

**Soil Dynamics**  
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**Lecture no. 37**  
**Analysis of Machine Foundations**  
**(Elastic Half Space Method – Part 1)**

Hello everyone, in last class, we have discussed about the design criteria of machine foundation. Today, we will study the analysis of machine foundation using elastic half space method.

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**Introduction**

➤ A block foundation can undergo following six modes of vibrations under the action of unbalanced forces:

1. Translation along z axis – Vertical vibration
2. Translation along y axis – Longitudinal or sliding vibration
3. Translation along x axis – Lateral or sliding vibration
4. Rotation about z axis – Yawing motion
5. Rotation about y axis – Rocking vibration
6. Rotation about x axis – Pitching or rocking vibration

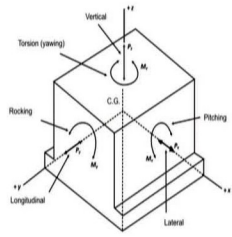


Fig. 37.1 Different modes of vibration of a rigid block foundation

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So, in this diagram that is figure 1 what we can see, here we can see the different modes of vibration of a rigid block foundation. So, there are 6 modes of vibration under the action of unbalanced forces, what are those 6 modes of vibration first one is translation along z axis alright? So, this is the first mode of vibration, second mode of vibration sorry second mode of vibration is translation along x axis and third is translation along y axis or we can take said y and x in this order as well.

So, for first mode of vibration which is translation along Z axis, we can measure the vertical vibration. Likewise, when we are focusing on translation along y axis, we are getting longitudinal or sliding vibration for x axis we are getting lateral or sliding vibration as well. Other than these 3, there are 3 other modes of vibrations which are rotation about Z axis, which is called as yawing motion.

Second is rotation about y axis, which is called rocking vibration and the third one is rotation about x axis which is called pitching or rocking vibration as well.

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## Vertical Vibration of Circular Foundation Resting on Elastic Half Space

➤ In 1936, Reissner analysed the problem of vibration of a *uniformly loaded circular area* resting on an elastic half-space as shown in Fig. 37.2. The solution was obtained by integration of Lamb's solution for a point load. The vertical displacement at the centre of the flexible loaded area can be given by:

$$z = \frac{Q_0 e^{i\omega t}}{Gr_0} (f_1 + if_2) \quad \dots (1)$$

where  $Q_0$  = amplitude of the exciting force acting on the foundation  
 $z$  = periodic displacement at the center of the loaded area  
 $\omega$  = circular frequency of applied load  
 $r_0$  = radius of the loaded area  
 $G$  = shear modulus of the soil  
 $Q$  = exciting force, which has an amplitude of  $Q_0$ .  
 $f_1$  and  $f_2$  = Reissner's displacement functions (Table 37.1)

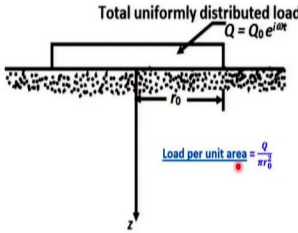


Fig. 37.2 Vibration of a uniformly loaded circular flexible area

Now, let us see how we can get the, how we can do that analysis of a circular foundation resting on elastic half space and subjected to vertical vibration. So, in 1936 Reissner analysed this type of problem of vibration where uniformly loaded circular area was considered which is resting on an elastic half space.

Let us see the figure for this here you can see the figure for the problem. So, this is the foundation which is circular in shape and its radius is  $r_0$ . So, if the uniformly distributed load is  $Q$ , which can be expressed by  $Q_0 e^{i\omega t}$  where  $Q_0$  is the amplitude of vibration or amplitude of loading  $\omega$  is the operating frequency of the

vibration or if machine is placed in that case we can say operating frequency of the machine then what will be the load per unit area? That is  $q$  divided by the area on which the load is acting which is circular area. So, area will be  $\pi r_0^2$  in this case alright?

Now, what will be the vertical displacement? So Reissner said that the vertical displacement at the center of the flexible loaded area can be given by the equation 1, you can see equation 1 which says that  $Z$  is equal to the load which is  $Q_0$  times  $e^{-i\omega t}$  divided by  $G$  times  $r_0$  this entire thing is multiplying with  $f_1 + i f_2$  What is  $G$  here?  $G$  is dynamic shear modulus of the soil.

And if you see  $\omega$  as I said this is the circular frequency of the applied load,  $z$  is periodic displacement at the center of the circular loaded area. Other than that, there are 2 new parameters  $f_1$  and  $f_2$  which are called as Reissner displacement function, you can see table 37.1.

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**Table 37.1 Values of Displacement Function of Flexible Foundations (Bowles, 1977)**

Poisson's Ratio $\mu$	Values of $f_1$	Values of $(-f_2)$
0	$0.318310 - 0.092841 a_0^2 + 0.007405 a_0^4$	$0.214474 a_0 - 0.029561 a_0^3 + 0.001528 a_0^5$
0.25	$0.238733 - 0.059683 a_0^2 + 0.004163 a_0^4$	$0.148594 a_0 - 0.017757 a_0^3 + 0.000808 a_0^5$
0.5	$0.159155 - 0.039789 a_0^2 + 0.002432 a_0^4$	$0.104547 a_0 - 0.011038 a_0^3 + 0.000444 a_0^5$

Here is table 37.1 where the values of displacement functions for flexible foundations are provided what we can see in this table here, it depends  $f_1$  and  $f_2$  depends upon the Poisson's ratio of the soil alright.

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> Let's consider a flexible circular foundation of weight  $W$  (mass =  $m = W/g$ ) resting on an elastic half-space as shown in Fig. 37.3 and subjected to an exciting force of magnitude of  $Q_0 e^{i(\omega t + \alpha)}$  where  $\alpha$  is the phase difference between the exciting force and the displacement of the foundation.

> Reissner's provided the following solution of the amplitude of the vibration considering the displacement relationship given in Equation (1) and solving the equation of motion:

$$A_z = \frac{Q_0}{Gr_0} A_0 \quad \dots (2)$$

$A_z$  = the amplitude of the vibration

$$A_0 = \text{dimensionless amplitude} = \frac{f_1^2 + f_2^2}{\sqrt{(1 - ba_0^2 f_1)^2 + (ba_0^2 f_2)^2}}$$

$b$  = dimensionless mass ratio =  $\frac{m}{\rho r_0^3} = \frac{W}{\gamma r_0^3}$

$\rho$  = density of elastic material and  $\gamma$  = unit weight of the elastic material i.e. soil  
 $G$  = Dynamic shear modulus of soil

$a_0$  = dimensionless frequency =  $\omega r_0 \sqrt{\frac{\rho}{G}} = \frac{\omega r_0}{v_s}$

$v_s$  = velocity of shear waves in the elastic material on which the foundation is resting


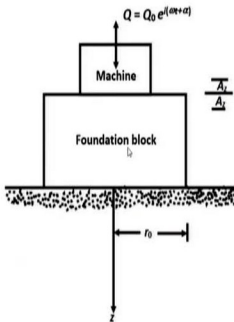




Fig. 37.3 Flexible circular foundation subjected to forced vibration



Now, let us consider a flexible circular foundation having weight capital  $W$  resting on an elastic half space we can show the figure here. So, in this figure what we can see? There is a circular foundation block on which machine is resting. Other than that, what we get machine is operating at a frequency, circular frequency  $\omega$  right and what about the dimension of the circular foundation? It has radius  $r_0$ .

So, with this Reissner provided the solution for the amplitude of vibration considering that displacement relationship which we have seen in that previous equation 1. So, he said that  $A_z$  is equal to  $q_0$  divided by  $G$  times  $r_0$  whole thing is multiplied by  $A_0$ . So, let us see what is  $A_0$  and  $A_z$ ?  $A_z$  is the amplitude of vibration, whereas,  $A_0$  is an dimensionless amplitude which can be expressed by the equation which is shown here.

So, here we can see that  $A_0$  which is a dimensionless amplitude depends upon  $f_1$  and  $f_2$  also it depends upon  $b$  and  $a_0$ . Now, the question then what is  $b$  and what is  $a_0$ ? So,  $b$  is dimensionless mass ratio, which can be determined by dividing the mass of the foundation to the  $\rho \times r_0^3$ , what is  $r_0$ ? That is known to us it is the radius of the circular loaded area or circular foundation.

$\rho$  and  $G$   $r$  density and the dynamic shear modulus of the soil. Another new parameter is  $a_0$  which is called as dimensionless frequency that can be expressed by or determined by  $\omega r_0$  which is called as dimensionless frequency that can be expressed by or determined by  $\omega r_0$  times square root of  $\rho$  by  $G$ . We already know that square root of  $G$  by  $\rho$  gives us the velocity of the shear wave. So, we can write  $a_0$  as  $\omega r_0$  divided by  $v_s$  as shown here.

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> The pioneering work of Reissner (1936) was extended later by Sung (1953) and Quinlan (1954) for the following three contact pressure distributions on circular loaded area:


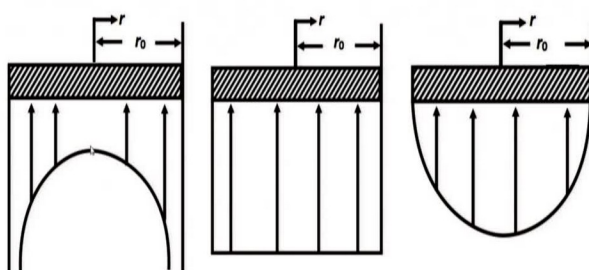
- Flexible circular base with uniform pressure for  $r \leq r_0$  as shown in Fig. 37.4 (a):

$$p_{cir} = \frac{Q_0 e^{i(\omega t + a)}}{\pi r_0^2} \quad \dots(3a)$$

- Rigid circular base for  $r \leq r_0$  as shown in Fig. 37.4 (b):


$$p_{cir} = \frac{Q_0 e^{i(\omega t + a)}}{2\pi r_0 \sqrt{r_0^2 - r^2}} \quad \dots(3b)$$

- Parabolic distribution below the circular base for  $r \leq r_0$  as shown in Fig. 37.4 (c):

$$p_{cir} = \frac{2(r_0^2 - r^2) Q_0 e^{i(\omega t + a)}}{\pi r_0^4} \quad \dots(3c)$$



(a) Flexible base circular foundation      (b) Rigid base circular foundation      (c) Parabolic pressure distribution at the base of circular foundation

Fig. 37.4 Pressure distribution under a circular



Now, the work of Reissner which we have already discussed was extended later by Sung in 1953 and Quinlan in 1954 for the following 3 contact pressure distributions on circular loaded area, what are those 3 contact pressure distributions? Let us see, for flexible circular base with uniform pressure at when  $r$  is less than or equal to  $r_0$  as shown in figure 37.4 a.

So, let us see the figure 37.4 a, you can see here for this case, you can see the pressure distribution. So, for this case, how we will get the magnitude, we can use this equation that pressure for circular base is equal to  $Q_0$  times  $e$  to the power  $i \omega t + \alpha$  divided by  $\pi r_0^2$ , simple case.

For rigid circular base, when we can see the uniform pressure distribution as you can see in figure b, what will be the equation for that case, we can calculate  $P$  circular as shown here,  $Q_0$  divided by  $e$  to the power  $i \omega t + \alpha$  divided by  $2 \pi r_0$  times square root of  $r_0^2 - r^2$  where  $r$  is any radial distance from the center of the circular loaded area, but it should be less than or equal to  $r_0$ .

For parabolic distribution function that means I can show you the figure this case, what will be the pressure distribution the pressure can be calculated by using equation 3 c, I hope all the terms are now known for us. So, I am not explaining it.

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Equation (2) i.e.  $A_z = \frac{Q_0}{G r_0} \frac{f_1^2 + f_2^2}{(1 - b a_0^2 f_1)^2 + (b a_0^2 f_2)^2}$  is valid for all three cases of contact pressure distributions with changed values of  $f_1$  and  $f_2$  as provided in following Table 37.2.

Table 37.2 Values of Displacement Function of Rigid Foundations (Bowles, 1977)

Poisson's Ratio $\mu$	Values of $(f_1)$	Values of $(-f_2)$
0	$0.250000 - 0.1011375 a_0^2 + 0.0101105 a_0^4$	$0.214470 a_0 - 0.311416 a_0^3 + 0.002444 a_0^5$
0.25	$0.187500 - 0.070313 a_0^2 + 0.006131 a_0^4$	$0.148594 a_0 - 0.023677 a_0^3 + 0.0012114 a_0^5$
0.5	$0.125000 - 0.046875 a_0^2 + 0.003581 a_0^4$	$0.104547 a_0 - 0.014717 a_0^3 + 0.007170 a_0^5$

Now, in equation 2, we get the expression for  $A_z$ . So, in that expression, what I have done I have directly written the value of  $A_0$ , which you can see here, this is nothing but  $A_0$  right, I can mark it. So, this is nothing but  $A_0$ . So, using we can use this equation for all the 3 cases of the contact pressures, which are shown in previous slide, only differences that we need to

change this time that displacement function  $f_1$  and  $f_2$ . And that we afford that we can use that table 37.2 to finally find out what will be the  $A_z$  for the three cases which are shown to you.

In this case also the value of  $f_1$  and the value of  $f_2$  depends upon the Poisson's ratio  $\mu$  you can see that.

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**Response of Rigid Circular Foundation Subjected to Vertical Vibration**

- Lysmer and Richart (1966) proposed a simplified mass-spring-dashpot analog to get the response of a rigid circular footing subjected to vertical vibration.
- The equation of motion of this model is shown below :

$$m\ddot{z} + c_z\dot{z} + k_z z = Q_0 e^{i\omega t} \quad \dots (4)$$

where:  $k_z = \frac{4Gr_0}{1-\mu}$  and  $c_z = \frac{34r_0^2}{1-\mu} \sqrt{G\rho}$

The diagram shows a 'Machine' on top of a 'Foundation block'. A vertical force  $Q = Q_0 e^{i\omega t}$  is applied to the machine. The foundation block is shown with a displacement  $z$  and a damping force  $c_z \dot{z}$  acting downwards. The diagram is labeled 'Fig. 37.5 Rigid circular foundation subjected to vertical vibration'.

Now, what is the response of rigid circular foundation subjected to vertical vibration? So, far we have discussed most of the cases for the flexible foundation, the initial equation which proposed by Reissner. Then, we now we will see what will be the response of rigid circular foundation which is subjected to vertical vibration as shown in the figure 37.5.

So, for these Reissner and Richart in 1966 proposed a simplified spring mass spring dashpot analog to get the response of the circular rigid circular foundation which is subjected to vertical vibration. For this the equation of motion is shown here this is a known form to all of us. Only thing which we need to know is what is  $c_z$ ? That means that damping and what is  $k_z$ ? That is the stiffness when the soil or foundation and soil is subjected to vertical vibration.

So, for these they proposed that we should take  $k_z$  is equal to 4 times  $G r_0$  divided by 1 minus  $\mu$  this one and for  $c_z$  they have proposed this relationship. So, here  $G$  is dynamic shear modulus for the soil  $\mu$  is Poisson's ratio  $r_0$  you can see here, the radius of the circular foundation and  $\rho$  which is the density of the soil.

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> For the analog model the natural frequency, damping ratio and the vertical response are calculated by using following Equations:

- ✓ Natural frequency:  $f_n = \frac{1}{2\pi} \sqrt{\frac{k_z}{m}} = \frac{1}{2\pi} \sqrt{\frac{4Gr_0}{(1-\mu)m}}$  ... (5)
- ✓ Damping ratio:
  - Critical damping  $c_{cz} = 2\sqrt{k_z m} = 2\sqrt{\frac{4Gr_0}{1-\mu}m} = \frac{8r_0^2}{1-\mu} \sqrt{GB_z \rho}$  ... (6a)
   
 $B_z = \frac{1-\mu}{4} \cdot \frac{m}{\rho r_0^3}$
  - Hence damping ratio  $D_z = \frac{c}{c_{cz}} = \frac{\frac{24r_0^2 \sqrt{G\rho}}{1-\mu}}{\frac{8r_0^2}{1-\mu} \sqrt{GB_z \rho}} = \frac{0.425}{\sqrt{B_z}}$  ... (6b)
- ✓ Resonance frequency: (for constant force type excitation):  $f_m = f_n \sqrt{1 - 2D_z^2} = \frac{1}{2\pi} \sqrt{\frac{4Gr_0}{(1-\mu)m}} \left[ \sqrt{1 - 2\left(\frac{0.425}{\sqrt{B_z}}\right)^2} \right]$  ... (7)
- ✓ For  $B_z \geq 0.3$  we can approximate  $f_m = \frac{1}{2\pi} \left( \frac{G}{\rho} \right)^{1/2} \left( \frac{1}{r_0} \right) \sqrt{\frac{B_z - 0.36}{B_z}}$  ... (8)
   
 $f_m < f_n$
- ✓ For rotating mass type excitation:  $f_m = \frac{f_n}{\sqrt{1 - 2D_z^2}} = \frac{1}{2\pi} \sqrt{\frac{4Gr_0}{(1-\mu)m}} = \frac{1}{2\pi} \left( \frac{G}{\rho} \right)^{1/2} \left( \frac{1}{r_0} \right) \sqrt{\frac{0.9}{B_z - 0.45}}$ 
  
 $f_m > f_n$

Where,  $B_z = \frac{(1-\mu)}{4} b = \left( \frac{1-\mu}{4} \right) \left( \frac{m}{\rho r_0^3} \right)$

Now, for the analog model we can calculate natural frequency, damping ratio and vertical vibration by using the following equations, we already actually know how to calculate the natural frequency for a mass spring dashpot system. The same way here natural frequency for the undamped system is first calculated, which is 1 divided by 2 pi times square root of k z by m. So, we are finding out the frequency in cycles per second that is a reason square root of k z by m is divided by 2 pi here all right?

Now, for damping ratio first we need to know what is the critical damping? So, we know critical damping c z is equal to 2 times square root of k z times m. So, if we will write the expression for k z which is 4 times G r 0 divided by 1 minus mu and that if we will multiply that by m and take the square root of that value and again multiply by 2 as written here, we can get the critical damping.

Also we can express these same the same thing in this form where B z is a term which actually B z is a term I can write here in the slide it is not written. So, I am writing here what is B z, B z is you can take 1 minus mu divided by 4 times m divided by rho r 0 Q. So, the B z which is used here can be calculated using this expression.

Now, after calculating critical damping, we can calculate the damping ratio which is the ratio of damping of the system to the critical damping, which is c cz you can see here just I am correcting it is c z, because right now, we are considering only the vertical vibration which is occurring in z direction.



Now, thereafter, we can calculate the resonance frequency  $f_m$ . If the force acting on the foundation is constant force type excitation, then we can use this formulation. We have already discussed it so I am not discussing it once again how it is coming. So,  $f_m$  which is the resonance frequency can be calculated by multiplying square root of 1 minus  $2Dz$  square to undamped natural frequency which is  $f_n$  here.

If it is rotating mass type excitation then what we can do then we can divide  $f_n$  by square root of 14 minus  $2Dz$  square. So, from this what we can see in case of constant force type excitation  $f_m$  is less than  $f_n$ , whereas, in case of rotating mass type excitation, we can see a  $f_m$  is greater than  $f_n$ . At the end actually this  $Bz$  which I have already written for you is given.

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✓ Amplitude of Vibration at Resonance (for constant force type excitation):  $A_z(\text{resonance}) = \frac{Q_0}{k_z} \left( \frac{1}{2Dz\sqrt{1-Dz^2}} \right) = \frac{Q_0(1-\mu)}{4G\gamma_0 \cdot 0.85\sqrt{\beta_z-0.18}} \dots (9)$

✓ For rotating mass type excitation:  $A_z(\text{resonance}) = \frac{m_1 e}{m} \frac{\beta_z}{0.85\sqrt{\beta_z-0.18}} \dots (10)$

✓ Amplitude of Vibration at Frequencies other than Resonance :

✓  $A_z = \frac{Q_0}{k_z}$  (constant force type excitation)  $\dots (11)$

✓  $A_z = \frac{(m_1 e/m)(\omega/\omega_n)^2}{\sqrt{[1-(\omega/\omega_n)^2]^2 + 4Dz^2(\omega/\omega_n)^2}}$  (rotating mass type excitation)  $\dots (12)$

Now, with all these information we can also calculate the amplitude at resonance for constant force type excitation, we will use this equation, in case of rotating mass type excitation, we will use this equation or what we can do instead of this we can directly use the equation which is known to us. You can use  $Bz$  you may not use  $Bz$  it is totally up to you.

So, if we are interested to know the amplitude of vibration at any other frequency than the resonance frequency, then what we will do? We will use this equation alright. So, this is basically for the constant force type excitation, what will be the amplitude of vibration for rotating mass type excitation? In that case we will use this equation.

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### Response of Rigid Circular Foundation Subjected to Rocking Vibration

➤ Arnold, Bycroft and Wartburton (1955) and Bycroft (1956) suggested the theoretical solutions for foundations subjected to rocking vibration. The contact pressure for a rigid circular foundations can be described by the following equation:

$$q = \frac{3M_y r \cos \alpha}{2\pi r_0^3 (r_0^2 - r^2)} e^{i\omega t} \quad \dots (13)$$

where  $q$  = pressure at any point defined by point  $a$  on the plan and  
 $M_y$  = the exciting moment about the  $y$ -axis =  $M_y e^{i\omega t}$

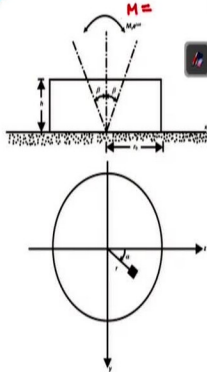


Fig. 37.6 Rigid circular foundation subjected to rocking

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So, with this will now move to the next one which is response of rigid circular foundation subjected to rocking vibration. So, here what we can see a foundation is subjected to rocking vibration, for that you can see Arnold, Bycroft and Wartburton 1955 and Bycroft 1956 suggested that theoretical solutions for foundations subjected to rocking vibration, what is the solution?

First they have said how to calculate the contact pressure you can calculate contact pressure using this equation where  $M_y$  is the exciting moment about the  $y$  axis and actually, you can I am just you just take it in alright. So, basically here it is  $M$  is equal to  $M_y$  times  $e$  to the power  $i \omega t$ .  $M$  means that exciting moment about  $y$  axis which can be expressed by this equation where  $M_y$  is the amplitude of exciting moment. Now, from this we can calculate the contact pressure  $q$ .

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➤ Hall (1967) developed a mass-spring-dashpot model for rigid circular foundations in the same manner as done earlier by Lysmer and Richart (1966) for vertical vibration.

➤ For this case the equation of motion of the foundation can be presented as:
 
$$I_0 \ddot{\theta} + c_\theta \dot{\theta} + k_\theta \theta = M_y e^{i\omega t} \quad \dots (14)$$

where  $k_\theta =$  static spring constant  $= \frac{8Gr_0^3}{3(1-\mu)}$  and  $c_\theta = \frac{0.8r_0^4 \sqrt{G\rho}}{(1-\mu)(1+B_\theta)}$

$B_\theta$  is called the inertia ratio  $= \frac{3(1-\mu)}{8} \frac{I_0}{\rho r_0^2}$


➤ In this case of rocking vibration of foundations the natural frequency, critical damping and damping ratio can be calculated as:  $f_n = \frac{1}{2\pi} \sqrt{\frac{k_\theta}{I_0}}$  ... (15)

➤ Critical damping  $c_{c\theta} = 2\sqrt{k_\theta I_0}$  where  $I_0 =$  mass moment of inertia about y-axis through its base ... (16)

➤ Damping ratio  $D_\theta = \frac{c_\theta}{c_{c\theta}}$  ... (17)

➤ Resonant frequency:  $f_m = f_n \sqrt{1 - 2D_\theta^2}$  (for constant force excitation) ... (18)

$f_m = \frac{f_n}{\sqrt{1 - 2D_\theta^2}}$  (for rotating mass type excitation) ... (19)



### Response of Rigid Circular Foundation Subjected to Rocking Vibration

➤ Arnold, Bycroft and Wartburton (1955) and Bycroft (1956) suggested the theoretical solutions for foundations subjected to rocking vibration. The contact pressure for a rigid circular foundations can be described by the following equation:

$$q = \frac{3M_y r \cos \alpha}{2\pi r_0^3 (r_0^2 - r^2)} e^{i\omega t} \quad \dots (13)$$

where  $q =$  pressure at any point defined by point  $\alpha$  on the plan and  $M_y =$  the exciting moment about the y-axis  $= M_y e^{i\omega t}$

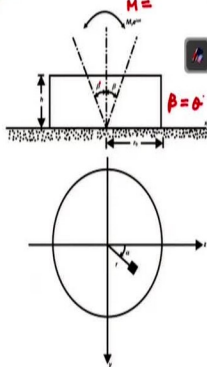



Fig. 37.6 Rigid circular foundation subjected to rocking



Now, after this Hall in 1967 developed a mass spring dashpot system for rigid circular foundations in the same manner as it was done by Lysmer and Richart for the foundations objected to vertical vibration which we just discussed. So, for these model when foundation is subjected to rocking vibration that time what will be the equation of motion? This is your equation of motion all right, what is theta if I will go back to the previous figure actually, in case of beta I have written there theta. So, you can take here beta is equal to theta also.

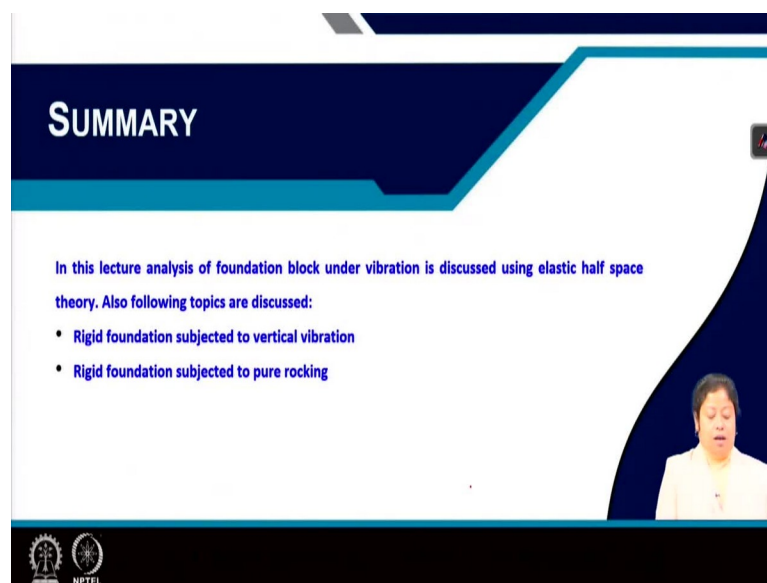
So,  $k_\theta$  is static spring constant here and that can be calculated by using this equation. So, here we can see  $k_\theta$  is a function of  $r_0$  to the power 3 that means,  $r_0$  is the radius of the circular foundation it also depends upon the  $\mu$  and  $G$ .

Next is damping  $c_\theta$ , which can be calculated using this expression that  $0.8$  times  $r_0$  to the power  $4$  times  $G$  rho divided by  $1 - \mu$  divided by  $1 + B_\theta$ , where you can see what is  $B_\theta$ . So, after knowing  $k_\theta$  after knowing  $c_\theta$  what we can calculate? First we can calculate that natural frequency of the undamped system which is  $f_n$  here using the equation 15.

Next to know the critical damping value which we can calculate using this. So, we have seen  $I_0$  a term that is mass moment of inertia about  $y$  axis through the base of the foundation. Now, thereafter, we can calculate the damping ratio, which is the ratio of  $c_\theta$  to critical damping  $c_c$ .

Rest of the process are same which we have already seen. So, we can calculate resonant frequency for constant force excitation by using equation 8 we can calculate resonant frequency for rotating mass type excitation using equation 19.

(Refer Slide Time: 28:00)



**SUMMARY**

In this lecture analysis of foundation block under vibration is discussed using elastic half space theory. Also following topics are discussed:

- Rigid foundation subjected to vertical vibration
- Rigid foundation subjected to pure rocking

And, we can also calculate the amplitude of vibration, rocking vibration as well. So, here is the summary of today's lecture. So, today we have discussed, how to do the analysis of block foundation under vibration using elastic half space theory. We also discuss the topic of rigid foundations subjected to vertical vibration and rigid foundation subjected to rocking vibration that means, moment about  $y$  axis or rotation about better I should say rotation about  $y$  axis, so in that case also we have discussed.

(Refer Slide Time: 29:00)


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So, let us see that references these are the references which I have used for today's class.

Thank you.