Soil Dynamics
Professor Paramita Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture 28
Determination of Dynamic Properties of Soils
(Seismic Refraction Survey-Inclined Layering)

Hello friends. Today we will discuss the seismic refraction survey considering inclined layering. In last class we have studied seismic refraction survey considering horizontal layering but today we will see what is the difference in the velocity or in the calculation of the velocity of P wave in the calculation of the thickness of layers when the layers are inclined to the horizontal plane.

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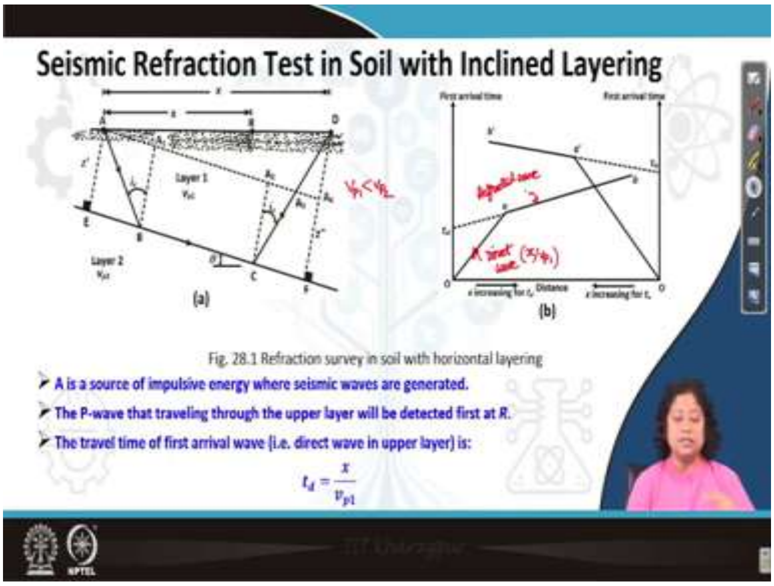

Let us see figure 28.1. So, in this figure A is the source of disturbance, D is any point at which geophones are placed to record the first arrival of the P wave. Now, what is happened, when we create disturbance at A, P wave starts to propagate and its wavefront is spherical wavefront.

So, up to some distance R, what we can see? Up to some distance R we see that the direct wave reaches the ground surface before the reflected wave or refracted wave. So, in this case we are considering refracted wave, so at point R which is not too far from the source A there direct wave will arrive before the refracted wave.

However, at point D the direct wave, before direct wave refracted wave will arrive. So, now if we will plot the first arrival time and the distance when A is the source of disturbance what we can get, we get two curves, one is oa this is for the direct wave and ab for the refracted wave, so this is for refracted wave, when a is the source.

Now, for the direct wave what is the arrival time? The arrival time of the direct wave is the distance a r divided by the velocity of the P wave in layer 1 which is v p 1. In this case one thing we should note is that the velocity of the P wave in layer 1 is lower than the magnitude of the velocity of P wave in layer 2. That means P wave moves faster velocity in layer 2.

So, the travel time of first arrival wave is in this case x divided by v p 1, I can write here also x divided by v p 1. Now, for the refracted wave what is the travel time? For the refracted wave the travel time is the travel time for a b, the travel time for B C and the travel time of C D if we will sum these three travel times together we will get the travel time for the refracted wave AB C D.

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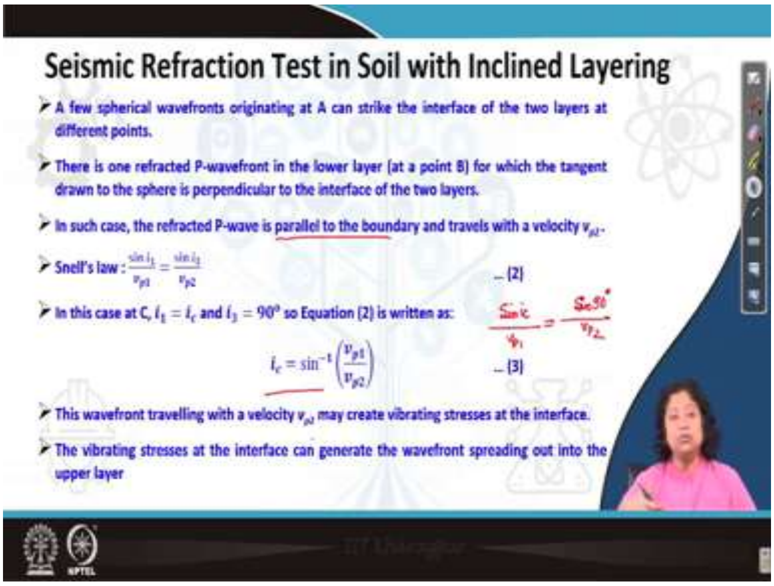

So as I said a few spherical wavefronts originating at A can strike the interface of the two layers at different points. There is one refracted P wavefront in the lower layer for which the tangent drawn to the sphere is perpendicular to the interface of two layers these things we have seen in last class also.

And in such case, the refracted P wave is parallel to the boundary, in this case it is inclined boundary and travels with a velocity v p 2. That means for the path B C P wave travels with a velocity v p 2. Then we can use Snell's law and using that we can write sin of i 1 divided by v p 1 is equal to sin of i 3 divided by v p 2.

Now, i 1 is equal to i c that means critical angle for i 3 is equal to 90 degree, that means when the refracted wave travels along the boundary, this case. So, that time what we can write? We can write actually sin i c divided by v p 1 is equal to sin, in place of i 3 I can write sin 90 degree divided by v p 2, so from this I can write i c is equal to sin inverse v p 1 divided by v p 2.

This wavefront traveling with a velocity v p 2 may create vibrate, vibrating stresses at the interface. As a consequence, the vibrating stress at the interface can generate the wavefront spreading out into the layer 1 once again.

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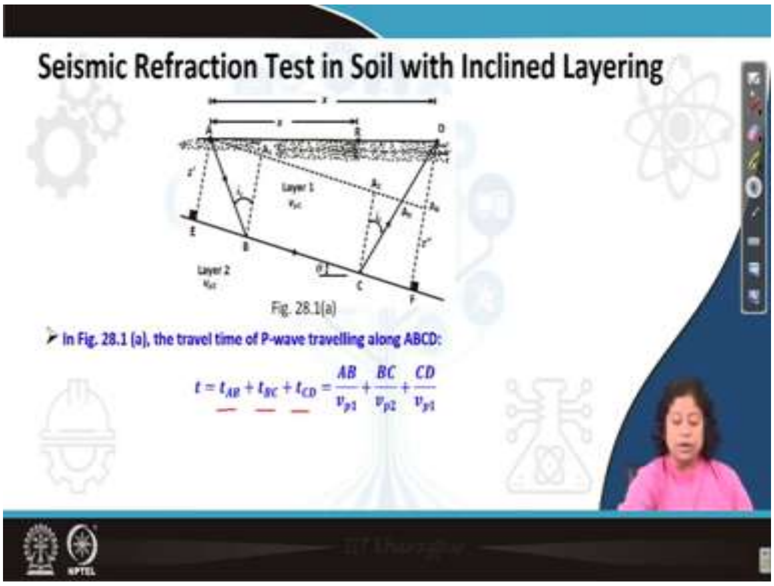

So, B C now change its path and follow the path C D. So, this time this case the travel time of the P wave is t is equal to travel time for the path AB plus travel time for the path BC plus travel time for the path CD, which is equal to AB divided by v p 1 plus BC divided by v p 2 plus CD divided by v p 1. So, now if I know the distance AB, BC and CD then I can get the travel time T easily.

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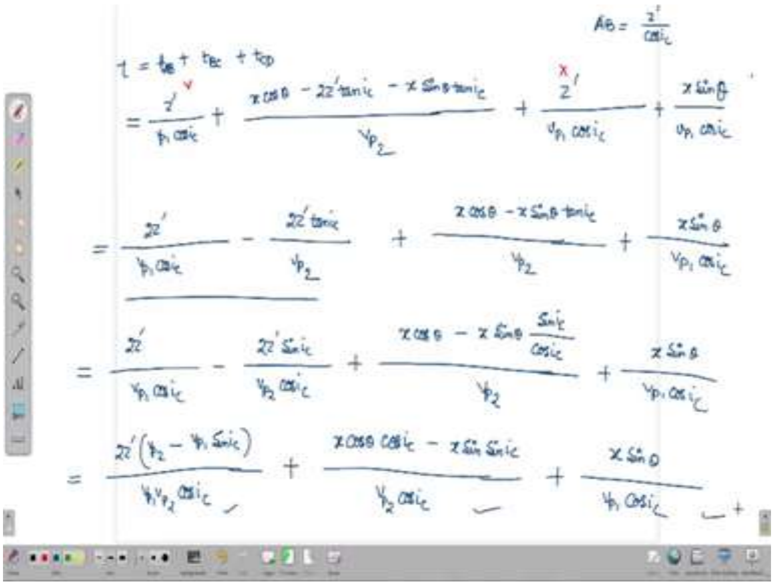

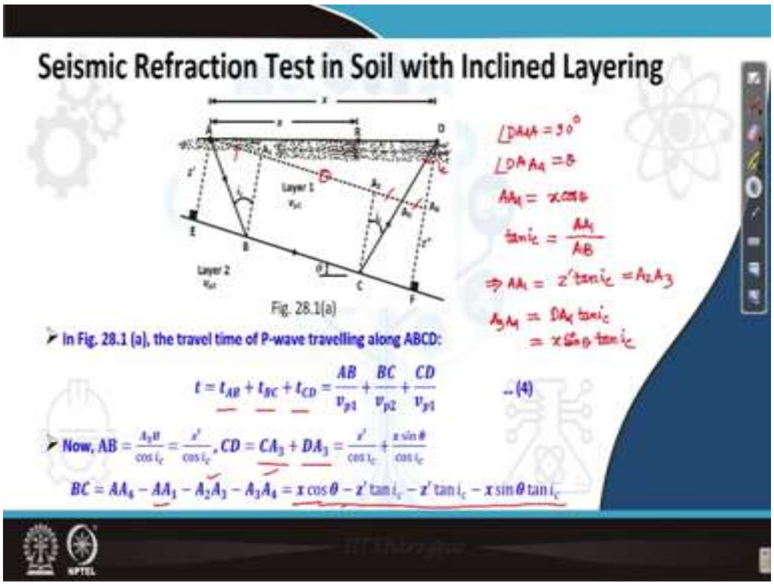

So, let us do this exercise here. So, t is equal to t AB plus t BC plus t CD, t AB means AB divided by v p 1 that is already written. So, what is AB here? We can write AB in terms of Z dashed so if I will write AB in terms of Z dashed then AB can be written as Z dash divided by cosine i c as per this diagram.

So, AB can be written as Z dashed divided by cosine i c. So, here in place of AB, I am just writing Z dashed divided by cosine i c divided by v p 1, so like this. Now, come to BC. BC means what? BC means I can write BC as AA4 minus AA1 minus A2 A3 minus A3 A4. So, I am just writing here itself or I can write it is here already written, so I am just writing.

So, you can see BC means AA4 minus AA1 minus A2 A3 minus A3 A4. So, AA1 that means this distance is how much? Sorry, not AA1, AA4. AA4 means this total distance which is equal to x times cosine theta as per this diagram the angle DA4 I am writing here angle DA4 is 90 degree and angle DA, yeah DA A4 is theta.

So, I can write in place of AA4, in place of AA4 I can write x cosine theta likewise in place of AA1 what we can write? In place of AA1 we can write Z dashed times tangent of i c because tangent of i c means what AA1 I am just writing here tangent of i c means AA1 divided by A1B, A1B is Z dashed so we can write AA1 is equal to Z dashed times tangent of i c.

Similarly for A2 A3 also we can write A2 A3 is equal to Z dashed times tangent of i c. Next is A3 A4. So, A3 A4 means as per this diagram if I am writing A3 A4 that is equal to what I can write is DA4 times tangent of i c this angle is also i c. Now, what is DA4? For DA4 we can consider the triangle DA A4, if we consider the triangle DA A4 then DA4 is equal to x times sin B sin theta so D4 is x times sin theta times tangent of i c.

Now, we know the expression for AA4 we know the expression for AA1, A2 A3, A3 A4. So, if we will write it, it becomes this, already it is, how it is coming that is written. So, I am just writing the what is t BC. So, in this case t BC is, t BC is x cosine theta minus 2Z dashed tangent of i c so I can write 2Z dashed tangent of i c then minus x sin theta tangent of i c divided by v p 2 because the wave travels with a velocity v p 2 along the path BC plus t CD.

So, t CD here is the path CA3 plus DA3, so CD means CA3 plus DA3 which is already written you can see Z dashed divided by cosine of i c plus DA3 means again we can write it as DA4 times DA4 divided by cosine i c so DA4 means already I think yes we have written it is equal to x sin theta. So, in place of CD we can use this expression.

So, I am writing here Z dashed v p 1 cosine i c plus x sin beta, sorry sin theta times tangent, sorry sin theta divided by cosine i c divided by v p 1, so cosine i c. Now, what I can do here, I can take the term Z dashed together. So, I have one term like this v p 1 cosine i c. So, the two terms which I have already consider are these two terms.

Now, again what is left is minus 2Z dashed tangent of i c divided by v p 2 and whatever is left that I am writing together. So, x cosine theta minus x sin theta times tangent of i c divided by v p 2 plus x sin theta divided by v p 1 cosine of i c. Now, these first two term what we can do for these first two term let us see.

So, here I am writing it as 2Z dashed divided by v p 1 cosine i c minus tangent i c can be written as sin of i c divided by cosine of i c. So, I am writing it test divided by cosine of i c plus the same thing I am doing here also in place of tangent of i c I will write cosine of i c, sorry sin of i c divided by cosine of i c this divided by v p 2 plus x sin theta divided by v p 1 cosine i c.

Now, the first two term can be written as, I can write it as v p 1 v p 2 times cosine i c, so 2Z dashed which is common for first two terms, so I am writing 2Z dashed and then multiplying the remaining components of first term and the second term within bracket as what it will be, it will be then v p 2 minus v p 1 sin i c plus I can write this also as x cosine theta cosine i c minus x sin theta sin i c plus this term as it is.

Now, here what I can do for the, now we have three terms basically, this is one, this is another, this is the third. So, for the first term what I can do, I can take v p 2 out of this bracket.

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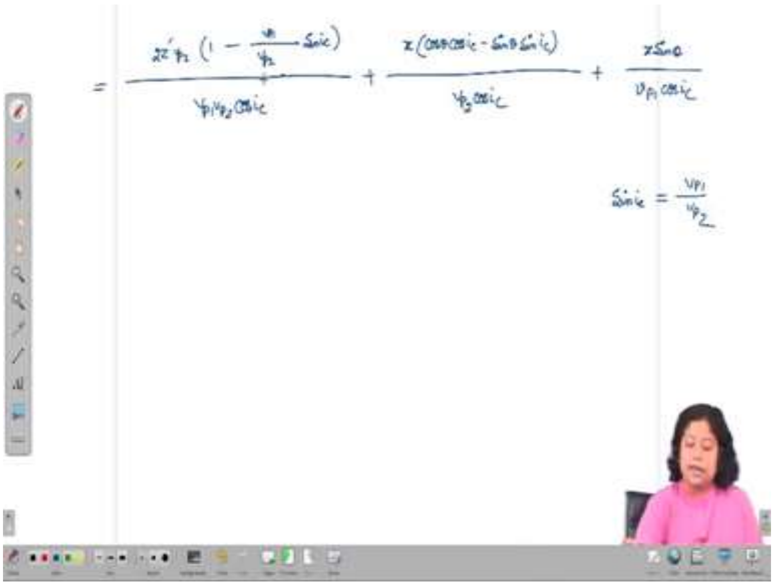
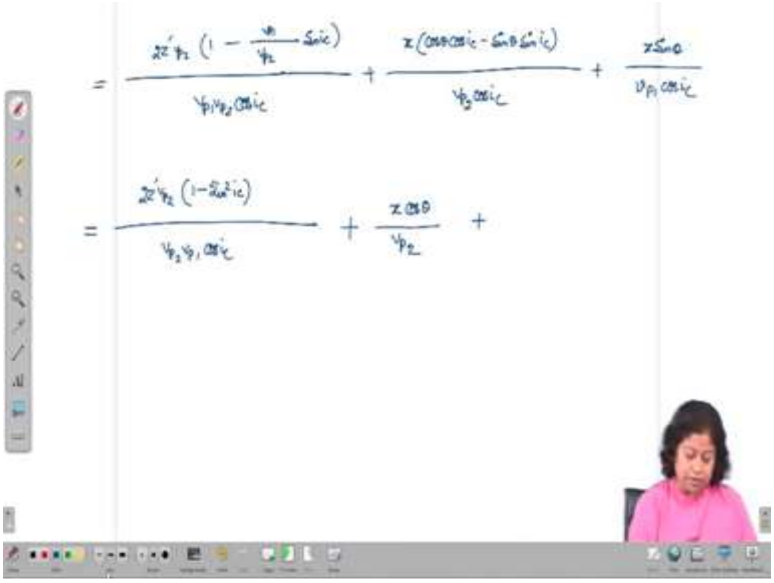
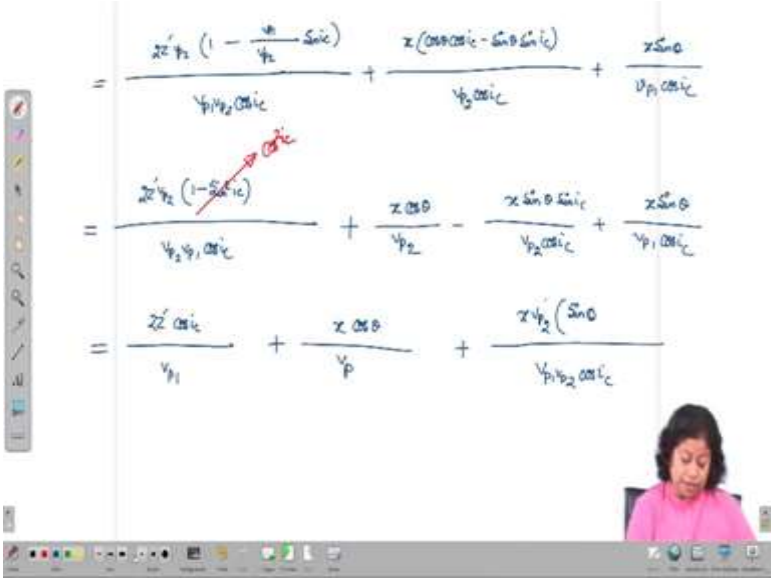
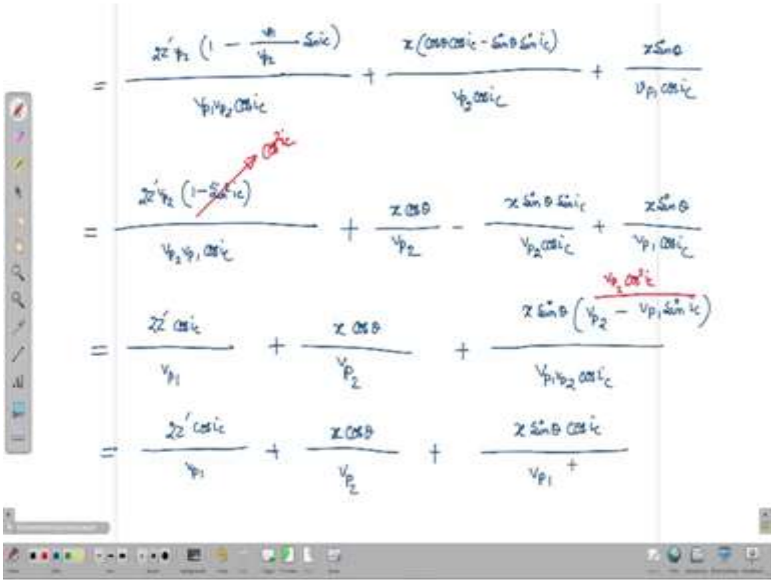
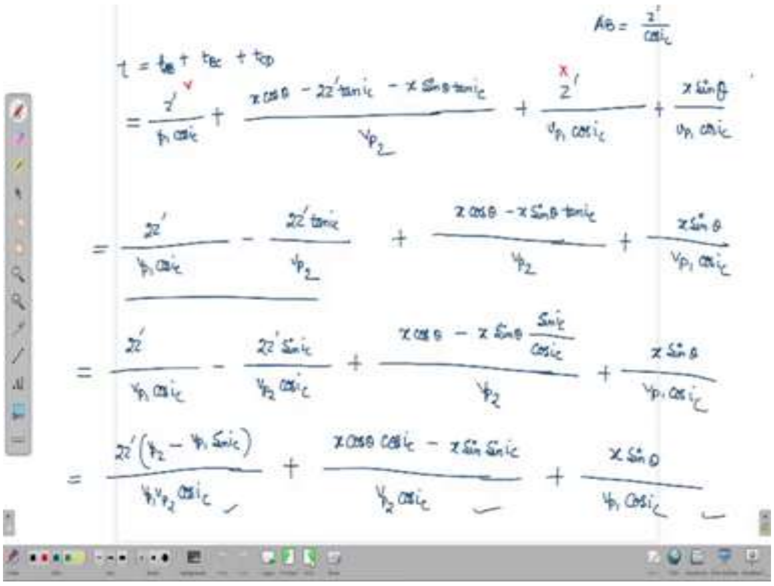



$= \frac{2z' v_{p_2} \left(1 - \frac{v_{p_1}}{v_{p_2}} \sin i_c\right)}{v_{p_1} v_{p_2} \cos i_c} + \frac{x(\cos\theta \cos i_c - \sin\theta \sin i_c)}{v_{p_2} \cos i_c} + \frac{x \sin\theta}{v_{p_1} \cos i_c}$
$= \frac{2z' v_{p_2} (1 - \sin^2 i_c)}{v_{p_2} v_{p_1} \cos i_c} + \frac{x \cos\theta}{v_{p_2}} +$


$= \frac{2z' v_{p_2} \left(1 - \frac{v_{p_1}}{v_{p_2}} \sin i_c\right)}{v_{p_1} v_{p_2} \cos i_c} + \frac{x(\cos\theta \cos i_c - \sin\theta \sin i_c)}{v_{p_2} \cos i_c} + \frac{x \sin\theta}{v_{p_1} \cos i_c}$
$\cos^2 i_c$
$= \frac{2z' v_{p_2} (1 - \sin^2 i_c)}{v_{p_2} v_{p_1} \cos i_c} + \frac{x \cos\theta}{v_{p_2}} - \frac{x \sin\theta \sin i_c}{v_{p_2} \cos i_c} + \frac{x \sin\theta}{v_{p_1} \cos i_c}$
$= \frac{2z' \cos i_c}{v_{p_1}} + \frac{x \cos\theta}{v_p} + \frac{x v_{p_2} (\sin\theta}{v_{p_1} v_{p_2} \cos i_c}$


$= \frac{2z' v_{p_2} \left(1 - \frac{v_{p_1}}{v_{p_2}} \sin i_c\right)}{v_{p_1} v_{p_2} \cos i_c} + \frac{x(\cos\theta \cos i_c - \sin\theta \sin i_c)}{v_{p_2} \cos i_c} + \frac{x \sin\theta}{v_{p_1} \cos i_c}$
$\cos^2 i_c$
$= \frac{2z' v_{p_2} (1 - \sin^2 i_c)}{v_{p_2} v_{p_1} \cos i_c} + \frac{x \cos\theta}{v_{p_2}} - \frac{x \sin\theta \sin i_c}{v_{p_2} \cos i_c} + \frac{x \sin\theta}{v_{p_1} \cos i_c}$
$v_{p_2} \cos^2 i_c$
$= \frac{2z' \cos i_c}{v_{p_1}} + \frac{x \cos\theta}{v_{p_2}} + \frac{x \sin\theta (v_{p_2} - v_{p_1} \sin i_c)}{v_{p_1} v_{p_2} \cos i_c}$
$= \frac{2z' \cos i_c}{v_{p_1}} + \frac{x \cos\theta}{v_{p_2}} + \frac{x \sin\theta \cos i_c}{v_{p_1}} +$



So, I am writing the first term here as 2Z dashed taking out v p 2 so one minus v p 1 divided by v p, sorry 2, v p 2 that is multiplying with sin i c divided by v p 1 v p 2 times cosine of i c, so this is the first term. Now, what about the second term? For second term I can take x out of this bracket.

So, taking x out cosine theta cosine i c minus sin theta sin i c that is divided by v p 2 cosine i c here if you see the second term it is v p 2 cosine i c. So, I am writing here v p 2 cosine i c. And the last term, x sin theta divided by v p 1 cosine i c. Now, come to the first term what is v p 1 divided by v p 2?

If you recall we have written already sin i c is equal to v p 1 divided by v p 2. So, here I can write directly v p 1 by v p 2 in place of v p 1 by v p 2 I am writing sin i c, 2Z dashed v p 2, so sin i c in place of v p 1 by v p 2 and there is another sin i c so it is becoming sin square i c divided by v p 2 v p 1 times cosine of i c.

Now, come to the second term. So, what I can write in place of second term now? So, first term is written like this, now come to the second term. For second term what we can write let us give a try for the second term. So, second term let us write it other way, instead of writing it as these I am just once again writing the second term as x of cosine theta divided by v p 2 plus, not plus if you see there is a negative sign here, so it is minus x sin theta sin i c divided by v p 2 cosine i c plus x sin theta divided by v p 1 cosine i c.

So, from this what I can write, for the first term now it is in what I can write this is nothing but cosine square i c and if you see that denominator we have v p 2 times v p 1 times cosine i

c. So, after simplifying what we can write? We can write then 2Z dashed times cosine i c divided by v p 1.

Second term I am writing as it is now third and fourth term I am trying to simplify here as v p 1 v p 2 times cosine i c, so it is now x if I will take out v p 2 also I can take out so sin theta or I can write sin theta out actually in instead of writing this way, I can write it x sin theta taking out so what is left here is v p 2 minus v p 1 sin i c already we have seen that these can be written as I can just write for you 2Z dashed cosine i c divided by v p 1 plus x cosine theta divided by v p 2 plus what I can write in whatever written within bracket.

In place of that I can write v p 2 times 1 minus sin square i c which is cosine square i c, so if it is like that then we can write x sin theta times cosine i c divided by v p 1 that is something is left or missing, no, it is all right. So, now in the next line what I will do I will take second and third term together and will keep first term as it is.

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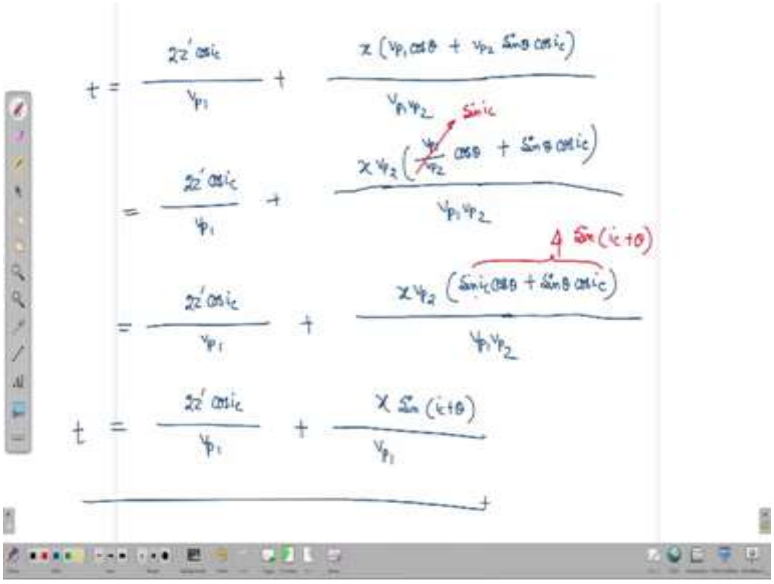

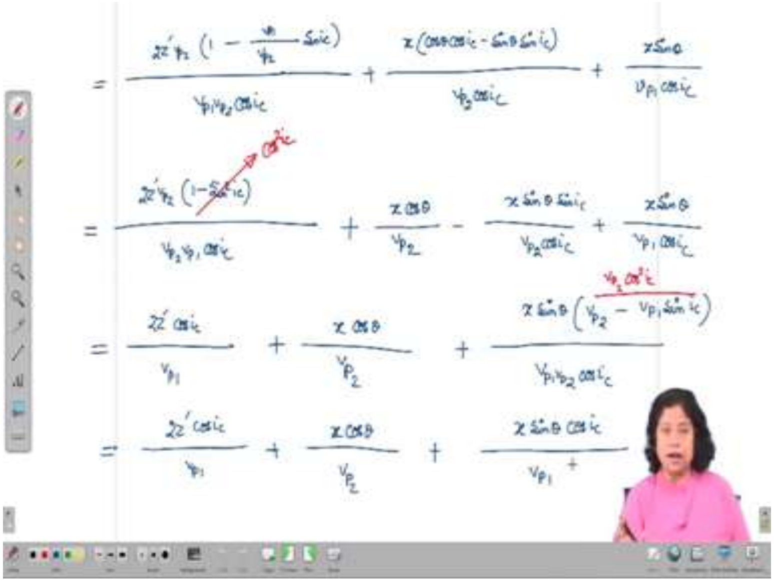

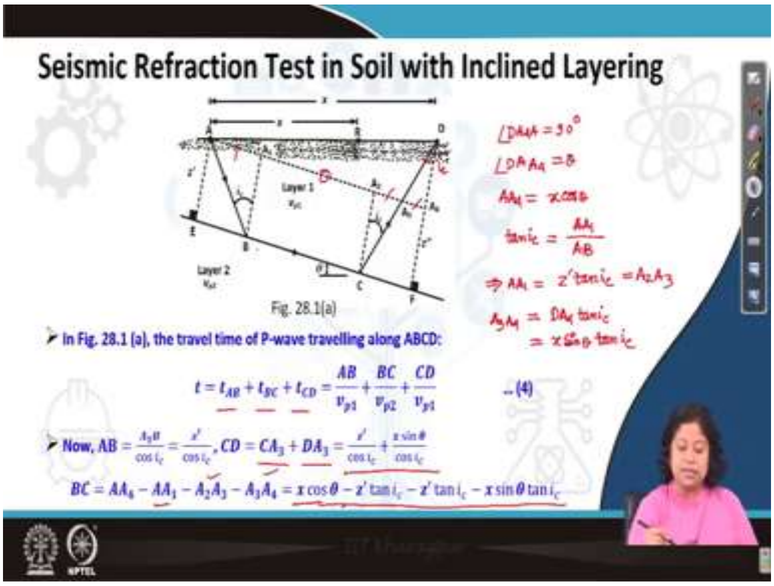

So, then travel time is now 2Z dashed cosine i cm 2Z dashed cosine i c divided by v p 1 plus here what I will do I will now add second and third term together. Then we are getting x I will take out, so v p, previous page, it is v p 1 cosine theta, v p 1 cosine theta then plus v p 2 sin theta cosine i c.

So, we can write now the new second term first term as it is, the new second term here we can take, we need to take one term out, so v p 2 I can take out then I can write it as x v p 2 times v p 1 divided by v p 2 times cosine theta plus sin theta cosine i c, this is divided by v p 1 v p 2.

Now in place of v p 1 v p 2 what I can write, I can write it as once again sin i c, so in this way the second term can be written here as x times v p 2 times sin i c cosine theta plus sin theta cosine i c and that is divided by v p 1 times v p 2. Now, here you can see in the numerator and denominator we can get v p 2 and v p 2 is a non zero number.

So, we can simplify here, the second term and finally we get x divided by v p 1 times whatever written in the bracket, that means x sorry sin i c times cosine theta plus sin theta times cosine i c. And what it is exactly? Whatever written within bracket in place of that what we can write is sin of i c plus theta. So, here I will write then i c plus theta. So, what I have shown here is that the total travel time for the path A B C D, A B C D is this one. So, now go back to the PPT and here you can see the total travel time in the next slide.

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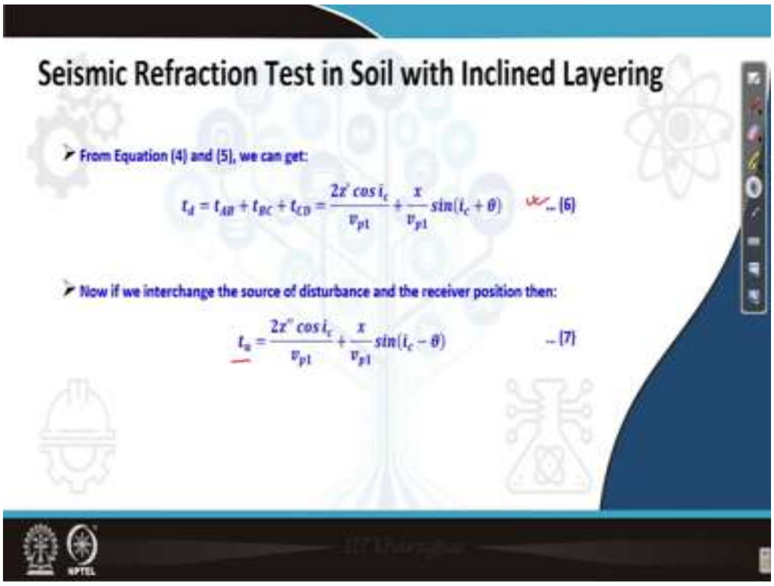

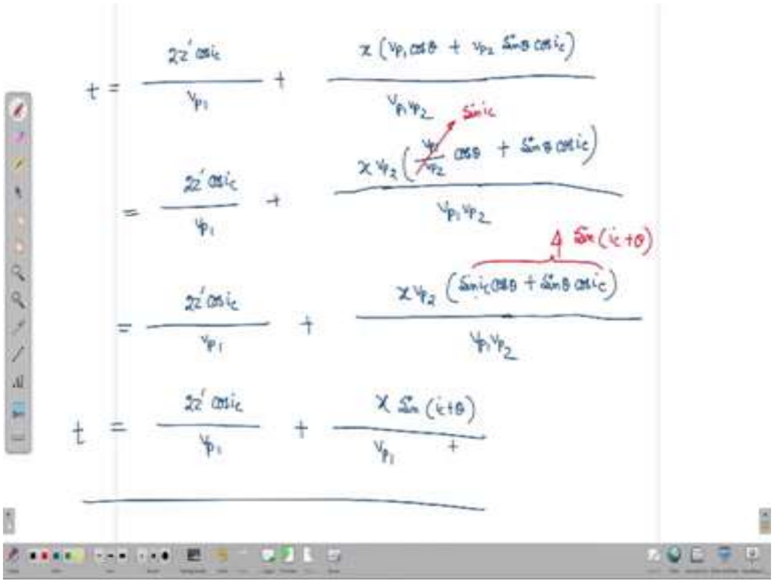

Yeah, so this thing if you see we have already expressed, just let me check, yeah, same thing we have already derived. Now, if we will interchange the position of the source and the receiver then the travel time will become t u which is equal to 2Z double dot times cosine i c divided by v p 1 plus x divided by v p 1 times sin of i c minus theta.

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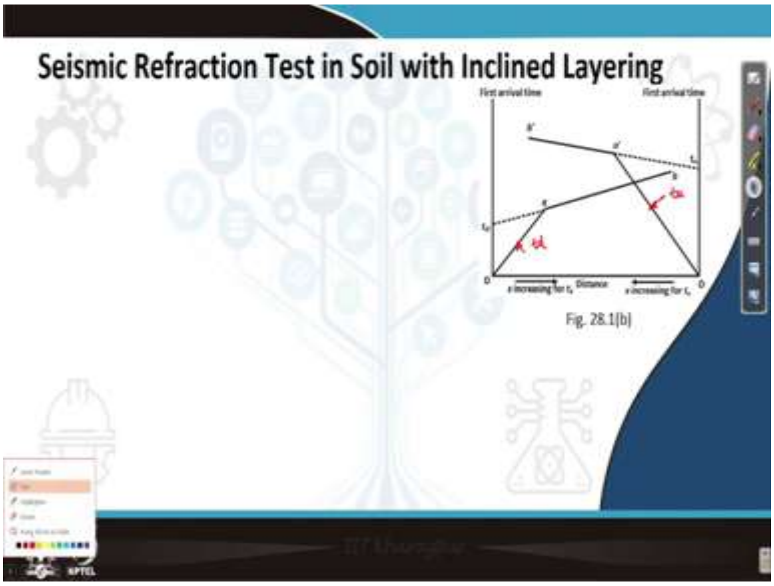

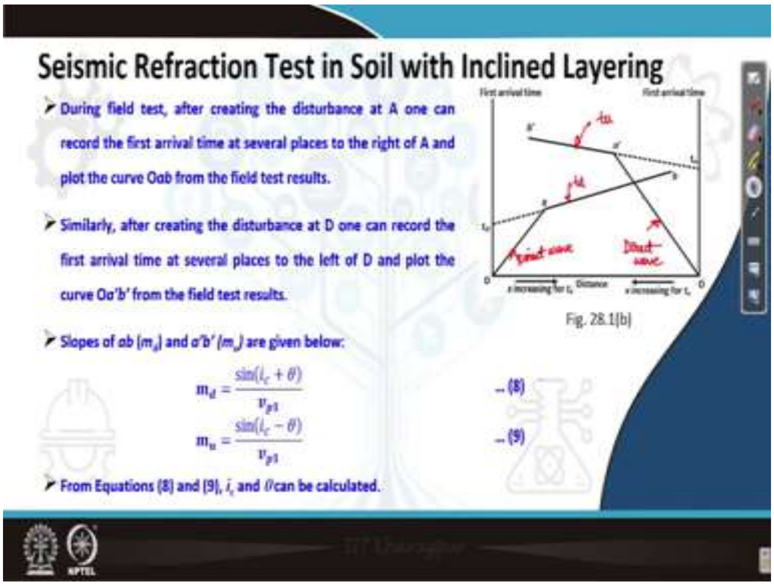

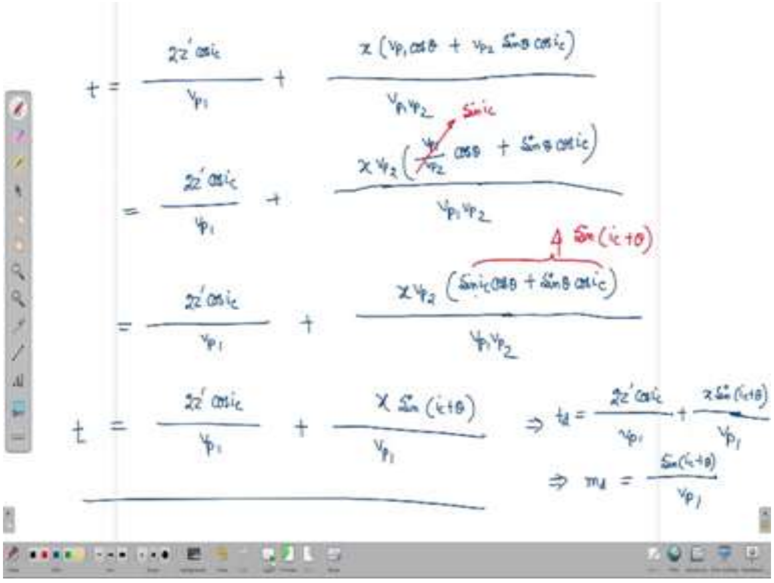

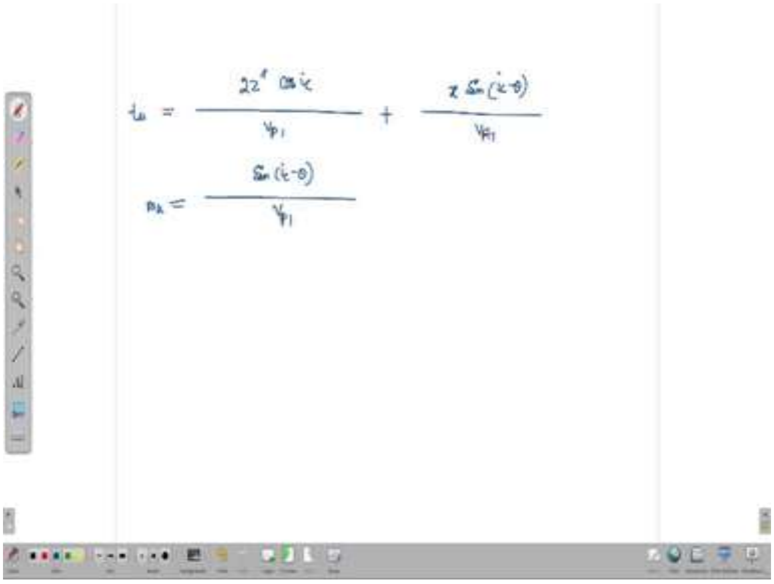

So, that means now if I will write the two travel times that means this is t d and this one t u. Of course we need to take care of one thing here which is better I should not show t d and t u like this way better I show this is t d and this is t u, the expression of t d and t u I have already shown.

And o a and o a dashed represent the equation for the direct, for the travel time of the direct wave not the refracted one. So, this is for direct wave, same thing here also this is for direct wave. So, during field test, after creating the disturbance at A one can record the arrival time T at several places from the right of A, from the A towards the right of A and plot the curve Oab like this.

Similarly, after creating the disturbance at D that means we are interchanging now the position of the receiver and the position of the source and the receiver and recording the first arrival time, one can record the first arrival time at several places to the left of D and plot the curve Oa dashed b dashed as shown this one.

Then the slope m d that means the slope of the line ab and the slope of the line a dash b dashed which is m u can be written as what is md if you see the equation from this is for the t d I can write it also the same for t d I am just writing plus x times sin of i c plus theta divided by v p 1 so when it is for, then what is m d here? m d is nothing but sin of i c plus theta divided by v p 1.

Likewise if you see the equation for t u it says that 2Z double dashed cosine i c divided by v p 1 plus x times sin i c minus theta divided by v p 1. And in that case m u is slope of this straight line which is sin of i c minus theta divided by v p 1. So, with these we can write then here m d is equal to sin of i c plus theta divided by v p 1 and in place of m u we can write sin of i c minus theta divided by v p 1. So, from equation 8 and 9 we can calculate we can determine i c which is the inclination of the boundary of the two layers and i c is the incident angle of the wave, P wave at the boundary and theta is the inclination of the boundary between two layers so that these two parameters can be determined by solving equation 8 and 9.

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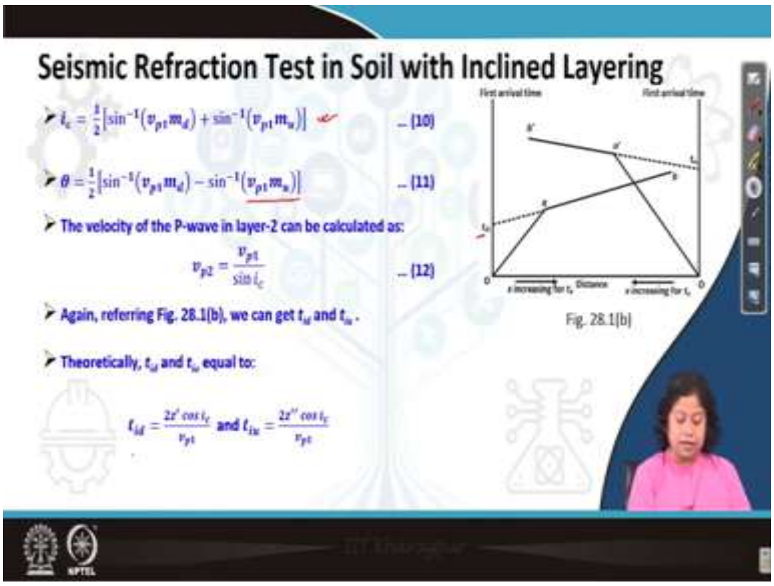

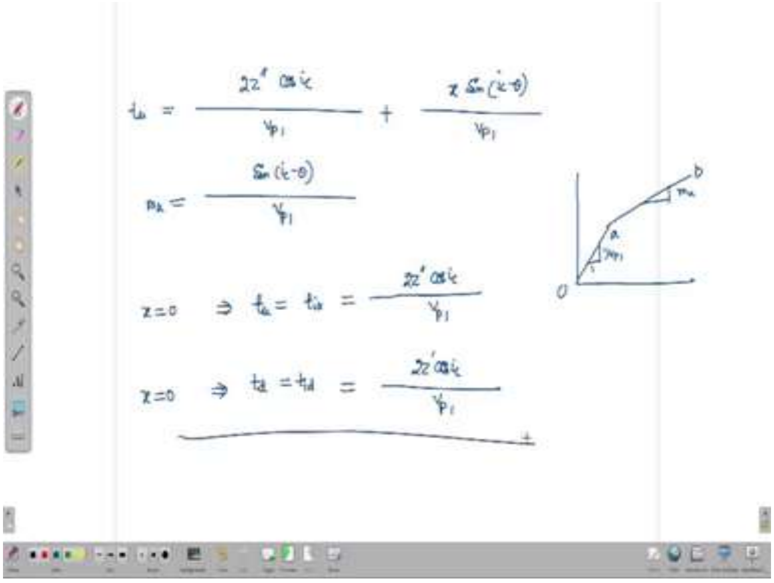

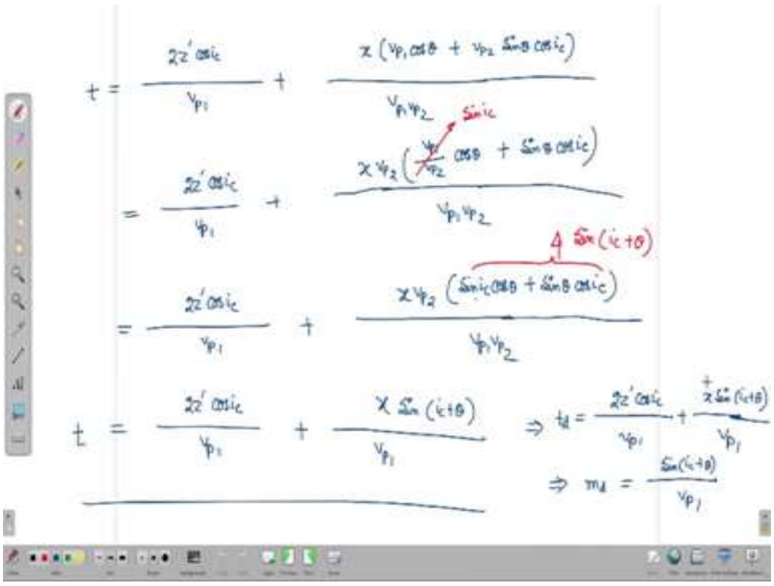

So, if you solve you will get i c is equal to half of sin inverse, sin inverse of md times v p 1 plus sin inverse of m u times v p 1. And theta is equal to half of sin inverse of m d times v p 1 minus sin inverse of m d, m u times v p 1. So, from this, now what we know? We know three things, first we know v p 1 which is the velocity of the P wave in layer one, we know i c, we know theta.

So, I am just once again drawing this curve this is for o a, this is a b, so o a, a b, so this slope of o a is nothing but v p 1 and the slope of this a b is m u. From these we can now calculate what is the value of v p 2. So, for that we if, when we know v p 1 we can directly use Snell's law and find out v p 2 which is equal to v p 1 divided by sin of i c already we have determined i c from equation 10.

Again if you see this figure 28.1 b what we can see if we extend back the curve a b then it intersect the vertical axis at some point which is t i d. Similarly, if we extend back the curve b dash a dash that will intersect the time axis at t i u, that means for at t i d x is equal to 0 for the wave following the path A B C D and for t i u x is equal to 0 for the wave traveling the path D C B A.

Then what is the value of t i d? So, in this equation if we put x is equal to 0 then we get t u is equal to t i u which is 2Z double dash times cosine i c divided by v p 1 likewise for the first equation if I will write x is equal to 0 then I will get t d is equal to t i d which is equal to 2Z dash times cosine i c divided by v p 1. I am just going back to the previous, yes. Here you can see if I will write x is equal to 0 here then it will be this expression. So, that is written at the end of this slide.

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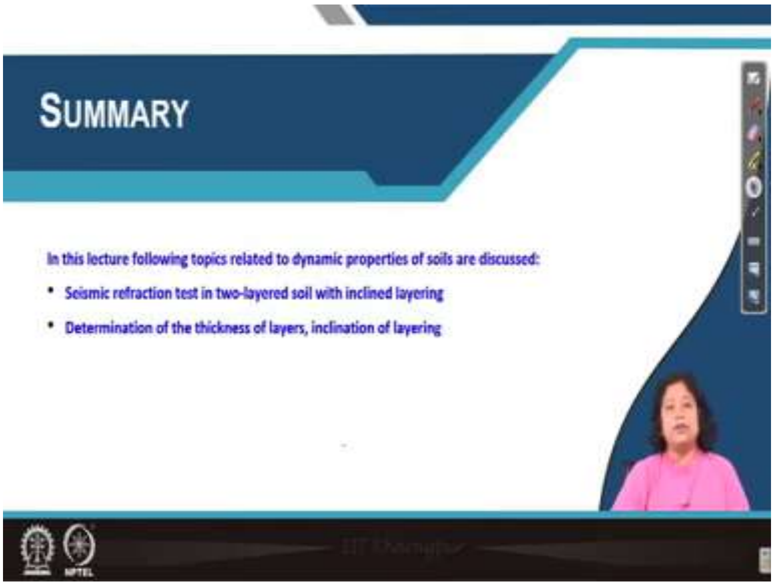

So, come to the summary of today's class. Today we have discussed the refraction, seismic refraction test in two layered soil considering inclined layering and here we have studied how to find out the velocity of P wave in layer 1, in layer 2, how to find out the thickness of the layers and how to find out the inclination of the boundary between two layers.

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These are the references which I have used for today's class. So, thank you.