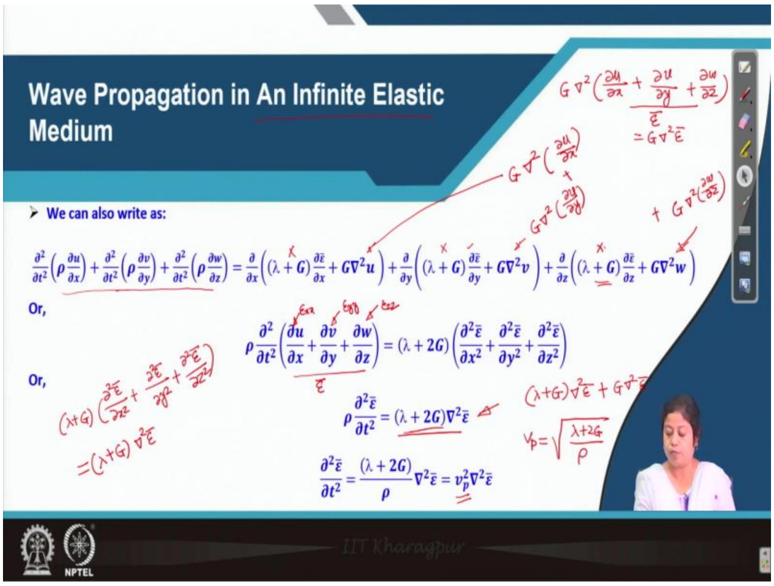

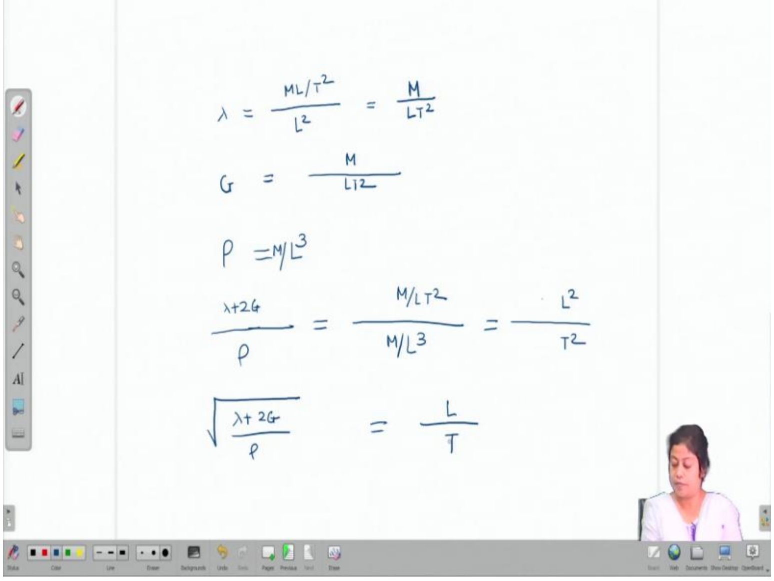

So, left hand side now, we can write we just take del 2 by del t 2 outside because that is common for all the three terms on the left hand side and then we have written this way and right hand side what I have already shown. So, the left hand side now can be written as if I will take rho also outside then we can write rho times del 2 of del 2 of these term del u of del x plus del v by del y plus del w by del z this is what, first term is epsilon xx, second term is epsilon yy and third term is epsilon zz. So, it is nothing but epsilon bar.

So, left hand side we are getting rho times del 2 of epsilon bar divided by del t 2, what about the right hand side? Right hand side if you see we have a term if I am taking the second term of all the three terms, so, second term of each terms here that means these how what we can write here g times grad 2 times del u del x. This is this one. This one I can write G times grad 2 del v del y and this term I can write as G times grad to del w by del z.

Now, when we are adding these three terms, what we are getting? G times grad 2, I am just writing here what we are getting is G times grad 2 del u del x plus del v it is del v plus del v del y plus del w del z. So, this is nothing but again epsilon x plus epsilon y plus epsilon z or I can write epsilon bar. So, finally what we are getting is G times grad 2 epsilon bar and from the first term already we have these from the first term now I am putting cross mark at the top of first term of each term within bracket.

So, what we can write here for first term? For first term if I will write, I can write here, since just it is a rough work kind of, so, we can write it as lambda plus G times del 2 epsilon bar of del tau x 2 plus from this one I can write del 2 epsilon bar del y 2 plus third one means this one. So, from these I can write del 2 epsilon bar by del z 2 or I can write it as lambda plus G times grad 2 epsilon bar.

So, eventually, now, all the terms on the left hand side finally, give us lambda plus 2 G times grad 2 epsilon bar, how it is coming? It is actually coming like this lambda plus G grad 2 epsilon bar plus G grad 2 epsilon bar. If we will write hand side this way, then finally, we are getting this one. So, from this what we can write? We can write it as del 2 epsilon bar divided by del t 2 is equal to lambda plus 2 G divided by rho times grad 2 epsilon bar.

Now, what is the dimension of lambda plus 2 G divided by rho? Let us see that lambda dimension is force per area force by area. So, force means mass times acceleration divided by area. So, what we are getting is mass times divided by LT 2. Same thing we will get for G also and rho means L cube. So, lambda plus 2 G by rho has a dimension which is sorry rho means mass divided by L cube here mass divided by L cube.

So, now, finally, what we are getting is L squared divided by T square. So, if I will take square root of L by lambda plus 2 G divided by rho that has dimension L by T. L by D is the dimension for velocity. So, we can in place of lambda plus 2 G divided by rho we can write one velocity component squaring of the velocity component which is v p squared.

So, what is v p here? V p represents the velocity of the longitudinal wave when propagating through and infinite elastic medium, that time the soil element is subjected to the stresses in 3 orthogonal stresses of three orthogonal directions. So, from these finally, we can write before going to the summary I am just writing it that v p is nothing but by root lambda plus 2 G divided by rho.

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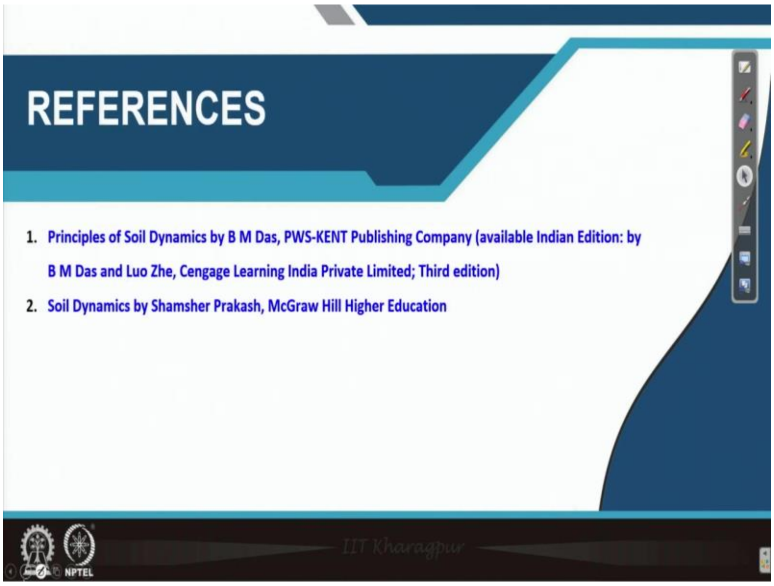

So, from today's class, what we have learned is that, we learned about the wave propagation in an elastic infinite medium and what will be the resulting, in these cases we have consider only the irrotational wave. There is another wave in these type of cases, that is called rotational wave that we will discuss in next plus and also we have learned what will be the velocity of the P-wave when it is propagating in an elastic infinite medium. So these are the references which I have followed for this class. Thank you.