Soil Dynamics
Professor Paramita Bhattacharya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur
Lecture no. 37
Analysis of Machine Foundations
(Elastic Half Space Method – Part 1)

Hello everyone, in last class, we have discussed about the design criteria of machine foundation. Today, we will study the analysis of machine foundation using elastic half space method.

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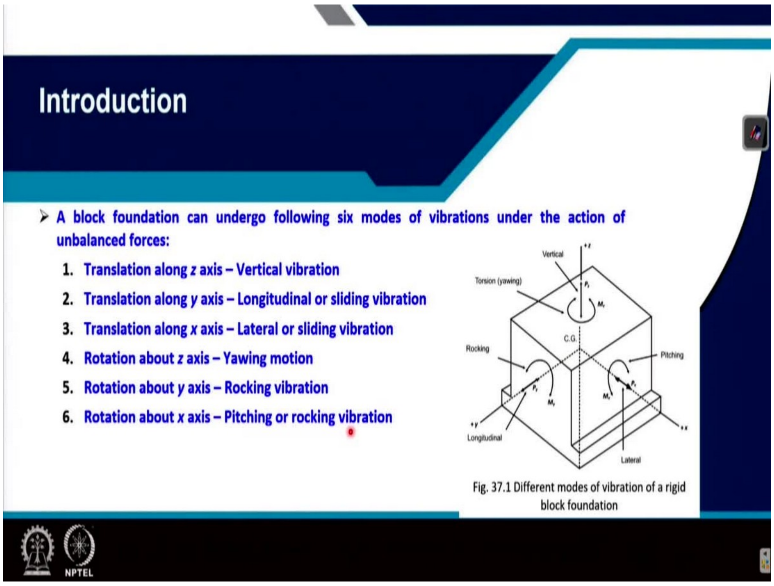

So, in this diagram that is figure 1 what we can see, here we can see the different modes of vibration of a rigid block foundation. So, there are 6 modes of vibration under the action of unbalanced forces, what are those 6 modes of vibration first one is translation along z axis alright? So, this is the first mode of vibration, second mode of vibration sorry second mode of vibration is translation along x axis and third is translation along y axis or we can take said y and x in this order as well.

So, for first mode of vibration which is translation along Z axis, we can measure the vertical vibration. Likewise, when we are focusing on translation along y axis, we are getting longitudinal or sliding vibration for x axis we are getting lateral or sliding vibration as well. Other than these 3, there are 3 other modes of vibrations which are rotation about Z axis, which is called as yawing motion.

Second is rotation about y axis, which is called rocking vibration and the third one is rotation about x axis which is called pitching or rocking vibration as well.

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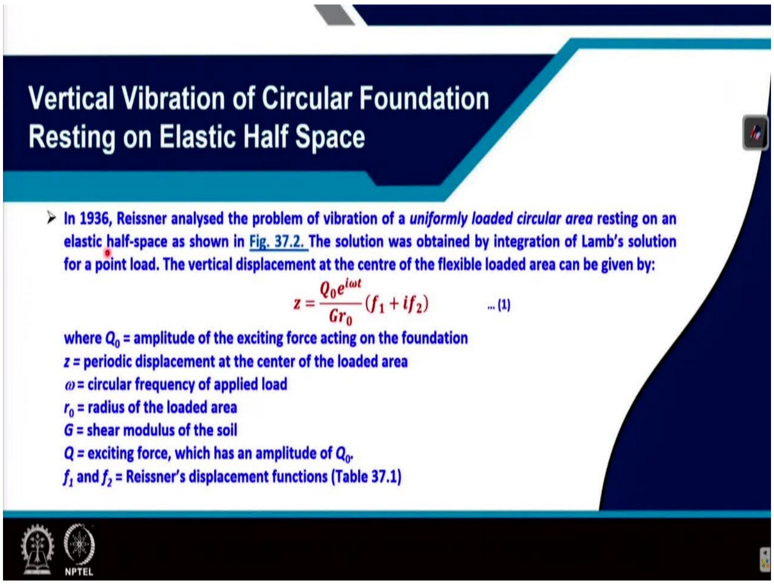

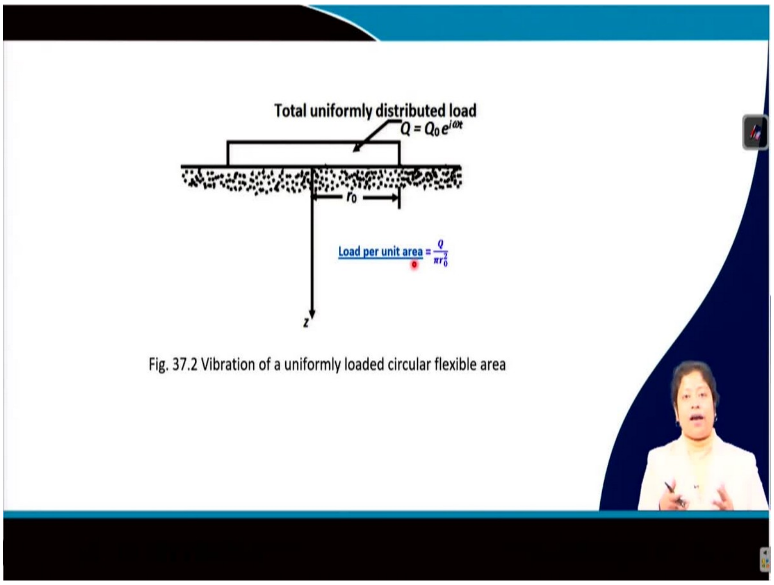

Now, let us see how we can get the, how we can do that analysis of a circular foundation resting on elastic half space and subjected to vertical vibration. So, in 1936 Reisner analysed this type of problem of vibration where uniformly loaded circular area was considered which is resting on an elastic half space.

Let us see the figure for this here you can see the figure for the problem. So, this is the foundation which is circular in shape and its radius is r 0. So, if the uniformly distributed load is Q, which can be expressed by Q 0 times e to the power i omega t where Q 0 is the amplitude of vibration or amplitude of loading omega is the operating frequency of the vibration or if machine is placed in that case we can say operating frequency of the machine then what will be the load per unit area? That is q divided by the area on which the load is acting which is circular area. So, area will be pi r 0 square in this case alright?

Now, what will be the vertical displacement? So Reissner said that the vertical displacement at the center of the flexible loaded area can be given by the equation 1, you can see equation 1 which says that Z is equal to the load which is Q 0 times e to the power i omega t divided by G times r 0 this entire thing is multiplying with f1 plus i times f2 What is G here? G is dynamic shear modulus of the soil.

And if you see omega as I said this is the circular frequency of the applied load, z is periodic displacement at the center of the circular loaded area. Other than that, there are 2 new parameters f1 and f2 which are called as Reissner displacement function, you can see table 37.1.

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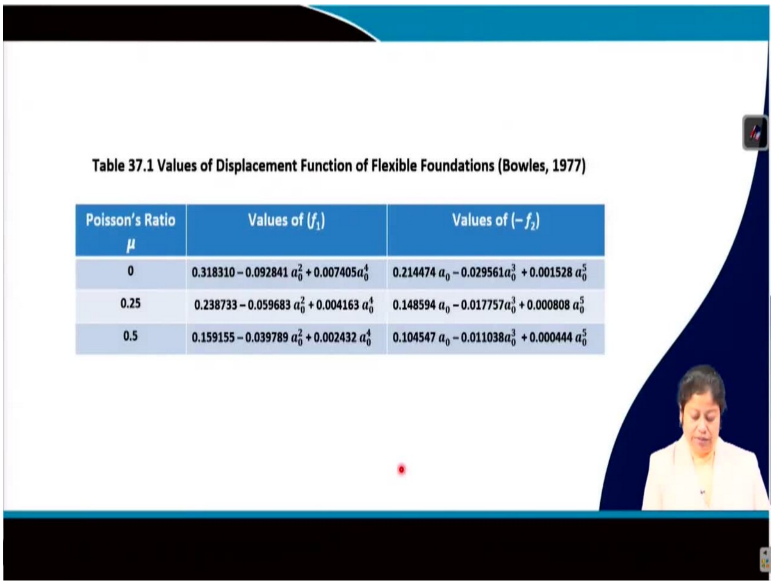

Poisson's Ratio μ	Values of (f_1)	Values of $(-f_2)$
0	$0.318310 - 0.092841\, a_0^2 + 0.007405 a_0^4$	$0.214474\, a_0 - 0.029561 a_0^3 + 0.001528\, a_0^5$
0.25	$0.238733 - 0.059683\, a_0^2 + 0.004163\, a_0^4$	$0.148594\, a_0 - 0.017757 a_0^3 + 0.000808\, a_0^5$
0.5	$0.159155 - 0.039789\, a_0^2 + 0.002432\, a_0^4$	$0.104547\, a_0 - 0.011038 a_0^3 + 0.000444\, a_0^5$

Here is table 37.1 where the values of displacement functions for flexible foundations are provided what we can see in this table here, it depends f1 and f2 depends upon the Poisson's ratio of the soil alright.

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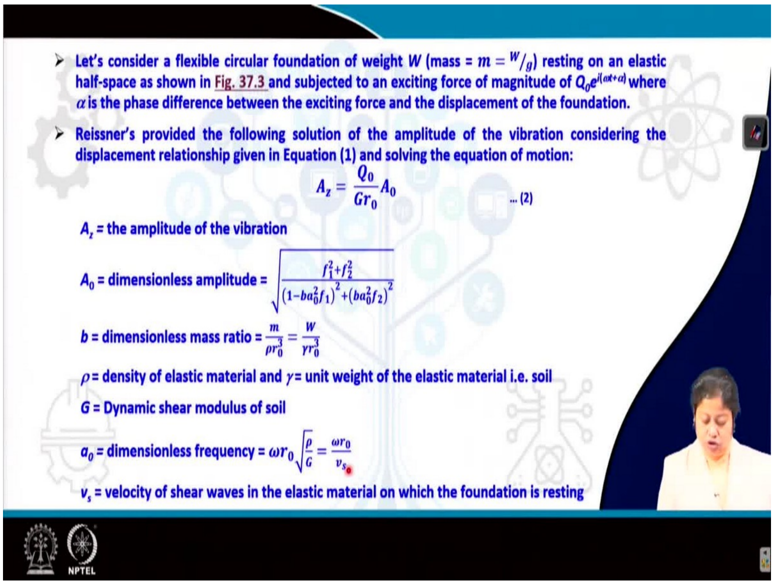

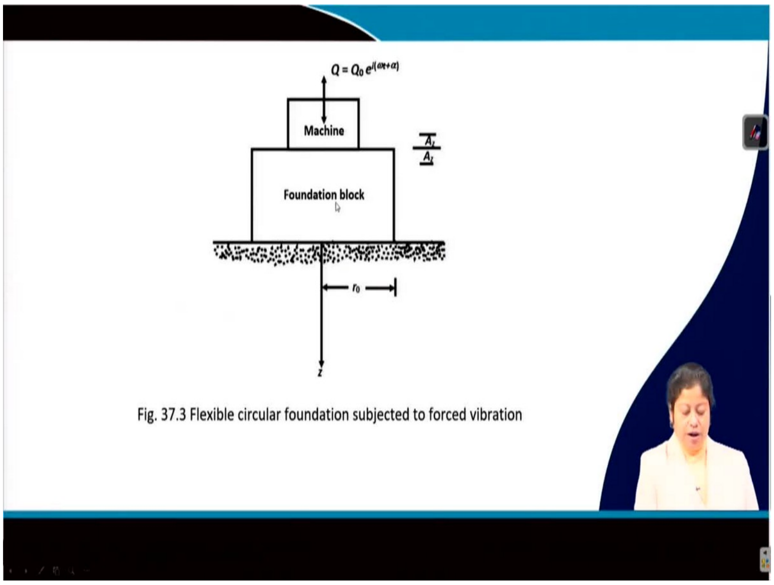

Now, let us consider a flexible circular foundation having weight capital W resting on an elastic half space we can show the figure here. So, in this figure what we can see? There is a circular foundation block on which machine is resting. Other than that, what we get machine is operating at a frequency, circular frequency omega right and what about the dimension of the circular foundation? It has radius r 0.

So, with this Reissner provided the solution for the amplitude of vibration considering that displacement relationship which we have seen in that previous equation 1. So, he said that A z is equal to q 0 divided by G times r 0 whole thing is multiplied by A 0. So, let us see what is A 0 and A z? A z is the amplitude of vibration, whereas, A 0 is an dimensionless amplitude which can be expressed by the equation which is shown here.

So, here we can see that A 0 which is a dimensionless amplitude depends upon f1 and f2 also it depends upon b and a 0. Now, the question then what is b and what is a 0? So, b is dimensionless mass ratio, which can be determined by dividing the mass of the foundation to the rho times r 0 cube, what is r 0? That is known to us it is the radius of the circular loaded area or circular foundation.

Rho and G r density and the dynamic shear modulus of the soil. Another new parameter is a 0 which is called as dimensionless frequency that can be expressed by or determined by omega r 0 times square root of rho by G. We already know that square root of G by rho gives us the velocity of the shear wave. So, we can write a 0 as omega r 0 divided by v s as shown here.

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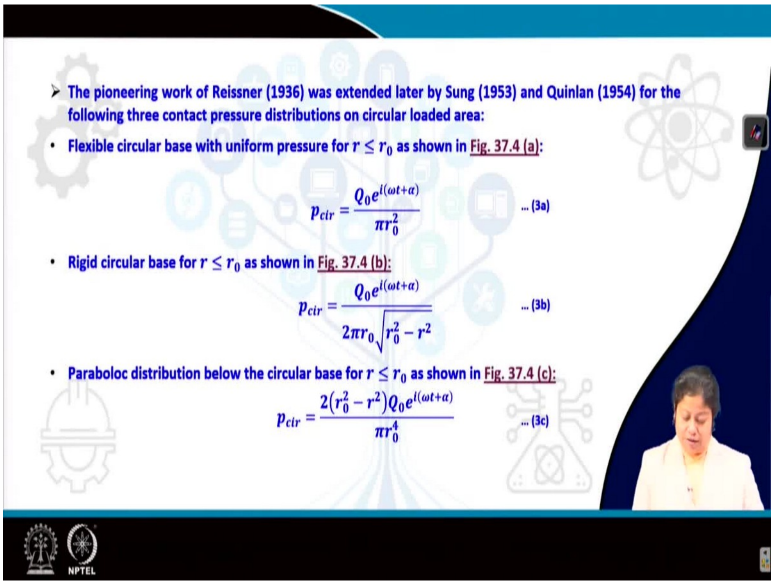

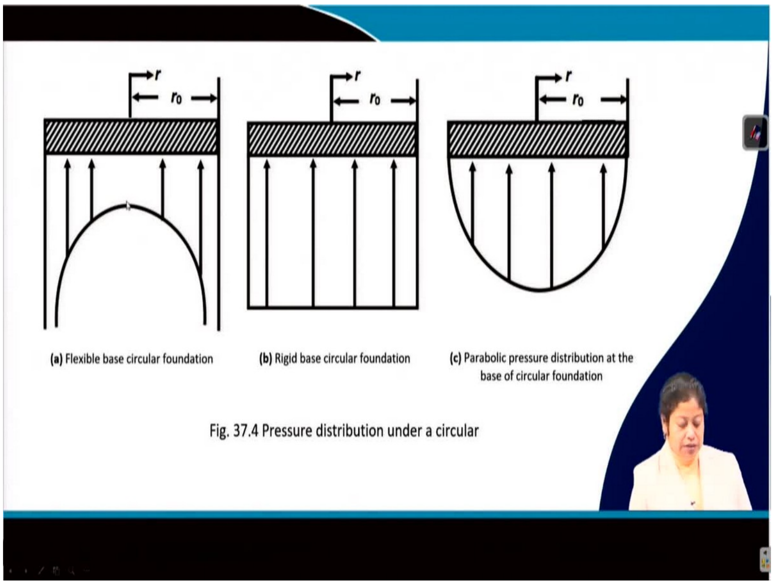

Now, the work of Reissner which we have already discussed was extended later by Sung in 1953 and Quinlan in 1954 for the following 3 contact pressure distributions on circular loaded area, what are those 3 contact pressure distributions? Let us see, for flexible circular base with uniform pressure at when r is less than or equal to r 0 as shown in figure 37.4 a.

So, let us see the figure 37.4 a, you can see here for this case, you can see the pressure distribution. So, for this case, how we will get the magnitude, we can use this equation that pressure for circular base is equal to Q 0 times e to the power i omega t plus alpha divided by pi r 0 square, simple case.

For rigid circular base, when we can see the uniform pressure distribution as you can see in figure b, what will be the equation for that case, we can calculate P circular as shown here, Q 0 divided by e to the power i omega t plus alpha divided by 2 pi r 0 times square root of r 0 square minus r square r is any radial distance from the center of the circular loaded area, but it should be less than or equal to r 0.

For parabolic distribution function that means I can show you the figure this case, what will be the pressure distribution the pressure can be calculated by using equation 3 c, I hope all the terms are now known for us. So, I am not explaining it.

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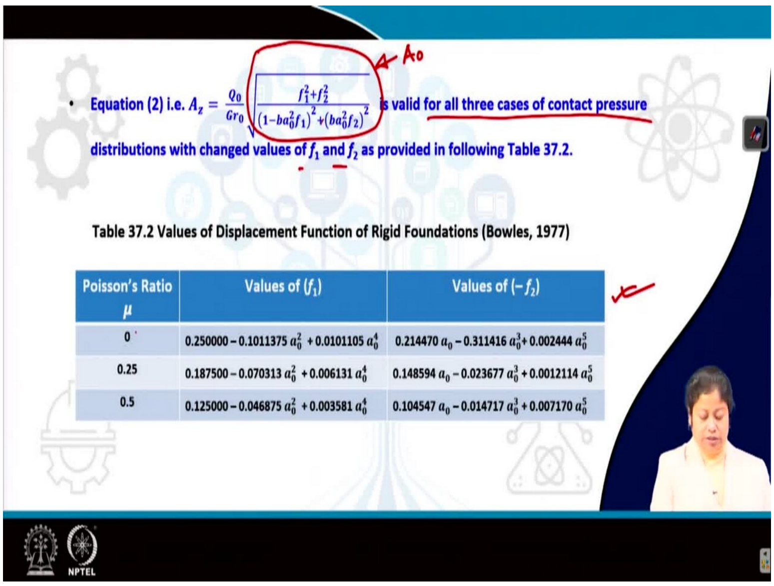

Now, in equation 2, we get the expression for A z. So, in that expression, what I have done I have directly written the value of A 0, which you can see here, this is nothing but A 0 right, I can mark it. So, this is nothing but A 0. So, using we can use this equation for all the 3 cases of the contact pressures, which are shown in previous slide, only differences that we need to change this time that displacement function f1 and f2. And that we afford that we can use that table 37.2 to finally find out what will be the A z for the three cases which are shown to you.

In this case also the value of f1 and the value of f2 depends upon the Poisson's ratio mu you can see that.

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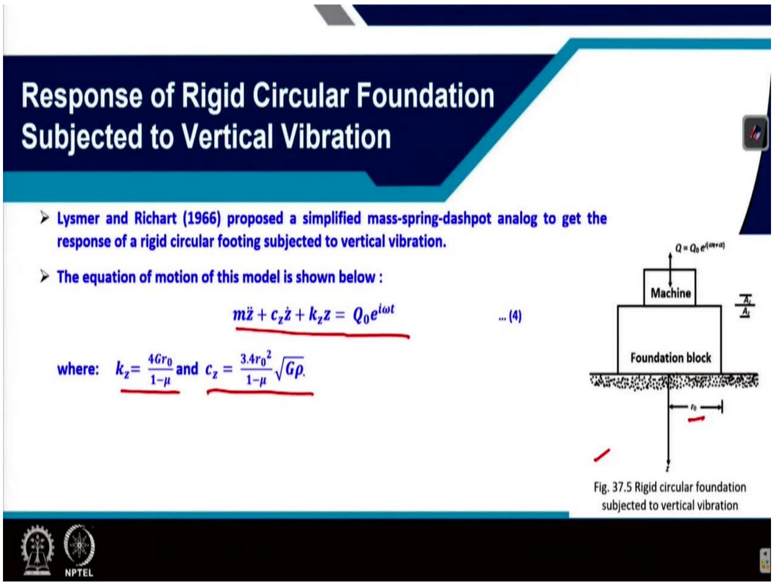

Now, what is the response of rigid circular foundation subjected to vertical vibration? So, far we have discussed most of the cases for the flexible foundation, the initial equation which proposed by Reissner. Then, we now we will see what will be the response of rigid circular foundation which is subjected to vertical vibration as shown in the figure 37.5.

So, for these Reissner and Richart in 1966 proposed a simplified spring mass spring dashpot analog to get the response of the circular rigid circular foundation which is subjected to vertical vibration. For this the equation of motion is shown here this is a known form to all of us. Only thing which we need to know is what is c z? That means that damping and what is k z? That is the stiffness when the soil or foundation and soil is subjected to vertical vibration.

So, for these they proposed that we should take k z is equal to 4 times G r 0 divided by 1 minus mu this one and for c z they have proposed this relationship. So, here G is dynamic shear modulus for the soil mu is Poisson's ratio r 0 you can see here, the radius of the circular foundation and rho which is the density of the soil.

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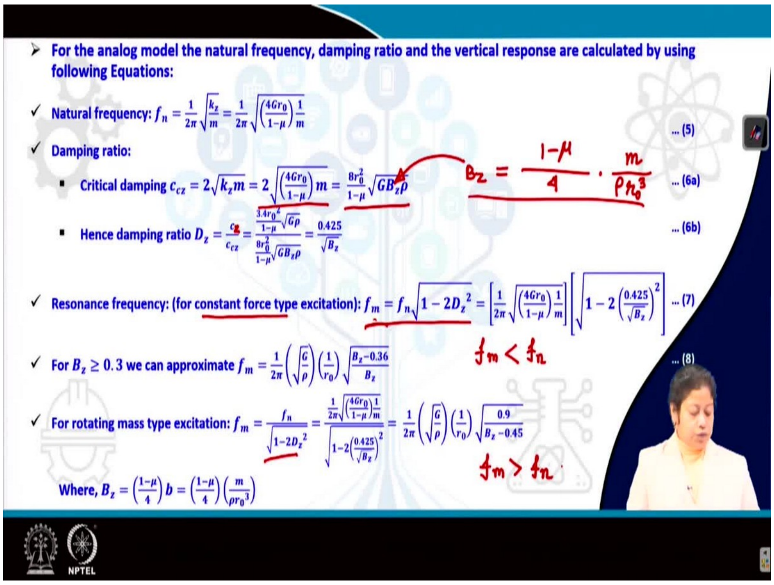

Now, for the analog model we can calculate natural frequency, damping ratio and vertical vibration by using the following equations, we already actually know how to calculate the natural frequency for a mass spring dashpot system. The same way here natural frequency for the undamped system is first calculated, which is 1 divided by 2 pi times square root of k z by m. So, we are finding out the frequency in cycles per second that is a reason square root of k z by m is divided by 2 pi here all right?

Now, for damping ratio first we need to know what is the critical damping? So, we know critical damping c z is equal to 2 times square root of k z times m. So, if we will write the expression for k z which is 4 times G r 0 divided by 1 minus mu and that if we will multiply that by m and take the square root of that value and again multiply by 2 as written here, we can get the critical damping.

Also we can express these same the same thing in this form where B z is a term which actually B z is a term I can write here in the slide it is not written. So, I am writing here what is B z, B z is you can take 1 minus mu divided by 4 times m divided by rho r 0 Q. So, the B z which is used here can be calculated using this expression.

Now, after calculating critical damping, we can calculate the damping ratio which is the ratio of damping of the system to the critical damping, which is c cz you can see here just I am correcting it is c z, because right now, we are considering only the vertical vibration which is occurring in z direction.

Now, thereafter, we can calculate the resonance frequency f m. If the force acting on the foundation is constant force type excitation, then we can use this formulation. We have already discussed it so I am not discussing it once again how it is coming. So, f m which is the resonance frequency can be calculated by multiplying square root of 1 minus 2 D z square to undamped natural frequency which is f n here.

If it is rotating mass type excitation then what we can do then we can divide f n by square root of 14 minus 2 D z square. So, from this what we can see in case of constant force type excitation f m is less than f n, whereas, in case of rotating mass type excitation, we can see a f m is greater than f n. At the end actually this B z which I have already written for you is given.

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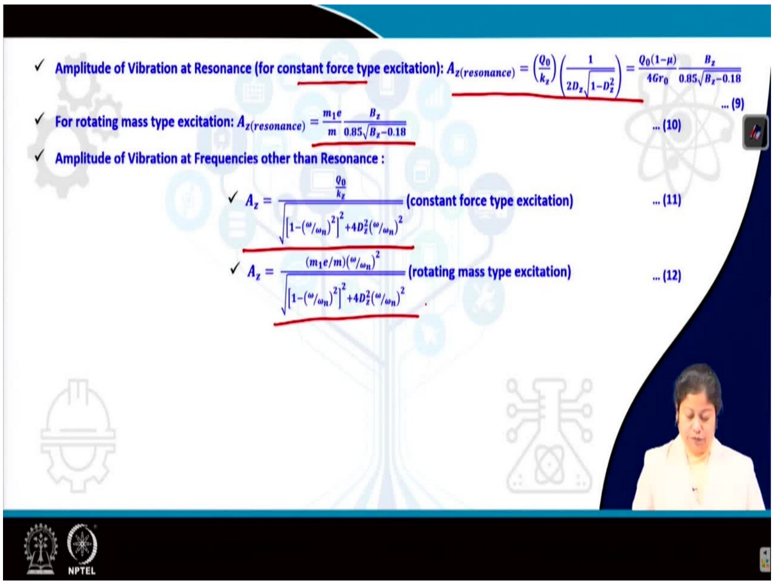

Now, with all these information we can also calculate the amplitude at resonance for constant force type excitation, we will use this equation, in case of rotating mass type excitation, we will use this equation or what we can do instead of this we can directly use the equation which is known to us. You can use B z you may not use B z it is totally up to you.

So, if we are interested to know the amplitude of vibration at any other frequency than the resonance frequency, then what we will do? We will use this equation alright. So, this is basically for the constant force type excitation, what will be the amplitude of vibration for rotating mass type excitation? In that case we will use this equation.

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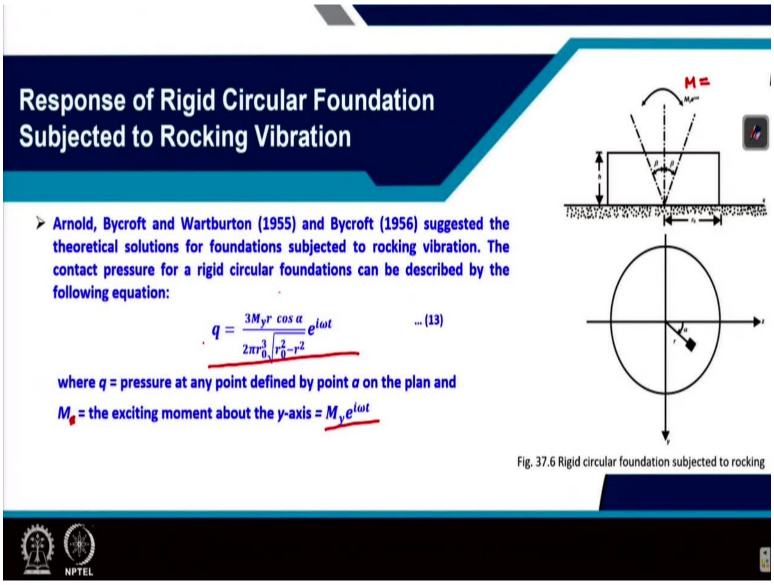

So, with this will now move to the next one which is response of rigid circular foundation subjected to rocking vibration. So, here what we can see a foundation is subjected to rocking vibration, for that you can see Arnold, Bycroft and Wartburton 1955 and Bycroft 1956 suggested that theoretical solutions for foundations subjected to rocking vibration, what is the solution?

First they have said how to calculate the contact pressure you can calculate contact pressure using this equation where M y is the exciting moment about the y axis and actually, you can I am just you just take it in alright. So, basically here it is M is equal to M y times e to the power i omega t. M means that exciting moment about y axis which can be expressed by this equation where M y is the amplitude of exciting moment. Now, from this we can calculate the contact pressure q.

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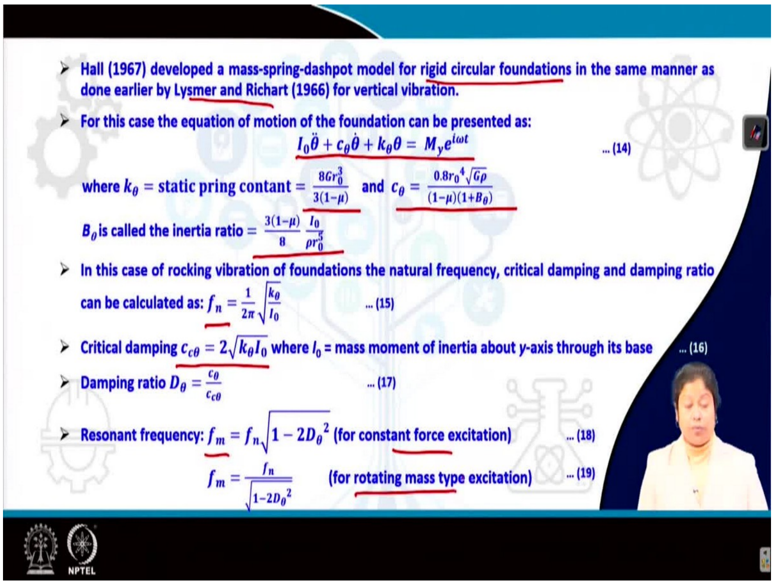

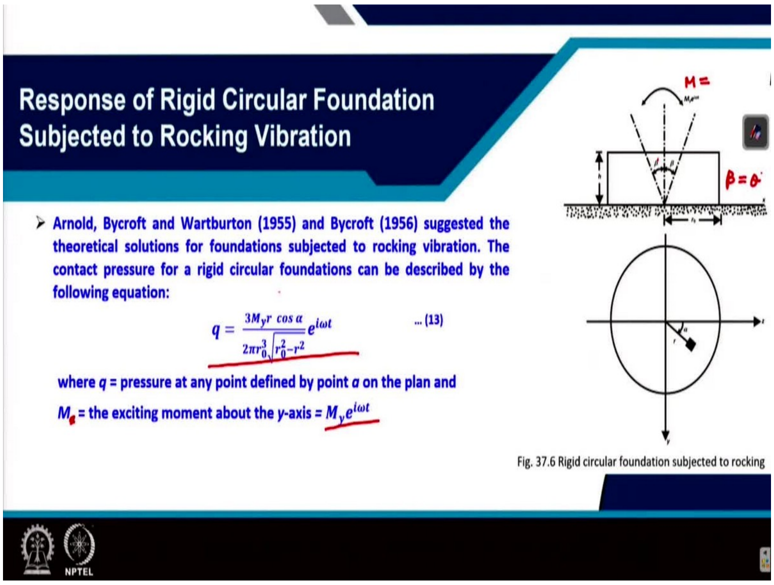

Now, after this Hall in 1967 developed a mass spring dashpot system for rigid circular foundations in the same manner as it was done by Lysmer and Richart for the foundations objected to vertical vibration which we just discussed. So, for these model when foundation is subjected to rocking vibration that time what will be the equation of motion? This is your equation of motion all right, what is theta if I will go back to the previous figure actually, in case of beta I have written there theta. So, you can take here beta is equal to theta also.

So, k theta is static spring constant here and that can be calculated by using this equation. So, here we can see k theta is a function of r 0 to the power 3 that means, r 0 is the radius of the circular foundation it also depends upon the mu and G.

Next is damping c theta, which can be calculated using this expression that 0.8 times r 0 to the power 4 times G rho divided by 1 minus mu divided by 1 plus B theta, where you can see what is B theta. So, after knowing k theta after knowing c theta what we can calculate? First we can calculate that natural frequency of the undamped system which is a f n here using the equation 15.

Next to know the critical damping value which we can calculate using this. So, we have seen I 0 a term that is mass moment of inertia about y axis through the base of the foundation. Now, thereafter, we can calculate the damping ratio, which is the ratio of c theta to critical damping c c theta.

Rest of the process are same which we have already seen. So, we can calculate resonant frequency for constant force excitation by using equation 8 we can calculate resonant frequency for rotating mass type excitation using equation 19.

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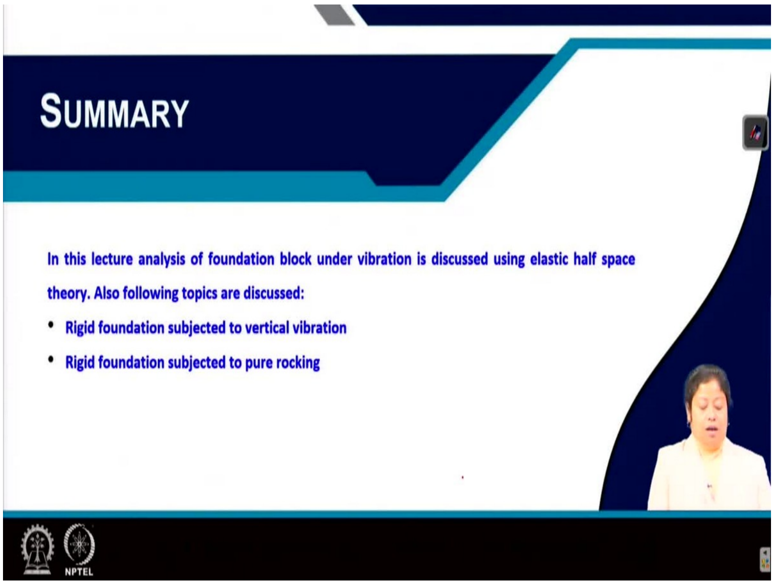

And, we can also calculate the amplitude of vibration, rocking vibration as well. So, here is the summary of today's lecture. So, today we have discussed, how to do the analysis of block foundation under vibration using elastic half space theory. We also discuss the topic of rigid foundations subjected to vertical vibration and rigid foundation subjected to rocking vibration that means, moment about y axis or rotation about better I should say rotation about y axis, so in that case also we have discussed.

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So, let us see that references these are the references which I have used for today’s class. Thank you.