Soil Dynamics
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Lecture - 3
Single Degree of Freedom System (SDOF) - Part 1

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Hello friends. So, today's class we will study single degree of freedom system. Already we have studied the fundamental elements of theory of vibrations, using that knowledge and some other concept of soil mechanics and soil dynamics we will developed our understanding on single degree of freedom system.

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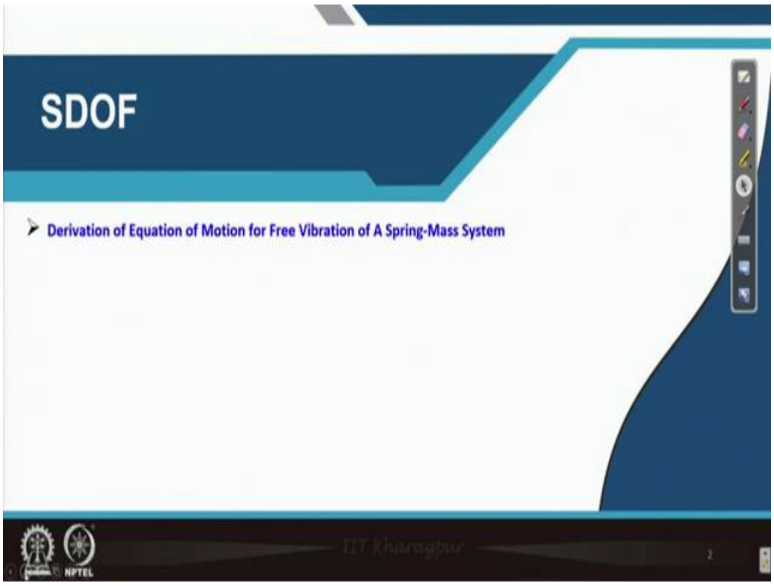

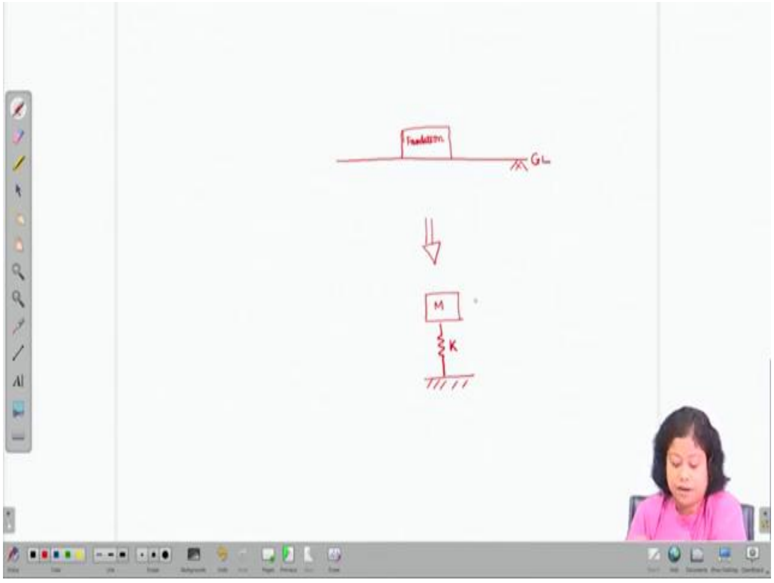

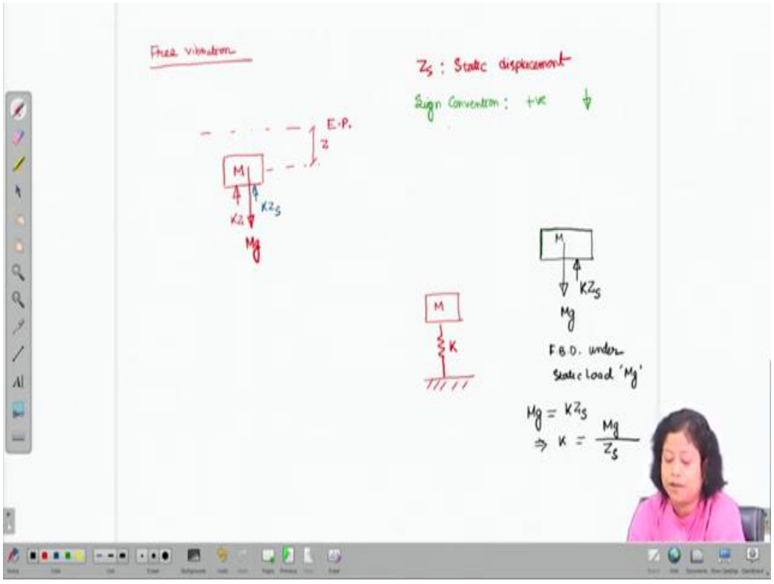

So, first case which we will study today is the derivation of equation of motion for free vibration of a spring mass system. The question why I am taking spring mass system here? The reason is that let us go to whiteboard. In our geotechnical engineering problem what we see, generally we have block foundation, we construct block foundation for the machine.

Generally the foundation is embedded at some depth but for the simplicity right now I am taking, I am taking it on the ground surface. So, this is the foundation, this is our ground level. Now, we can model the soil by spring and dashboard, when we are considering dashboard that means we are considering the damping effect of the soil, we will discuss damping effect in detail later.

For today's lecture we will consider only the spring. So, soil will be represented by a spring which is massless and the foundation will be represented by a mass of m. So, let us draw the thing. I can represent this as mass, this is the mass of the foundation, if there is machine then we sometimes take if the machine of the, if the weight of the machine is not too large then we can include the weight of the machine also here.

Now, this is resting on the soil K is the stiffness of this soil. So, this is the diagram which we are currently choosing for the problem. So, we have a mass which is resting on the spring. Now, we are interested to write the equation of motion and then the solution for, solution means the response of this mass.

We are considering the, for this analysis we are considering it is free vibration, that means free vibration means the system is vibrating due to the inherent forces within the system and no external forces is present at this term. So, if we will draw the free body diagram of this mass how does it look?

So, let us take the mass M, so this is our mass M. What are the forces acting on this mass M? Now, first of all we need to show the equilibrium position of this mass. That means when there is, there was no vibration that time what is the equilibrium position. So, let us take this is the equilibrium position. Now, the mass is lower the, at some depth, so Z is the displacement.

Now, what are the forces acting on this mass M? Since it is connected to the spring K when the mass M will push the spring K what will be happen? The spring will also exert same amount of the force to the mass. So, how much will be the magnitude of that force? Mass will exert the force K Z to the spring so as per Newton's third law the spring will also exert the force K Z it to the mass.

What are the other forces? Because of the action of gravity there should be self-weight which is capital M times g. And because of this self-weight there should be some static displacement of the system which is let us take Z S, so I am writing Z S means static displacement.

Then the, because of static displacement what will be happened? Spring will be pushed by the mass because of the force M g and how much is that force which mass M will exert to the spring, that is K times Z S, so the equal and opposite force will be exerted by the spring during static displacement. So, that force is K times Z S.

Now, we need to consider also the sign convention. So, I am writing here sign convention. We will consider downward forces are positive. Now, before analysing the vibrating system first I would like to discuss something about this mass which is because of the load M g. So, we have a mass M subjected to the weight M g, because of which the spring, because of which there will be a static displacement Z S and as a consequence the spring K will exert a force K times Z S to the mass M.

So, this is the free body diagram under static load which is M times g. Now, in this case under equilibrium what we can write? We can write M g is equal to K times of Z S or we can also write K is equals to M g divided by Z S. So, from static equilibrium we can see that M g is equal to K times Z S.

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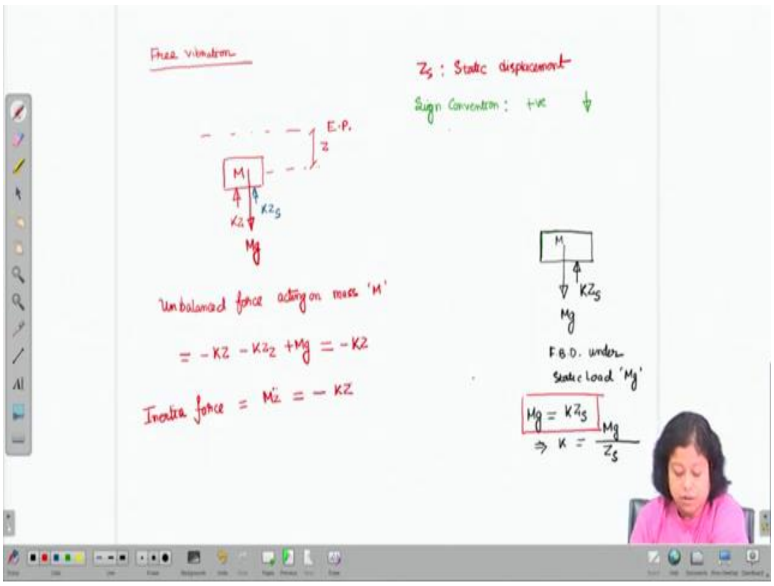

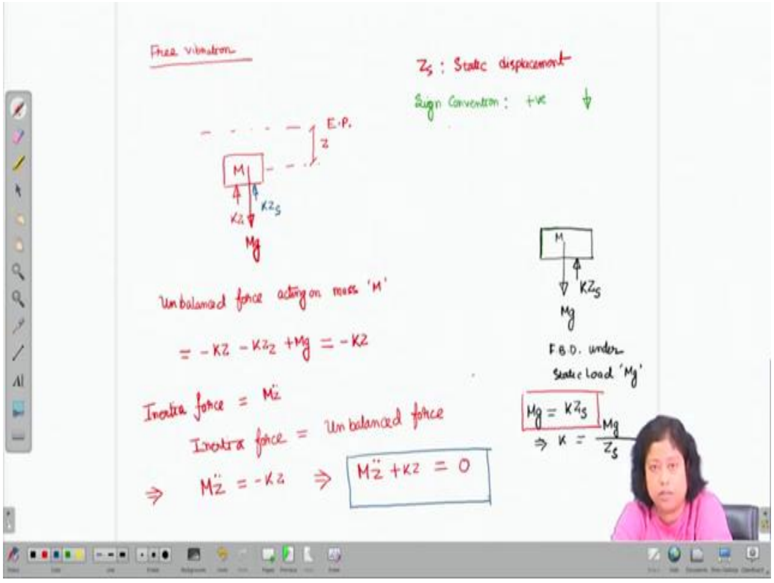

Now, what about the dynamic equilibrium equation? So, here first we need to write the unbalanced force. So, unbalanced force acting on mass M is how much? Minus K Z minus K times Z S plus M g, there is no external force because we are considering free vibration. So, finally the unbalanced force is minus K times Z because we have already seen M g is equal to K times of Z S from the static equilibrium equation for this mass.

So, now these unbalanced force should be equal to the inertia force. Now, how much is the inertia force in this case? Inertia force we know is equal to mass times acceleration. So, in this case mass time acceleration will be M Z 2 dot, so M Z 2 dot is equal to minus K Z. I can write this in next line that will be better.

So, the inertia force is equal to the unbalanced force. So, we can write M Z 2 dot is equal to minus K Z or we can write M Z 2 dot plus K Z is equal to 0. So, this is our equation of motion under free vibration. I hope till this the thing is clear to all of us. Now, we will find out the general solution for this equation of motion under free vibration.

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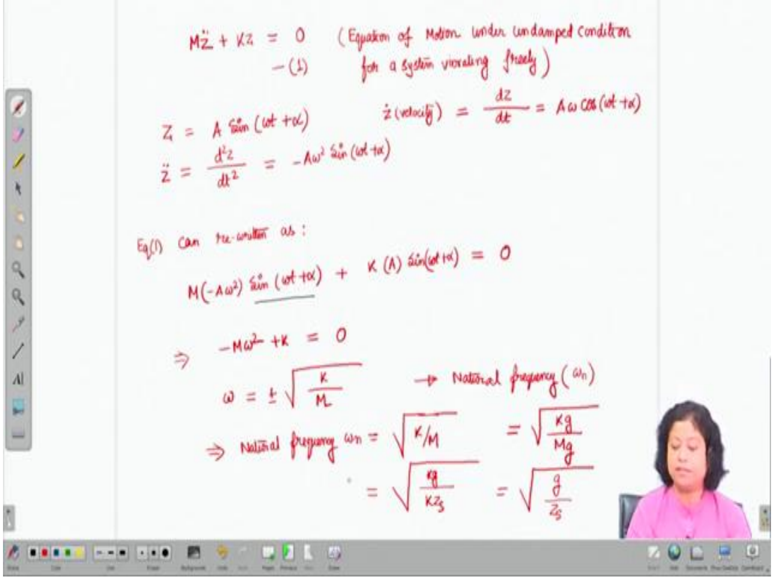

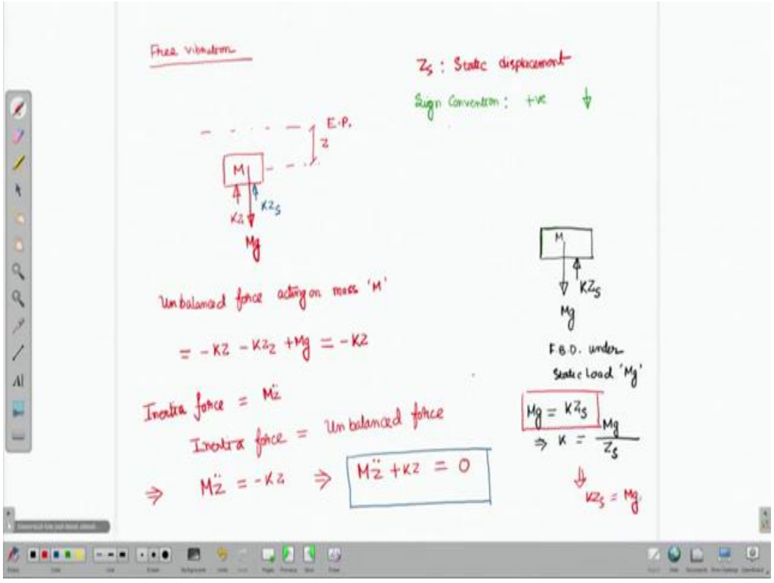

So, go to the next page I am once again writing the equation of motion M Z 2 dot that means mass times acceleration plus K Z that means spring force is equal to Z, this is our equation of motion. Under which condition? Under undamped condition for a system vibrating freely. Now, we are interested to find out the solution for this system.

So, for this type of problem, this is a homogeneous equation we know that, so we can take a solution, trial solution. In this case I am taking Z is equal to A times sin of omega t plus alpha. So, what will be then the Z 2 dot which is the second derivative of Z with respect to the time?

It will be minus A omega square times sin of omega t plus alpha. Actually it is good if I first calculate Z dot which is the velocity which is nothing but the first derivative of Z with respect to time and that is A times omega times cosine of omega t plus alpha. And from this we can calculate acceleration which is Z 2 dot.

Now, with this what we can do? We can use Z, the value of Z and the value of Z 2 dot to our governing equation of motion, let us give a number to this equation as equation 1. So, now we can rewrite equation 1, I can write it as, it can be rewritten as. What we will write? M times minus A omega square times sin of omega t plus alpha plus K times A times sin omega t plus alpha and that is equal to 0.

So, since this cannot be 0, there is alpha. So, what we can write, minus M omega square plus K is equal to 0. So, from this we can see that omega is equal to plus minus square root of K by M. So, after this, next step is to write the general solution for this problem but I would like to mention here that the omega which is the frequency of this, of the response of this system is called as the natural frequency and symbolically we can represent natural frequency by omega n.

So, I can write next step that natural frequency omega n is equal to square root of K by m, sorry this one is capital M. Now, once again I am repeating K is the stiffness of the soil which is represented by a spring, M is the mass of the foundation which is resting on the ground surface for this case.

Now, if we will do some more simplification we will see we can avoid the term K and M in this expression, how, let us see. So, what I will do here, I will multiply numerator and denominator by g. So, what is g here we all know, I am not repeating. Now, we have already seen in the previous slide that you can see here K is equal to M g divided by Z S.

So, now in the, in this expression that means I can or I am just going back once again the previous slide. What I can write in place of M g, I can write here in place of M g as K Z S equal to M g. So, let us do this task in place of M g I will write K Z S. So, K g divided by K Z S. So, what I can write then?

Omega n which is the natural frequency is equal to g divided by Z S under square, so square root of sg divided by Z S can give us the estimate of the natural frequency of a system under free vibration.

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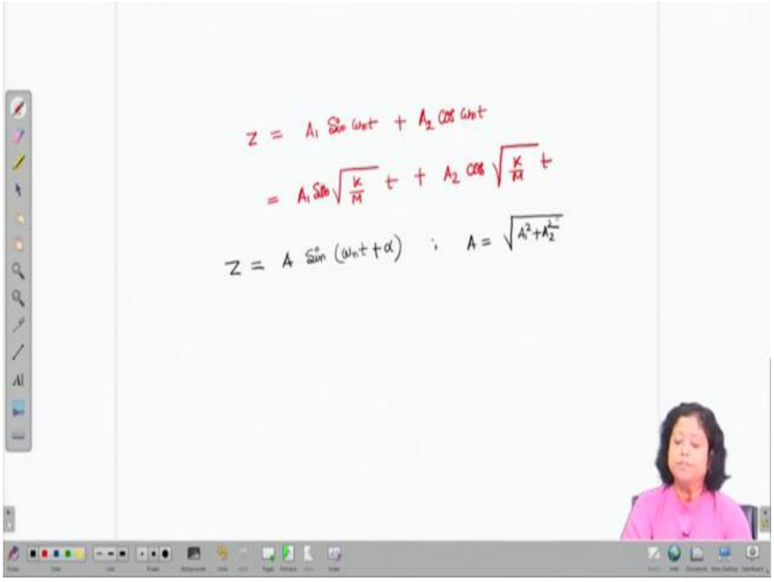

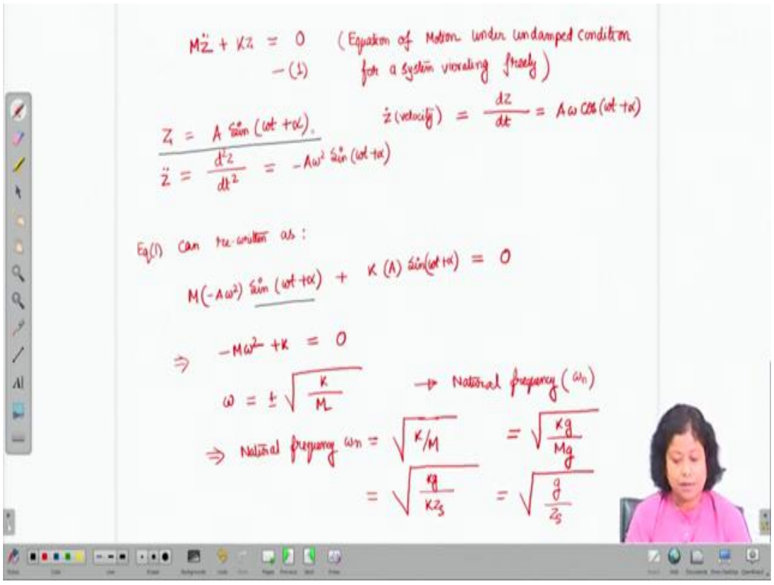

So, now the general equation, so the general equation if I go back to the previous slide not the general equation I am talking about the general solution, that means this one, so we can rewrite also it as another form like Z is equal to A1 sin omega n t plus A2 cosine omega n t or I can write it as A1 times sin of square root of K by M t plus A2 times cosine of square root of K by M times t.

So, this is the general equation A1 and A2 can be calculated from the given boundary conditions. Now, what is A1 and A2 when we compare this general solution to the previous form, I am just writing here the previous form. So, previous form was Z is equal to A times sin of omega t in this case I am writing omega n t plus theta, that was the, sorry alpha it was.

So, now if I will compare these two, I can say what is A and what is rest of the terms. So, I am just writing here A. So, in this way we can calculate all other terms.

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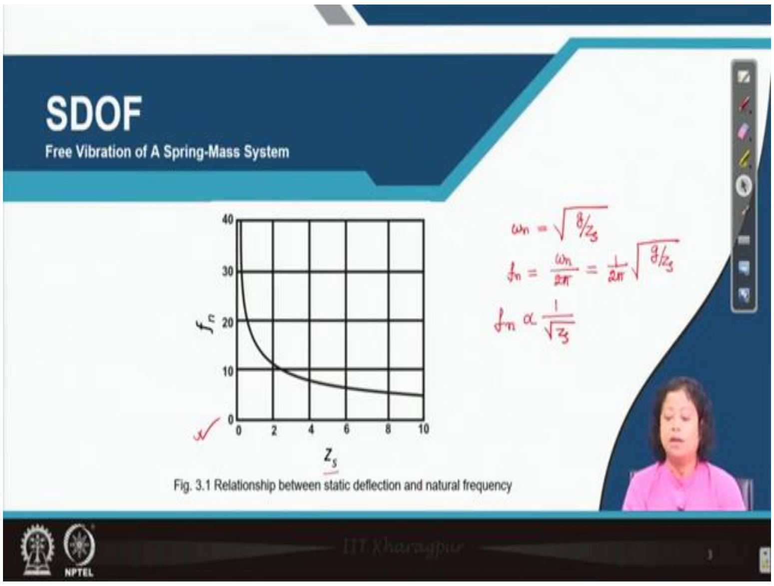

Now, we can go back to the PowerPoint. Here what we can see then? Already we have derived the relationship for omega n which says that omega n is equal to square root of g by Z S. That means if I will write fn that means frequency in hertz what I can write here, omega n divided by 2 pi which is 1 divided by 2 pi g divided by Z S.

So, basically f n is proportional to square root of or 1 divided by square root of Z S. So, the same thing you can see in this figure when n is, when fn, sorry when Z is which is the static displacement is low that time we can expect very high natural frequency, likewise when the Z S value that means static displacement is very large that time we, for that type of material we can expect very low natural frequency.

So, basically Z S gives us some idea about the material from which we can say whether the natural frequency of that material or system will be very high or very low.

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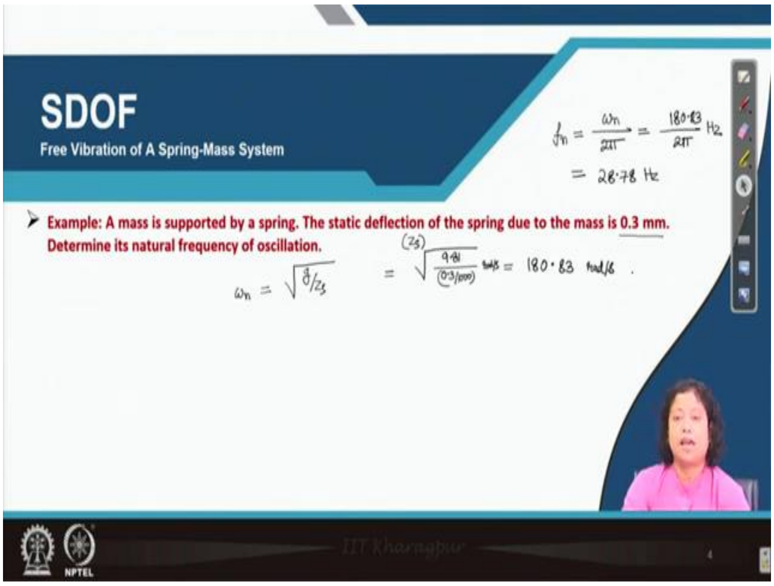

So, with this we can take one simple example of free vibration system. So, in this problem what is given that let me read. A mass is supported by a spring, the same thing which we have just discussed. In this case the static deflection that means Z S is given which is 0.3 in millimetre. You are asked to determine the natural frequency of the oscillation.

So, already we have calculated how we, already we have seen how we can calculate the natural frequency. So, the natural frequency, that means circular frequency is nothing but square root of g divided by Z S. So, I can calculate in this case, first omega n which is g means in this case 9.81 meter per second square and Z S is 0.3 millimeter, that means we can write it as 0.3 divided by 1000 to convert it in meter.

Now, what we will get that will be in radian per second. So, what we will get in this case let us check. We have 9.81 divided by 0.3 divided by 1000 or I can take this they are also so and we need to calculate square root of this quantity. So, how much it is coming let me check, 9.81, yes it is coming 180.83 in radian per second. So, what we have calculated is circular frequency.

Now, if I am interested for frequency in hertz what we will do, we will divide circular frequency divided by 2 pi. So, 180.83 divided by 2 pi that will be in hertz. So, I think it will be let me check it is coming 28.78 in hertz. So, with this I am concluding today's class. So, today what we have learned? We have learned what will be the equation of motion for a single degree of freedom system under free vibration and the system is also undamped, so no damping component is present in our analysis.

Then we have studied how, what will be the natural frequency of the same system and how we can calculate it if we know the static displacement of the system under static loading. So, then we, with this knowledge we have solved a numerical problem, we will continue this topic in our next lecture. Thank you, have a nice day.