Soil Dynamics
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Lecture - 2
Theory of Vibrations

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Hello friends, after brief introduction to the subject Soil Dynamics its objective and to learn about the definition of the fundamental elements of theory of vibrations. Now, we will start our next class which is the continuation of theory of vibrations.

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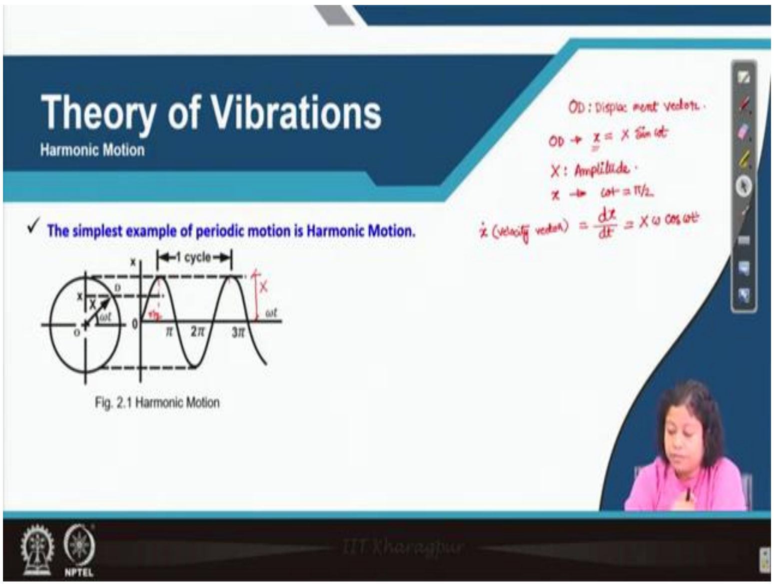

So, today we will discuss what is harmonic motion, we all actually know that the simplest example of periodic motion is harmonic motion. So, in this diagram we can see the displacement versus time or yes displacement versus time curve and also see the vector diagram of the harmonic motion. So, in this diagram what we can see? The left hand diagram on the left hand side shows OD which is the displacement vector.

So, I am writing here for understanding OD is the displacement vector and that OD can be represented by small x is equal to capital X times sin omega t as per this diagram. Now, at position OD, if we know the angle of OD with respect to the horizontal position, then which is omega t, then we can find the value of small x.

Now, what other things we can note here? What is capital X? Capital X is the amplitude of displacement. So, in this diagram capital X means this one, I am writing what is capital X, it is amplitude. Now, when it reaches to the maximum when small x becomes maximum that means reaches to the peaks small x reaches to the peak when Omega t is equal to Pi by 2, that means at this position Omega t is pi by 2.

Now, this is displacement vector, what about velocity vector, we all know velocity x dot I am writing in bracket also, the full form velocity vector, it is nothing but the first derivative of the displacement vector. So, I can write it as dx of dt, so in this case what will be the velocity equation for this velocity vector, it will be capital X times Omega times cosine of Omega t am I right? So, now I need to do some more calculation, so let us go to the board.

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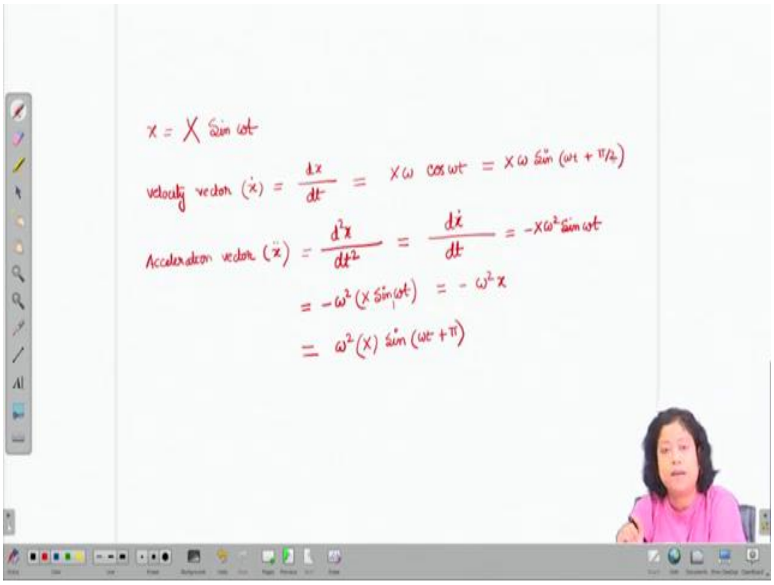

So, I am writing once again what I have written already, I have written there, so small x which is very displacement vector is equal to capital X times sin Omega t, capital X is the amplitude and small x is the displacement vector. Now, when we are calculating velocity vector, so I am writing here velocity vector which is symbolically represented by x dot and it is nothing but the first derivative of x with respect to time and that is equal to x times Omega times cosine of Omega t.

Now, we can also calculate the acceleration from this, so at next is acceleration vector which is represented by x double dot and x double dot is nothing but the second derivative of small x with respect to time. So, I can also write it as first derivative of velocity vector with respect to the time. So, what will be this, then minus x Omega square times sin of Omega t. The interesting thing in this case is that I can rewrite this expression once again as minus Omega square times X capital X times sin Omega t.

So, what is our capital X sin Omega t? It is nothing but the displacement vector, so I can write it as minus Omega square times small x. So, what we can see from this derivation? Acceleration vector is that if we think about the magnitude, first what we can see is that acceleration vector is proportional to displacement vector.

But it is not exactly same its magnitude is Omega square times the displacement and the direction of the displacement vector and the acceleration vector is just opposite to each other that means if I think I can write it once again as Omega square times x times sin of Omega t plus pi. So, I am taking care of this minus sign in the expression of sin Omega t plus pi. What does it mean? It means that the angle between displacement vector and the acceleration vector is 180 degree.

Similarly, for the velocity vector which we have already determined for that also we can find out the angle between displacement vector and the velocity vector, how much is this angle, first I will rewrite velocity vector once again, so x times Omega times, we have cosine Omega t, so which is nothing but sin Omega t plus Pi by 2. So, the angle between displacement vector and the velocity vector is pi by 2 that means 90 degree.

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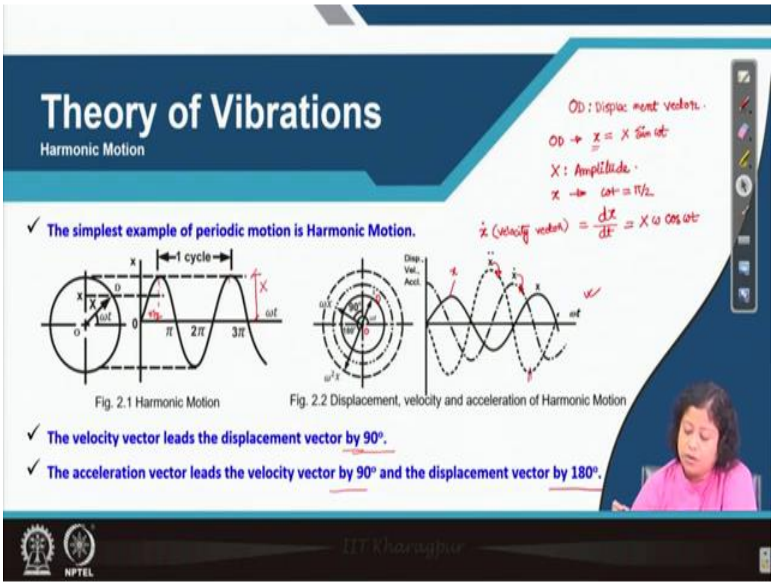

So, now we can see the diagram for these velocity vectors and the acceleration vector, as I already told here that velocity vector leads the displacement vector by an angle 90 degree, likewise acceleration vector leads the velocity vector by an angle 90 degree and the same leads the displacement vector by an angle 180 degree that we have already seen here.

So, we can now draw the vector diagram for displacement velocity and acceleration also we can show it here. So, first diagram this one for the displacement vector which is x, then second one we can see this one so x, I have already marked here so x dot means velocity, this is this one and x double dot means acceleration which is this diagram and here we can see the angle between displacement vector, so this is our OD, this is velocity vector.

So, you can see the amplitude I can say amplitude of the velocity is this is Omega times capital X and for acceleration vector that means from this diagram we can get the amplitude of the acceleration.

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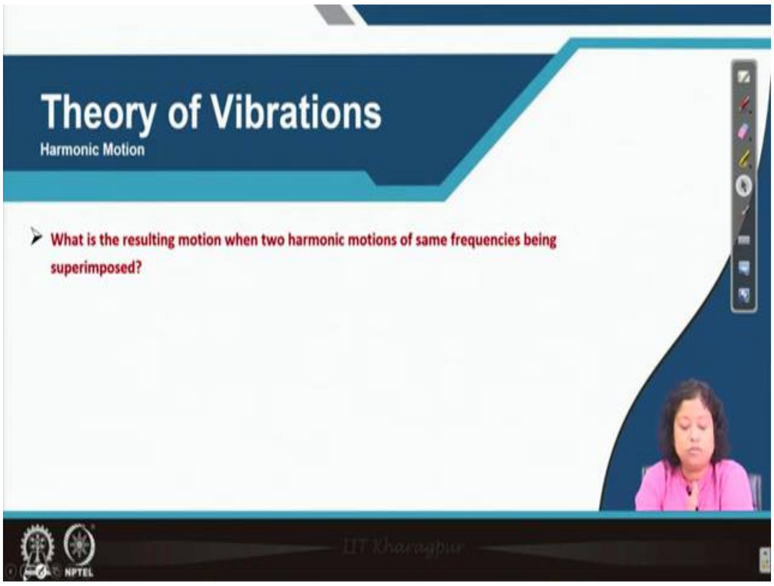

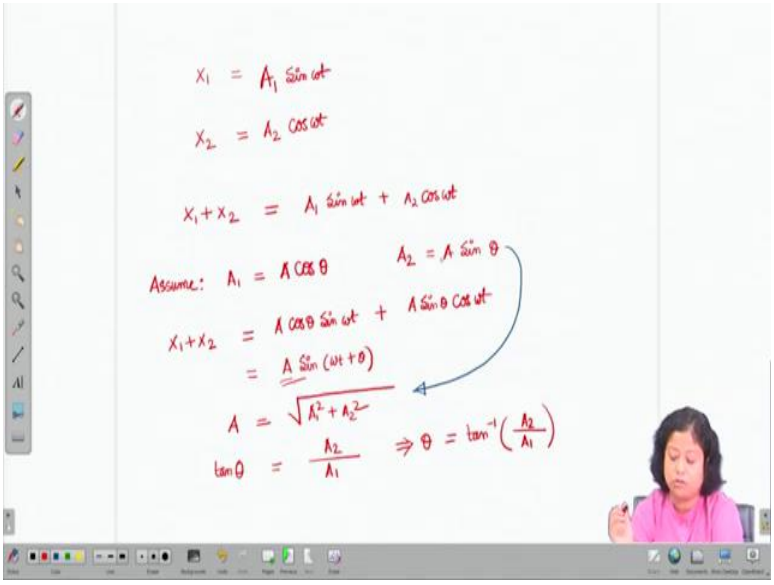

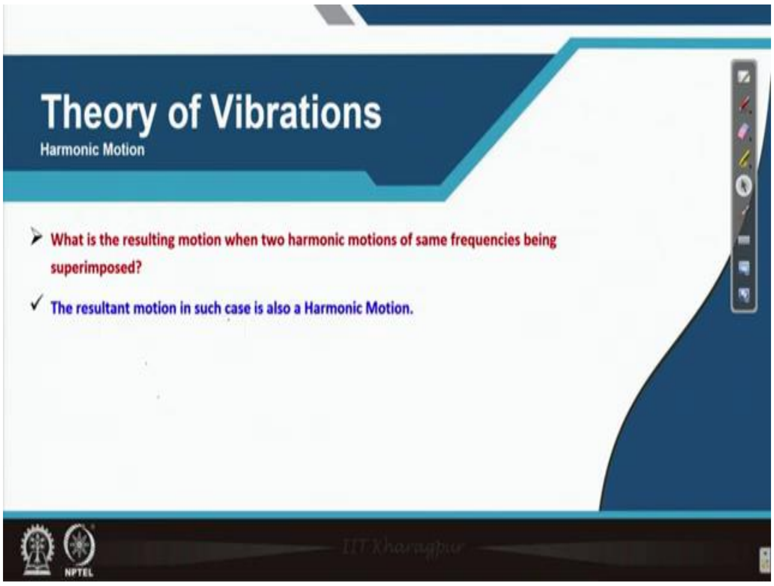

Now, we will check what is happened what is the resultant when two harmonic motions of same frequency being superimposed? So, what I am seeing I am going to the next page. So, suppose we have two harmonic motion, one is x 1, I can write it as small x 1 that will be better probably or capital X 1, fine. So, this can be represented by this equation A 1 times sin Omega t. Another harmonic motion which we have is suppose A 2 times cosine Omega t.

So, what we can note here? The frequency of x 1 and x 2 are same both the cases frequency is omega. However, for first harmonic motion X 1 the amplitude is A 1, whereas for the second harmonic motion X 2 amplitude is A 2. Now, when these two that means X 1 and X 2 will be superimposed what will be the resultant that means now we are interested to add X 1 and X 2. So, X 1 plus X 2 nothing but A 1 times sin omega t plus A 2 times cosine Omega t.

Now, we can assume that we are assuming that A1 is equal to a times cosine theta, theta may be any angle, likewise, we can take A 2 which is also equal to A times sin theta, then what will be X 1 plus X 2, then our X 1 plus X 2 will be A times cosine theta times sin Omega t plus A times sin theta times cosine Omega t, which we can write as A times sin of Omega t plus theta. Now, what we can see from here?

It is or another harmonic motion, so the resultant of the two harmonic motion, if we are superimposing two harmonic motions of the same frequency then the resultant will also be another harmonic motion, the frequency will be different but amplitude also we can find out but resultant is always a harmonic motion.

So, in this case quickly we can find out what is the value of theta which is the phase angle, we can find out what is the amplitude of the resultant harmonic motion. So, what will be A? If you see A is nothing but square root of A 1 square plus A 2 square, we can get it from these, likewise, we can calculate theta also. So, let us take tan theta, so tan theta will be how much tan theta in this case will be A 2 divided by A 1.

So, theta will be tan inverse of A2 by A 1, you can take A 1 and A 2 instead of taking it as A cosine theta and A sin theta respectively, you can take it some you can express A 1 and A 2 some other way, let us take both our you can take it as instead of A some b or some other amplitude and then also the resultant will be your harmonic motion only. So, the answer of this question is the resultant is the harmonic motion, the resultant motion in such case is also a harmonic motion.

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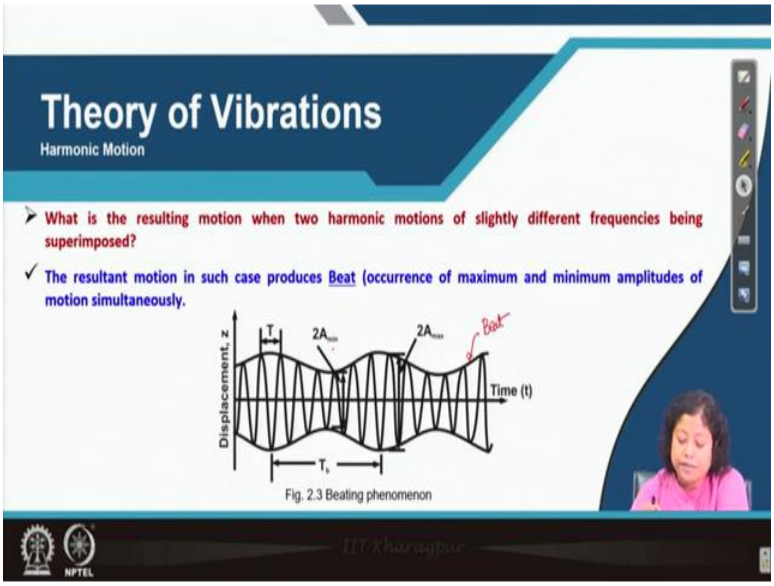

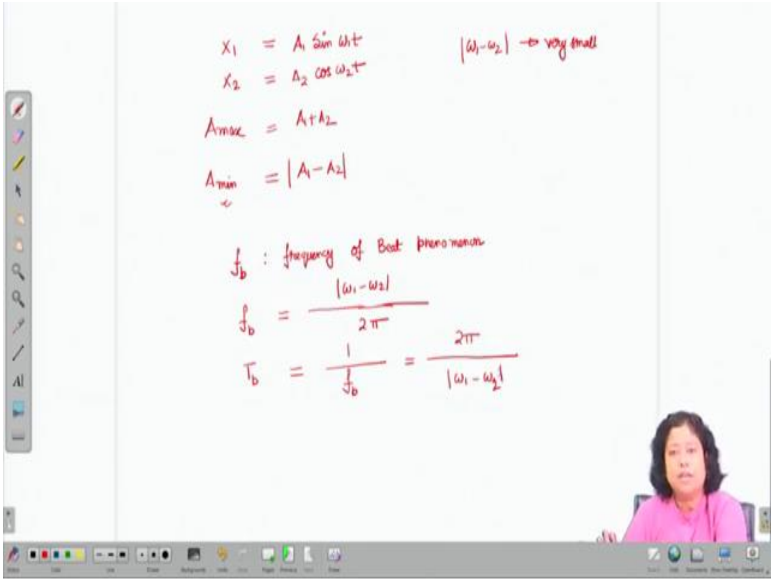

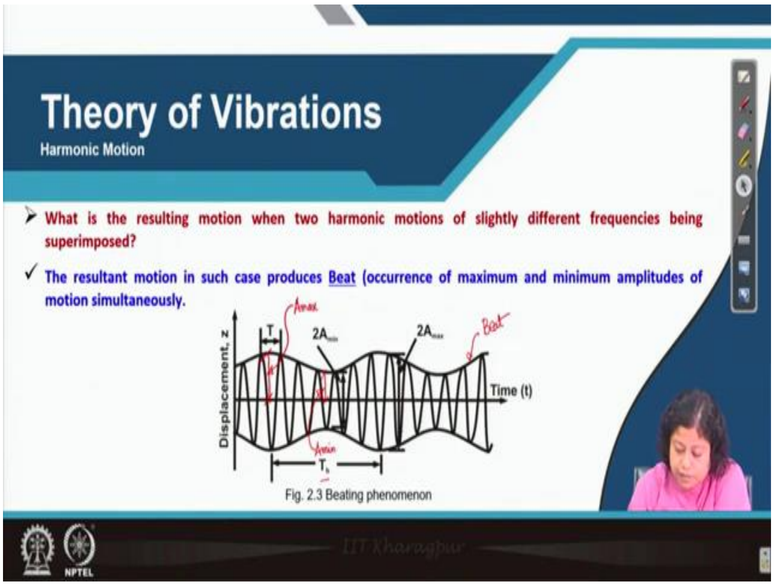

Fig. 2.3 Beating phenomenon

Now, let us see another question what is the resultant motion when two harmonic motions of slightly different frequencies being superimposed? So, earlier we have seen Omega is the frequency for two harmonic motions, now there is here it is asking that the frequencies are different, slightly different then what will be the resultant? Let us see, the resultant motion in such case produces Beat. What is beat? It is occurrence of maximum and minimum amplitudes of motion simultaneously.

So, one you will see the maximum amplitude after some time you will see minimum again you will see maximum after some time again minimum. So, how does it look? This kind of pattern so this is beat. So, now the question, how we can find out the maximum and minimum amplitude? So, let us take how we can calculate it, so I am going back to whiteboard.

So, A max is the maximum amplitude and A min is the minimum amplitude what are the two different harmonic motions that we have taken, in this case we have taken x 1 is equal to A1 times sin Omega 1 t, x 2 is equal to A2 times cosine Omega 2 t and the difference between Omega 1 and Omega 2 is very small. So, A max will be equal to A1 plus A2, whereas a min will be equal to the difference between A1 and A2, when we are reporting amplitude we will take the absolute value.

So, the absolute value of the difference between A1 and A2 will give us the magnitude of the minimum amplitude. Next in the figure if you see this is the figure, so what is T b? T b is the time period of the beat. So, already we know time period if we know the frequency, suppose for beat Fb is the frequency I am writing here Fb is the frequency of beat phenomena. Then how we can calculate Fb?

Fb is just the difference between Omega 1 and Omega 2 divided by 2 pi. So, let me write it better way Omega 1 minus Omega 2 divided by 2 pi, when we know the frequency of the beat phenomenon we can find out T b also which is 1 divided by Fb, so in this case it is 2 pi divided by the difference between Omega 1 and Omega 2. So, in this way we can calculate Fb, we can calculate T b and of course we can calculate A max and A min.

One thing just I did like to mention here that if we consider both halves in this diagram you can see so 2A max is called the double amplitude and if what is then A max at exactly, exactly A max is this much, this is A max, the same way A min is this much, so I can write it here as A min.

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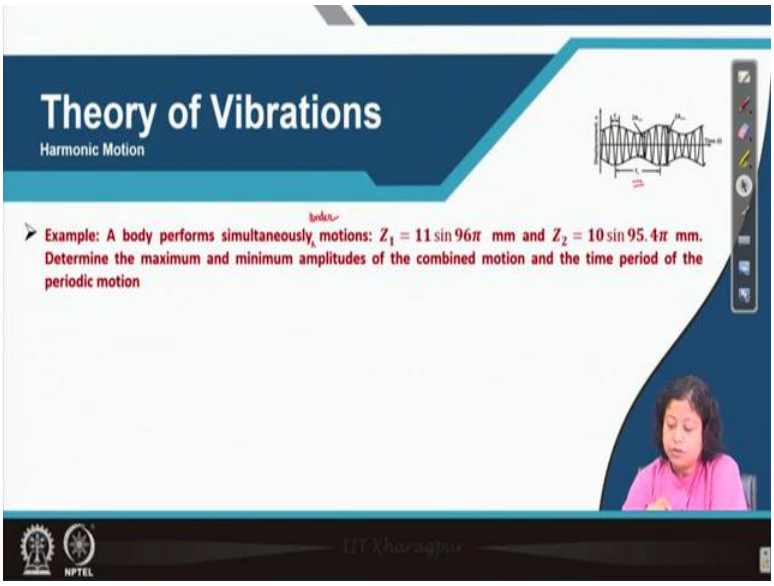

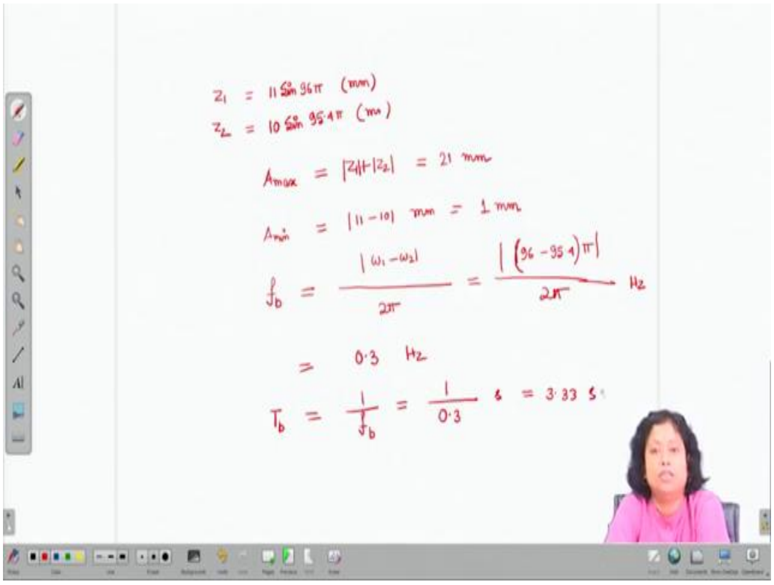

Now, let us take this numerical example, in this example what is said, a body performs simultaneously under motions Z1 and Z2, I can write here under motions Z1 and Z2 the equation for Z1 and Z2 are given. So, we are asked to determine the maximum and minimum amplitudes of the combined motion and the time period of the periodic motion. So, I am just trying to show the diagram which we have already seen in the previous slide also.

So, here the entire thing the resultant periodic motion we can see here, so we are asked to find out A max, A min and Tb. So, how we can calculate? Let us do this problem. So, Z1 and Z 2 are given Z1 let me see what is the Z 1 is 11 times sin of 96 pi, so that is in unit millimetre, likewise, Z2 is defined by the equation 10 times sin of 95.4 pi, so for this case also unit is in millimetre. Then what will be A max?

A max is the summation of the amplitude of Z1 and Z2. So, Z1 plus Z2 amplitude I am talking, so it is nothing but 21 millimetre. Likewise, A minimum is nothing but the difference of the amplitude, so in this case it is 11 minus 10 in millimetre which is 1 millimetre. Now, in order to calculate the time period for this periodic motion beat phenomena what we need to do, first we will calculate Fb which is the frequency.

So, Fb is equal to the difference between Omega 1 and Omega 2 divided by 2 pi. So, this is how much 96 minus 95.4, I can I am taking the absolute value divided by 2 pi, so finally what we are getting is 0.3 in hertz. Then T b which is the time period is equal to 1 divided by Fb, so 1 divided by 0.3 in second which is 3.33 second. So, in this way we can calculate the A max, A min, Tb for beat phenomena.

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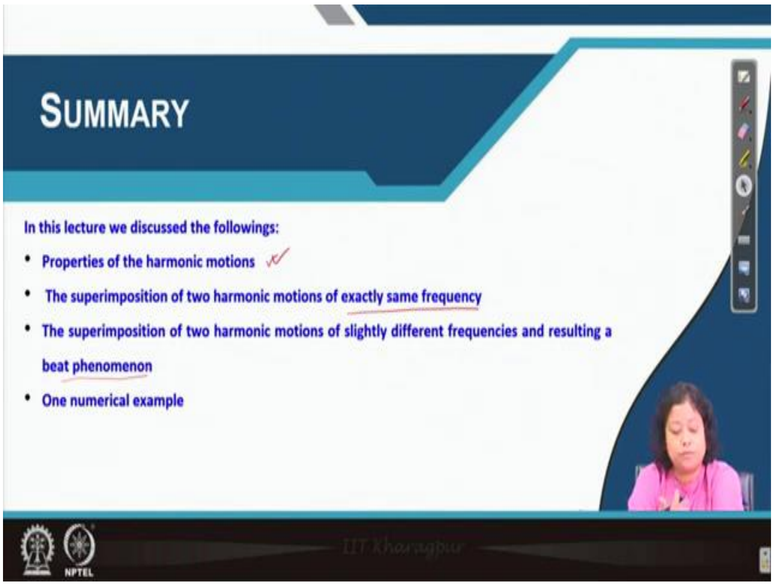

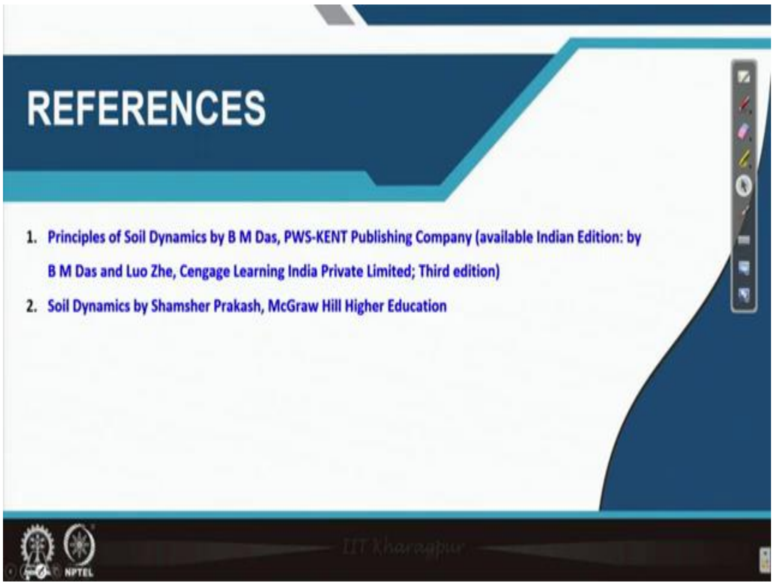

So, now we can conclude our today's lecture, in this lecture we discussed a several things, first we have discussed the properties of the harmonic motions, where we have learned that the displacement vectors leads by the angle 90 degree from velocity and I think I am just repeating the statement once again that angle between displacement vector and the velocity vector is 90 degree.

Similarly, the angle between the velocity vector and the acceleration vector is 90 degree that means in other words we can say the angle between displacement vector and the acceleration vector is 180 degree. Then what we have studied? We have studied the superimposed of two harmonic motions of exactly same frequency. What it will produce?

It will produce another harmonic motion. Now, when we will superimpose two harmonic motions of slightly different frequency what it will produce? It will result beat phenomena. Then we have solved one numerical problem. So, with this I think I can conclude today's lecture. These are the reference which we have used for this class. Thank you.