Soil Dynamics
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Lecture no. 38
Analysis of Machine Foundations
(Elastic Half Space Method – Part 2)

Hello friends, today, we will discuss the second part of the analysis of machine foundation using elastic half space method.

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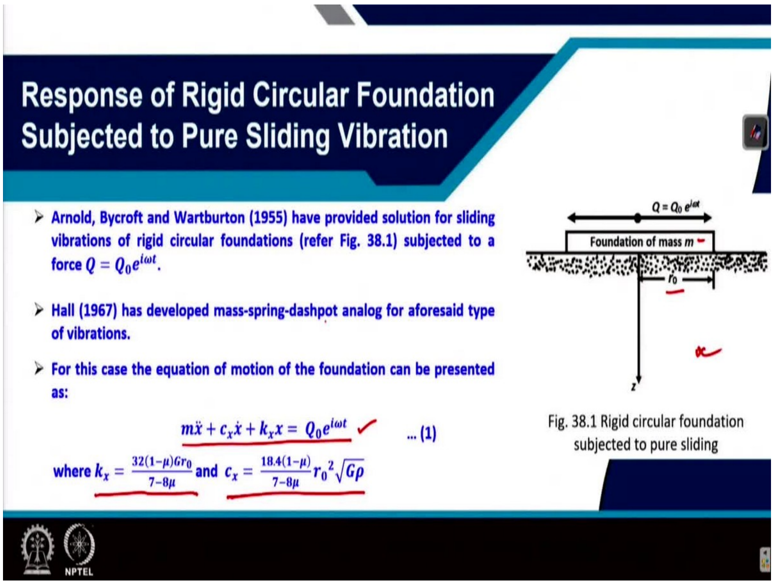

So, last class we have discussed how to get the response of the foundation when it is subjected to vertical vibrations only when it is subjected to rocking vibrations only. Now, today we will discuss how to get the response of rigid foundation of course, it is circular foundation when it is subjected to pure sliding vibration, and then we will discuss when it is subjected to torsional vibration.

So, let us start with pure sliding vibration. So, here you can see Arnold, Bycroft and Wartburton 1955 provided solution for sliding vibration of rigid circular foundations you can see the figure here. So, in this case, the foundation having mass m and radius r 0 is subjected to pure sliding vibration which can be expressed by Q is equal to Q 0 e to the power i omega t. Now, in this case Hall 1967 has developed mass spring dashpot analog for getting the solution.

So, for these that equation of motion can be expressed by the equation shown here. So, in order to solve this equation and to get the response and rest of the things here we need to know what is how we can calculate c x which is the damping coefficient when the soil or foundation is subjected to pure sliding and what is k x?

So, k x which is spring coefficient or we can call spring constant also can be calculated using this expression. So, here k x depends upon mu, G and r 0, mu and G are related to the soil whereas, r 0 is related to the dimension of the foundation. So, next is to find out c x so, far that Hall has given this expression to get the c x which is the coefficient of damping or I sorry it is better to say c x is the coefficient of the dashpot which is used to represent this system by us must be dashpot analog.

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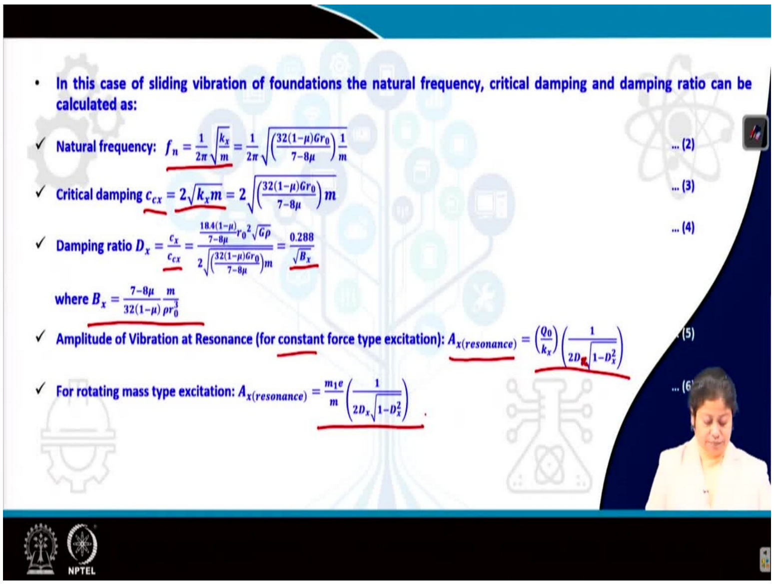

Now, if we know c x if we know k x then we can calculate the natural frequency. So, the natural frequency under undamped condition is equal to omega n divided by 2 pi where omega n is equal to square root of k x divided by m. So, from that we can calculate f n. Next target is to know the critical damping. So, c cx which is critical damping when the system is subjected to pure sliding, which occurs in x direction. That time c cx can be calculated by these expression 2 times square root of l x by m. Here if we put the expression of k x, then we can get the critical damping.

After doing critical damping and damping coefficient c x, we can calculate the damping ratio for the soil system which is nothing but c x divided by c cx. We can also express or simplify that expression for D x by introducing term B x, where B x is equal to this one 7 minus 8 mu divided by 32 times 1 minus mu this thing multiplied with m divided by rho r 0 cube.

After knowing all these parameter, we can calculate this amplitude at resonance for constant force type excitation, we will use this expression okay make it D x not D z, please correct equation 5 it is Q 0 divided by k x times 1 minus 2 D x 1 divided by 2 D x times square root of 1 minus D x square. Now, for rotating mass type excitation we will use this expression.

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All right after knowing these 2 we can also find out the amplitude of vibration at any other frequencies than resonance one for that, we can use this expression when we are dealing constant force type excitation we can use the equation 8 when dealing rotating mass type excitation. Actually, these equations are well known to us if you recall at when we study theory of vibrations we have gone through this exercise, so, I am not deriving it or explaining it in detail.

Only thing here which we need to know is how different researchers express the k x or k z, c x or c z and that we need to first find out then we can use our knowledge in theory of vibration and using those concepts, we can get the different parameters.

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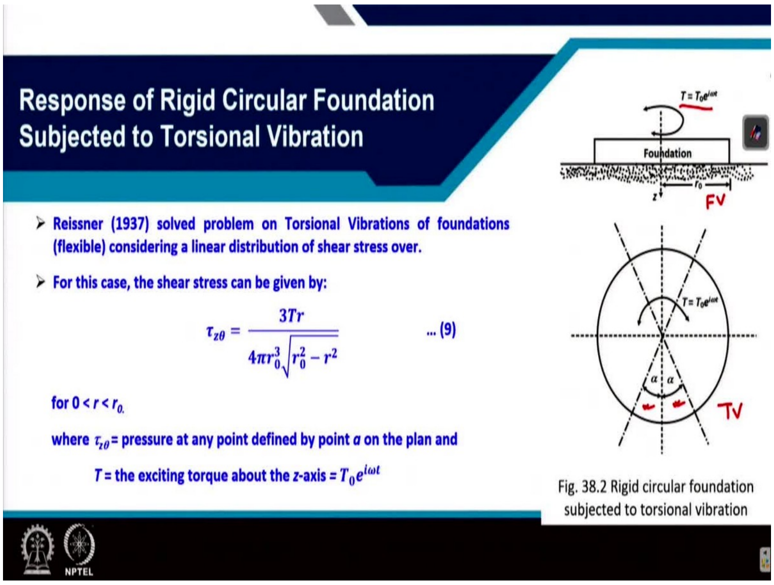

Next is the response of rigid circular foundation subjected to torsional vibration. So, in this case what is happening, let us see the figure a foundation block is resting on the ground surface. Now, it will be subjected to torsion. Torsion means rotation about Z axis and what is the magnitude of torsion? Torsion is expressed by T which is equal to T 0 times e to the power i omega t what is T 0 here? Here T 0 is the amplitude of the torsion or torsional vibration.

So, if you see that plan view of these foundation, so, this is actually your front view and this is the top view what we can see when these torsional vibration is applied to the foundation by the machine. What will happen? The circular foundation will rotate in by an angle alpha you can see this is in we can see plus alpha and minus alpha that way also like we used to see plus A z minus A z, the same way you can mention it here also.

However, now, we are interested to know the shear stress distribution. So, the expression to find out the shear stress is tau z theta, which is 3 times T r divided by 4 pi r 0 cube times square root of r 0 square minus r square. So, here T is the magnitude of the torsional vibration at any time t, r is the position or radial distance of the point at which we are interested to find the shear stress from the center of the circular foundation.

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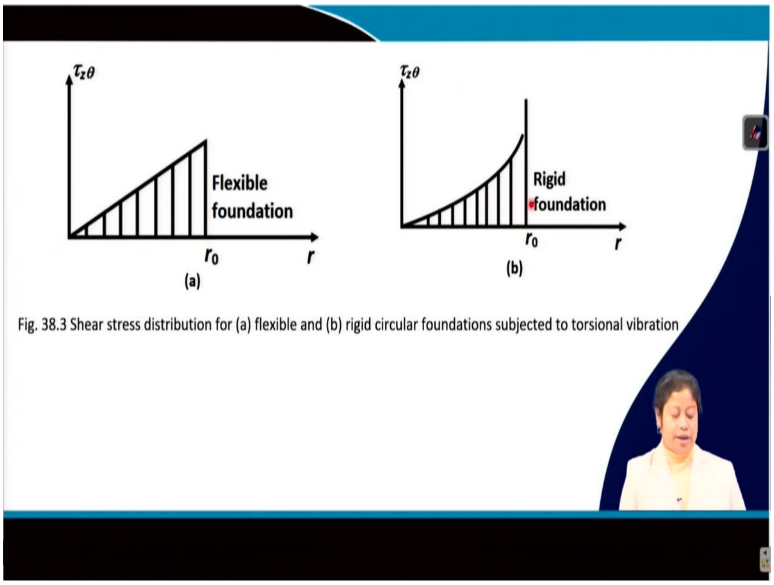

Now, we now, we can see in figure 3 that, how this distribution will look? When we are dealing the flexible foundation, that time we will see this kind of triangular distribution. So, this point is at the center of the circular foundation and we can see the maximum shear strength at the edges of the foundation, that means, at a distance r 0 from the center of the circular foundation.

In this case, in this case means, when we are dealing the rigid foundation, that time the stress, shear stress distribution is no longer linear, you can see a nonlinear distribution. However, one thing is common that at the center shear stress is 0 at the edge of the foundation, it is maximum and how to get that expression already we have discussed.

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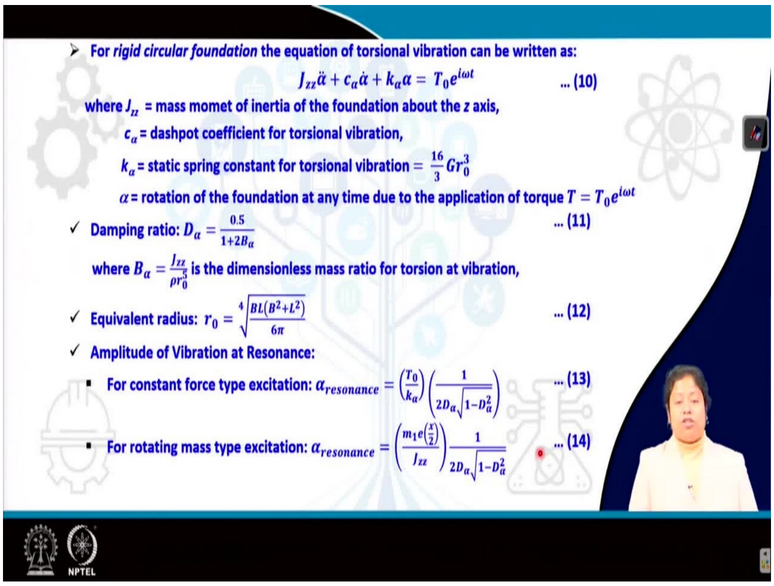

Now, for rigid circular foundation, the equation of torsional vibration or equation of motion for torsional vibration can be expressed by this equation. So, what are the meaning of the parameters which I have written here. So, J zz this is the mass moment of inertia of the foundation about Z axis, Z axis means in this case that is the vertical axis, c alpha is dashpot coefficient when the system soil is subjected to torsional vibration. I should say it when the foundation is subjected to torsional vibration that time the dashpot coefficient for the soil system.

Now, k alpha is static spring constant for torsional vibration and we can calculate it by using this expression 16 divided by 3 times G times r 0 cube. Alpha is rotation of the foundation at any time due to the application of torque which can be expressed by the expression shown here.

Next task is to find out the damping ratio which is D alpha here. So, the damping ratio can be expressed by that equation shown here. So, here we need to know what is B alpha right. So, B alpha is our dimensionless mass ratio for torsion at vibrating condition. So, when we will calculate B alpha we will use this expression that B alpha is equal to J zz divided by rho r 0 to the power 5. So, J zz is mass moment of inertia of the foundation about vertical axis or Z Z axis.

Now, in this case, not only just this case for all the cases which we have discussed so, far, there is a possibility that the foundation may not be circular, but it may be rectangular or square in shape. That time we need to find out the equivalent radius r 0. So, let us talk for this case how we will get the equivalent radius.

So, we can use this expression to get the equivalent radius. Amplitude of vibration at resonance. So, using equation 13, we can find out the amplitude of vibration at resonance when it is subjected to constant force type excitation that means, the amplitude of excitation does not depend upon the frequency if it depends upon the frequency that means, rotating mass type excitation, that time we can use equation 14 to get the amplitude of vibration at resonance.

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So, in the come to the summary of today’s class, today, we have discussed the rigid response of the rigid foundations subjected to pure sliding vibration and the response of the rigid circular foundation subjected to torsional vibration. Here I had like to mention an important thing that when we have discussed the vibration of when we have discussed rigid foundation or rigid circular foundation subjected to rocking, I have not considered the effect of sliding.

We need to understand that whenever a foundation is subjected to rocking, it will cause sliding as well. So, we need to take care of both rocking as well as sliding in that case.

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These are the references which I have used in today's class. Next class, we will solve a few numerical problems. Thank you.