Soil Dynamic
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Lecture 23
Determination of Dynamic Properties of Soils (Laboratory Tests-Part 3)

Hello friends today we will continue our discussion on cyclic triaxial test.

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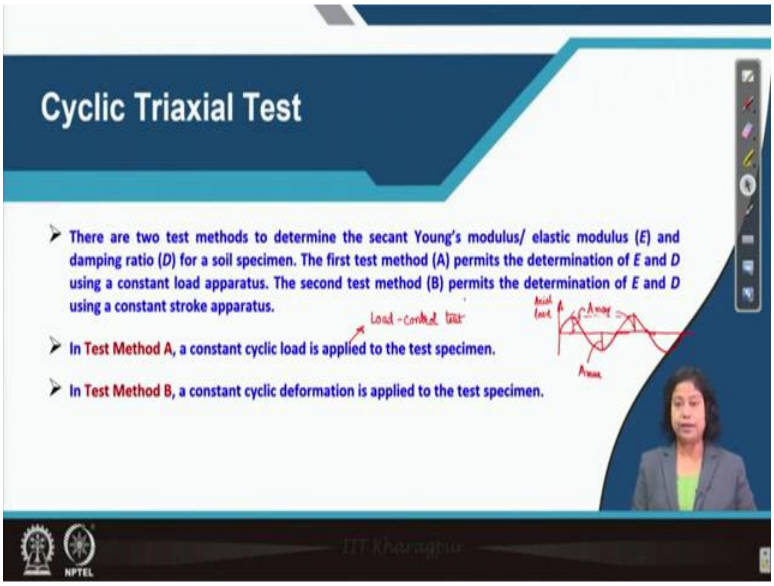

So last class we have started the cyclic triaxial test method. Today we will see the same little bit in depth and then we will see how to determine the different dynamic properties from this test. So for cyclic triaxial test actually there are two test methods to determine the secant Young's modulus or elastic modulus, which is symbolically represented here by E and the damping ratio D for a soil specimen. The first taste method we can call it as test method A permits the determination of E and D using a constant load apparatus whereas the second test method B permits the determination of E and D using a constant stroke apparatus.

So in test method A what is happen a constant cyclic load is applied to the test specimen actually, it is better to say that during test method A the amplitude of the load remains constant over the time. Because load may change over the time, but it is amplitude will remain same I can show it to you here. So if we will draw load versus time this is axial load here I am just showing the axial cyclic load so or it is not really good.

So here this is the amplitude of load so I can take it as A max it is also A max this is for the first cycle for the second cycle also these amplitudes will remain same that means you can see here I can remove so this is second cycle. So in both the cycle amplitude remains same.

So this is the characteristics of taste method A and this type of taste is also called as load control test where the amplitude of the load can be controlled and it is kept same.

In test method B a constant cyclic deformation is applied to the test specimen what does it mean instead of load here the amplitude of the deformation remains same over the time. So if for the first cycle amplitude is say 10 millimeters, then after 10 cycle also the amplitude of deformation will remain 10 millimeters. And I am not showing the diagram here I hope you will understand it.

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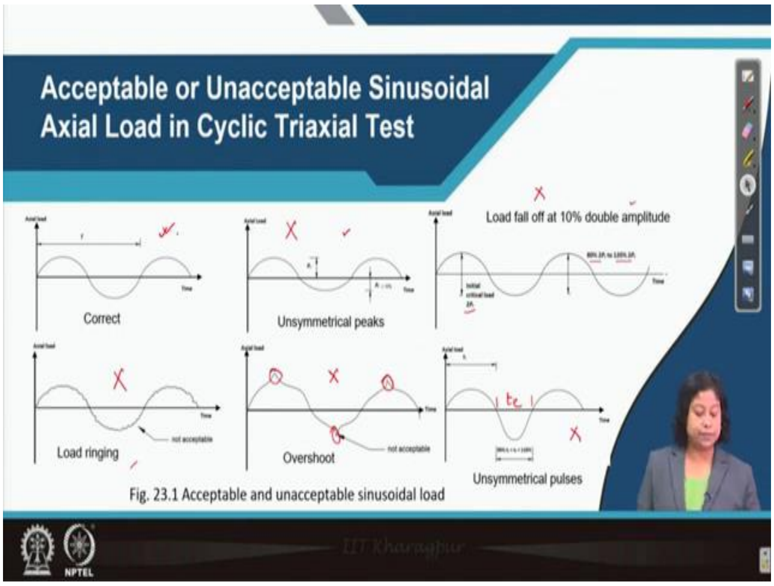

Now there is another important point which sinusoidal axial load will be considered will be acceptable and which sinusoidal axial load will not be acceptable that means unacceptable. So here you can see six figures. So the first figure is acceptable because the first figure if you see that it is for the load control test based that means, test method A. So here you can see that amplitude in extension and compression both part amplitude remains same moreover amplitude of loading over the time also remains same.

However, for the second case what we can see here if PC is the amplitude for the compression portion, then you can see in the extension portion the amplitude is reduced 10 percent of these PC that means the amplitude is not constant now for extension or I can say amplitude varies from compression to extension or compressive to tensile portion. So this type of case is not acceptable.

So come to the third case here what we can see initial critical load that is defined by the double amplitude to PC. Now after one cycle it is reduced to 80 percent of double amplitude of the load for first cycle. So here you can see it is reduced or it may increase also 1 to 20 percent of 2 PC. So this type of cases we cannot accept it in our cyclic triaxial test.

Come to the fourth case this one here what we can see? We can see load ringing you can see here so these type of cases also not acceptable. In the fifth case you can see there is a overshoot here or here which is also not acceptable in cyclic triaxial test so it is also unacceptable.

Now for the last case you can see the unsymmetrical pulses that means if you can see the time for first half cycle now the time for the second half cycle varies from that means for the extension portion, it varies from 0.9 tc to 1.1 times of tc that means no longer T E that T E is here this portion T E is no longer equal to Tc. So such kind of cases are also not acceptable only these first case where you can see that peaks are symmetrical, where we can see pulses also symmetrical where we cannot see any kind of ringing of loading that case is only acceptable that means the first case.

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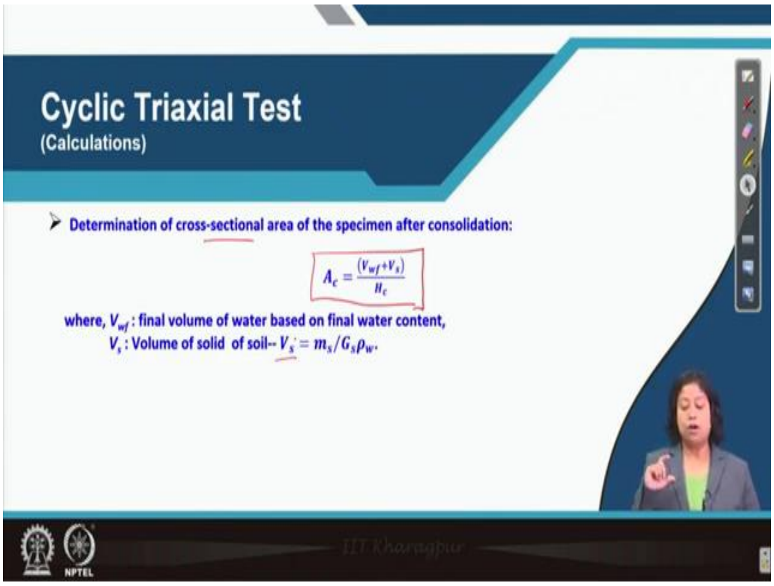

Now next task is to calculate the dynamic properties of soil from the raw data of the cyclic triaxial test we record axial load, axial deformation and pore water pressure over time. So from that now how we will calculate the dynamic elastic modulus. So first determination of cross sectional area of the specimen after consolidation it is one important thing.

So let us see the cross sectional area of the consolidated specimen can be determined using this equation is Ac is equal to volume of final volume of water present in the soil sample plus volume of the solid part of the soil divided by the Hc which is the height of the sample after consolidation.

Now for Vs at the starting of the experiment if we know the mass of the dry soil if we know that specific gravity G is of the solid part of soil if we know these two parameters, then we can easily calculate Vs which is the volume of the solid part of soil.

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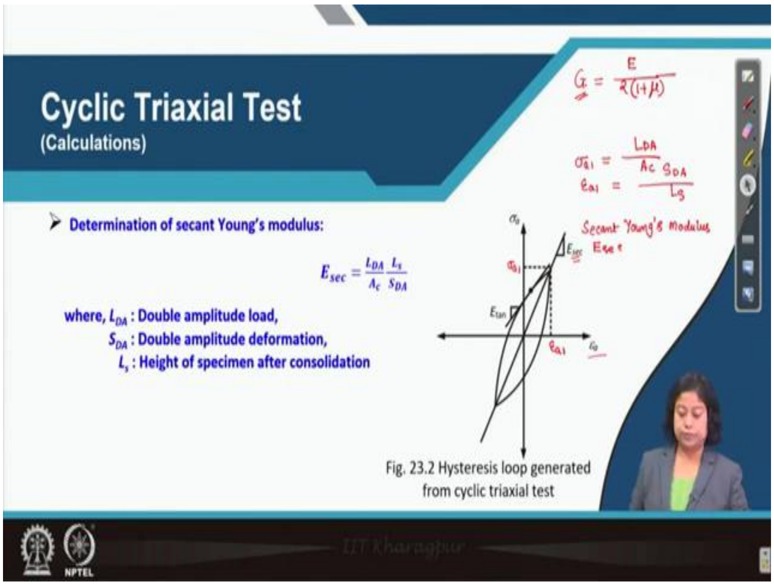

So here you can see the hysteresis loop which is generated from a cyclic triaxial test the horizontal axis represents the axial strength and the vertical axis here represents the axial stress. So if we will join the two ends of these loop by a straight line that will give us secant Young's modulus which we write as Esec.

Now that means if I know this value then I can get Esec value also. So how I will get sigma a 1 by sigma a 1 in this case is load double amplitude load divided by area of the consolidated sample epsilon a 1 is L s divided by sorry, epsilon just give me one minute time to erase, epsilon a 1 is basically deformation to the original length. So here deformation is SDA divided by origin length of the soil sample.

So you can see LDA is the double amplitude of the load Ac is the cross sectional area of the consolidated soil sample LS is the height of the specimen after consolidation anything else and SDA is the double amplitude deformation. So in this way we can calculate the E secant, after knowing the secant Young's modulus, we can also calculate the shear modulus G by using this equation. So here E is dynamics Young's modulus mu is the portions ratio of the soil sample. So using these two parameters, we can find out the G value.

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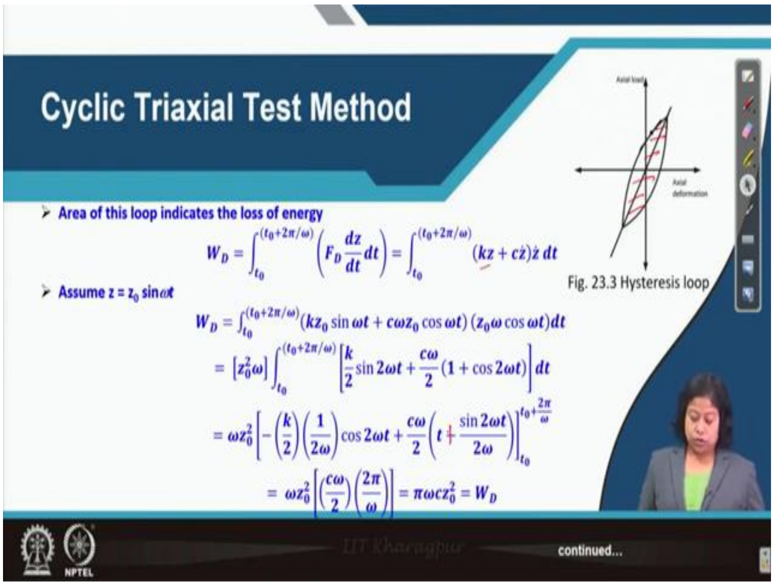

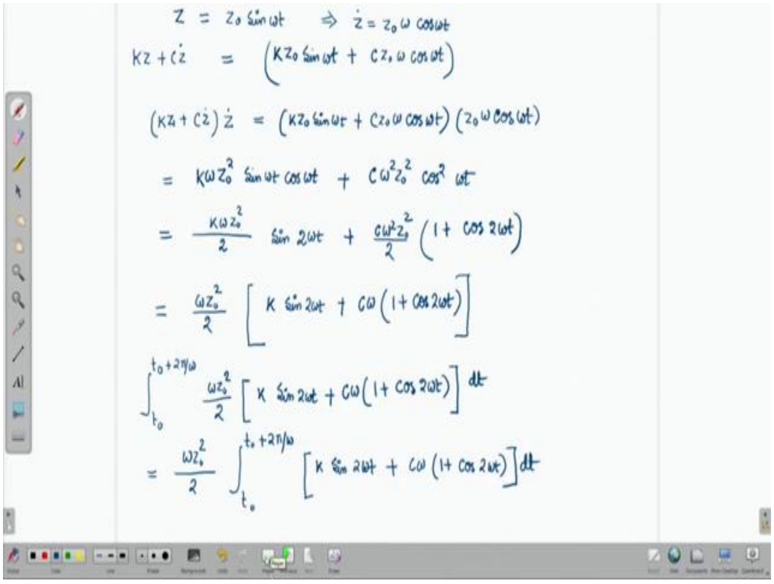

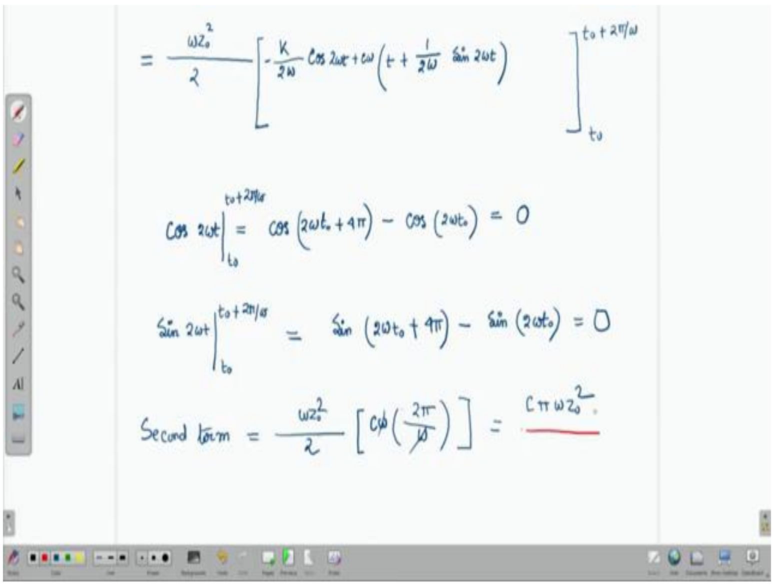

Now next one is how to find out the damping ratio of the soil sample. So from the hysteresis loop we will find out the hysteresis damping here you can see the horizontal axis represents the axial deformation whereas the vertical axis represents the axial load now the area of this hysteresis loop that means this area basically indicates the loss of energy from the system. So how we will calculate this area?

So this area means force times FD is the dynamic force times velocity which is dz by dt times dt. FD means dynamic force because of the spring and dashpot so spring force is kz and damping force is cz dot velocity is z dot times dt. Now if here we assume that z is equal to z 0 times sin omega t where z 0 is the amplitude of these displacements then what we can write here in place of kz we can write let me write on the board.

So we need to write here kz plus cz dot there is no 0 here. Now we have assumed z is equal to z 0 sin of omega t then from these we can also calculate z dot, which is equal to z 0 times omega times cosine of omega t then kz plus c times z dot is equal to k times z 0 sin omega t plus c times z 0 omega cosine omega t.

Now we need to find out kz plus cz dot times z dot which is equal to k times z 0 sin omega t plus cz 0 omega times cosine omega t times z 0 omega cosine omega t. Here of course z 0 omega cosine omega t is the velocity. Then what we will do we will multiply it. So if I multiply then the first term is k omega z 0 square sin omega t times cosine omega t plus c omega square z 0 square times cos square omega t only.

Now what we can do the first term can be written as k omega z 0 square by 2 times sin of 2 omega t this is the first term and from the second term what we can write is c omega square z 0 square by 2 times we can write 1 plus cosine 2 omega t. So finally in place of kz plus cz dot whole multiplying by z dot can be written like this or we can take also omega z 0 square by 2 out of these two term and then remaining term is k times sin 2 omega t plus c times omega times 1 plus cosine of 2 omega t.

So now these things can be written in the original expression, let us go back to the PPT. So here what I said already is written. So basically you can see here z 0 square by omega is taken out of this integration you can take by 2 also, but that is not done here no problem next day probably we can do it.

So now what is the limit of this integration this is for the 1 cycle. So 1 cycle means, what will be the upper and lower limit is of the integration if lower limit is t 0 then upper limit should be t 0 plus capital T which is the time period or we can write in place of capital T it is 2 pi by omega where omega is the circular frequency for the system. So these things now we will integrate I am once again going back to the board.

So basically what we are doing that I am writing here tc integration these multiplying with k sin of 2 omega t plus c omega times 1 plus cosine of 2 omega t times dt or I can take these omega z 0 square out of this integration as I already said. Then it is like this k sin 2 omega t plus c omega times 1 plus cosine of 2 Omega t. Now we will integrate it. So when we are integrating it what we will get that I can write in the next page.

So within bracket what are the terms are present let us see. So first we are integrating k times sin 2 omega t. So that will give K by 2 omega times cosine 2 omega t, K by 2 omega times cosine of 2 omega t this is the first term come to the second term now second term is of course, c omega is there times t plus not t plus now when we are integrating these what we are getting let us see. Here there is a negative sign probably please check.

So now it is c omega I have not written so let me write first that t plus 1 divided by 2 omega times sin 2 omega t. Now we can use the limit so for both the trigonometric functions cosine 2 omega t and sin of 2 omega t both the cases to omega terms the term to omega is present. So let us see when I am writing I am just doing a little bit rough work for you.

So this is 1 term now when I will put the limit t 0 to t 0 plus 2 pi by omega what it will be let us see cosine of 2 omega t 0 plus 4 pi I hope I have written it correctly minus cosine of 2 omega t 0 so it is 0 likewise if I will write sin of 2 omega t we need to t 0 to t 0 plus 2 pi by omega then what I will get sin of 2 omega t 0 plus 4 pi minus sin of 2 omega t 0. So this is also 0 here. So the first and last terms are becoming 0 when we are using the limit which is left is the second term only.

So what is the second term that I am just writing here that is omega z 0 square by 2 times c omega now t 0 plus 2 pi by omega minus t 0, which is nothing but 2 pi by omega. So what is left that I am trying to write now pi omega z 0 square anything else pi omega z 0 square c nothing in denominator that means 1 is in denominator I am not writing that just erasing this. So the second term is c pi omega you can see it is c pi omega z 0 square and that is the area of this hysteresis loop. So here you can see the same exercise here I have done.

And finally just let me correct one thing I think here when I am integrating just a minute there is a type o probably see this is I am integrating. So it is what is happened with the pen this is plus please check although it has no influence in the final form of this area of the hysteresis loop, but still the expression should be correct, I think this is a type o from my end. So please make it plus and this is the final expression for the area of this hysteresis loop.

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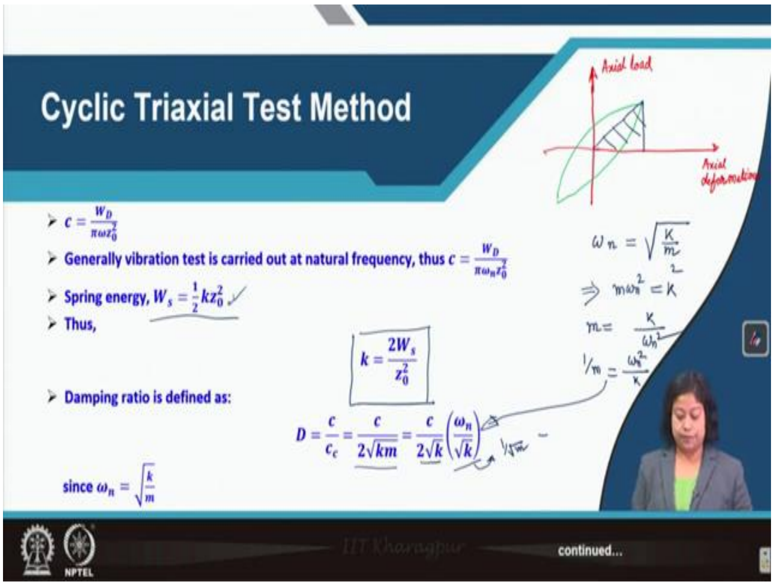

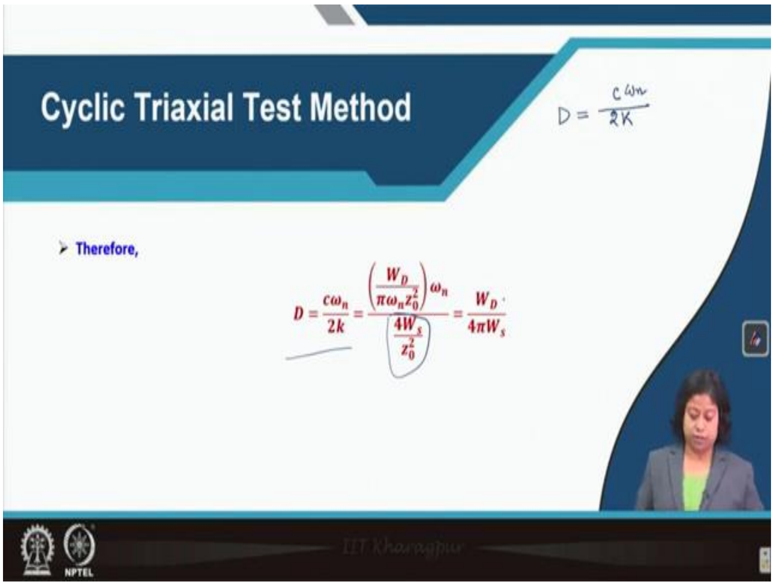

So from that expansion we can get that expression for c which is the coefficient of damping in terms of the area of hysteresis loop which is capital WD divided by pi omega z 0 square. Now generally the vibration test is carried out at the natural frequency. So in place of omega we can write the natural frequency which is omega n.

Now what is the spring energy? So spring energy if you recall what we have we have a hysteresis loop this is axial deformation I can write here at axial deformation and this is for axial load now the loop is something like this. So now for spring energy calculation what we can do? This area will give us the spring energy. And that can be calculated by Ws is equal to half of k times z 0 square where k is the spring stiffness. Basically kz 0 is the spring force.

So half times spring force times z 0 give us spring energy. Then from this expression we can write k which is the spring stiffness in terms of the spring energy Ws divided by z 0 square by 2 that means k is equal to 2 times of Ws is divided by z 0 square. Now damping ratio is defined as c divided by cc where c is the coefficient of viscous damping, cc is the critical damping.

So in case of critical damping we can write 2 times square root of k by m whereas, k is equal to square root of sorry not k I am making mistake. So let me correct myself sorry. So omega n is equal to square root of k by small m here we can write also k is equal to m times omega n square also. So or we can also write m is equal to I think here I made a mistake let me first erase this thing.

So from this what I can write is that m omega n square is equal to k or m is equal to k divided by omega n squared. So now here you can see C divided by 2 times square root of k is multiplied by 1 divided by square root of n. So actually this is nothing but 1 divided by square root of m. So 1 divided by square root of m means omega m divided by square root of k which we are getting from this expression. Now if I will simplify it what is what we have then D is equal to c times omega n divided by 2k.

So from the previous slide what I get I am writing here c times omega n divided by 2k so that is first written here now in place of c the expression which we get is used similarly the expression for k is also used here in you in terms of the spring energy and omega n as it is.

So finally, what we are getting is D which is the damping ratio is equal to WD divided by 4 pi times Ws what is WD? WD is the area of the hysteresis loop whereas, Ws the area of the triangular portion which I have shown in the previous slide or physically I can say Ws the spring energy whereas WD is the loss of energy which is represented by the hysteresis loop. So in this way we can determine the damping ratio for the hysteresis damping from our cyclic triaxial test.

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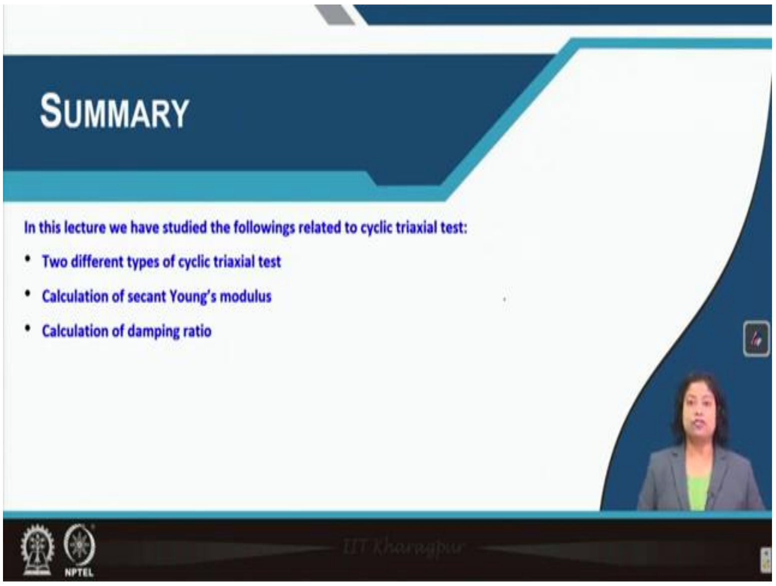

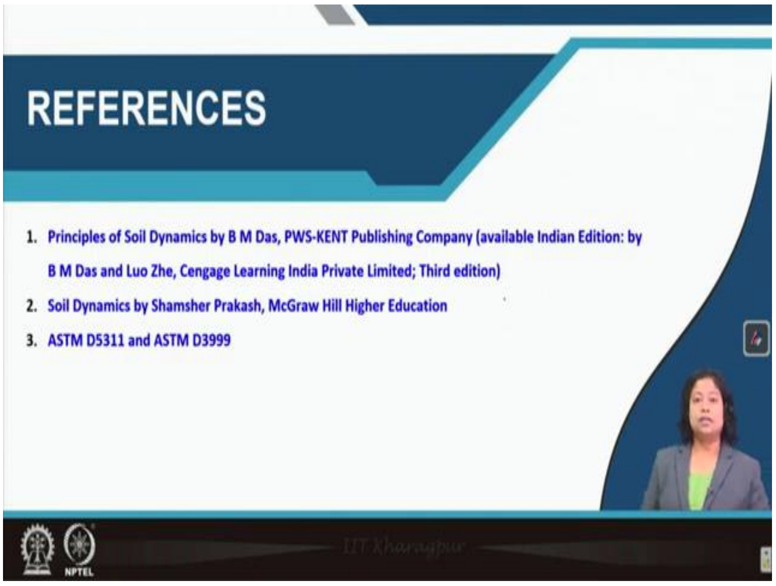

Come to the summary of today's class now in today's lecture we have studied two different types of cyclic triaxial test. One is called load control. The other one is called displacement controlled in case of load controlled test or test method A, the amplitude of the applying load remains constant over the time. Whereas, in case of displacement control test or test method B the amplitude of the displacement remains constant over the time.

Then we have studied how to calculate the secant Young's modulus. Then we have studied how to determine the damping ratio from the hysteresis loop generated from cyclic triaxial test. So with these references I would like to stop here. We will meet next class thank you.