Soil Dynamics
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Lecture - 04
Single Degree of Freedom System (SDOF) - Part 2

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Hello friends. Today we will continue our discussion on single degree of freedom system.

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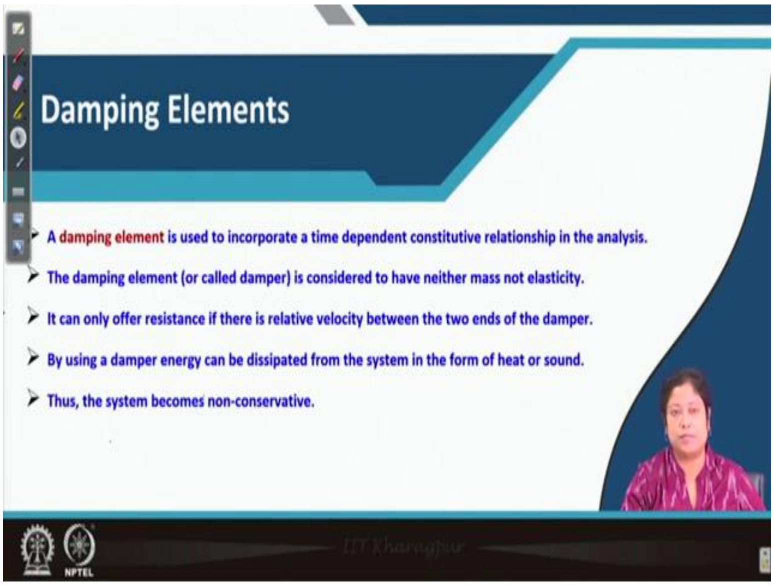

So, our today's topic is damping, so first we will see what is damping element. So, a damping element is generally used to incorporate a time dependent constitutive relationship in the analysis. The damping element sometime it is called also as damper is considered to have neither mass nor elasticity.

Now, it can only offer resistance if there is relative velocity between the two ends of the damper. By using a damper energy can be dissipated from the system in the form of either heat or sound. So, in this way the system becomes non-conservative.

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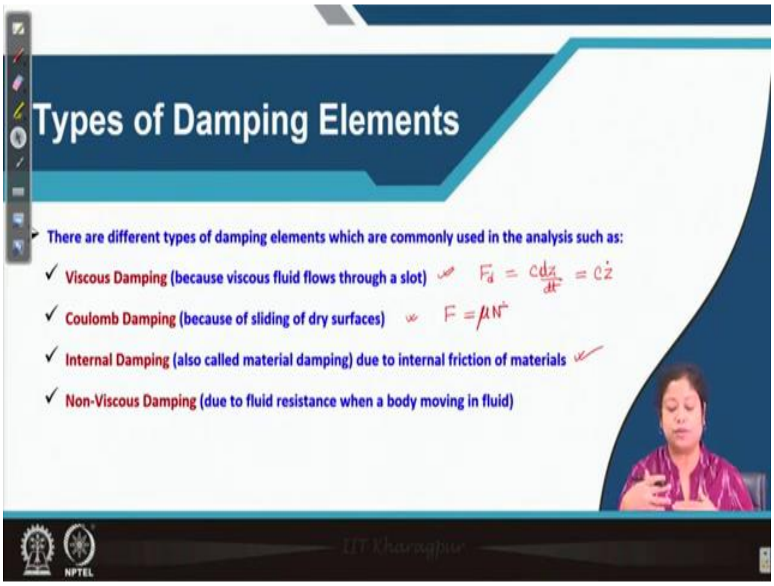

Now, let us see what are the different types of damping elements. There are different types of damping elements which are commonly used in the analysis such as, viscous damping it is present because of the flow of viscous fluid through a slot. Next is coulomb damping which is present because of sliding of dry surfaces on each other? Internal damping is also called as material damping which is due to internal friction of the materials.

Another damping is present in nature which is non viscous damping, this is mainly because of fluid resistance when a body moving in fluid. Now, we, in this subject we will discuss the viscous damping, then we will study coulomb damping also and at the end we will see what is, we will discuss internal damping.

Now, viscous damping, so the damping force for this case if I will take it as Fd this is equal to the C times velocity, so velocity I can write it as d Z by dt or I can write it as C times Z dot, I am assuming the displacement of the mass in the vertical direction. So, what we can see viscous damping is proportional to the velocity. Now, what is C? C is coefficient of damping.

Next is coulomb damping, so coulomb damping we can see it is equal to mu times N where N is the normal force acting between the two sliding surfaces and mu is the frictional coefficient between these two sliding surfaces.

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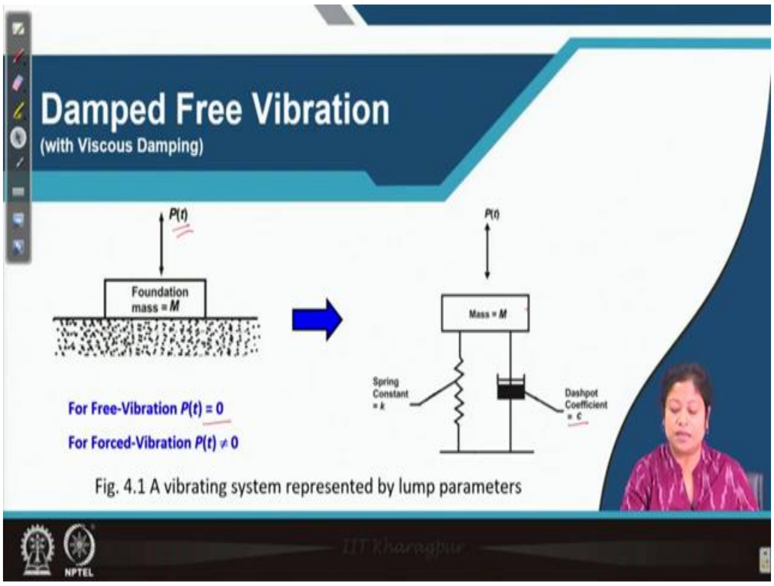

Now, let us see what will be the equation of motion for damped free vibration condition. So, in this figure we can see a foundation which is a block foundation, its mass is capital M is resting on the ground surface. Now, under free vibration condition the load P t will be equal to 0, in case of forced vibration P t will be non-zero. So, just for the sake of completeness I am just repeating what is free vibration which we have already studied.

So, free vibration system is possible when any system vibrated due to the inherent forces and in absent of the external forces that is the reason in this problem which we have taken here P t is equal to 0. Now, the problem can be represented by lump parameters K which is spring constant and dashboard coefficient C. That means the soil is represented by a spring and a dashboard coefficient, dashboard and M is mass of the foundation.

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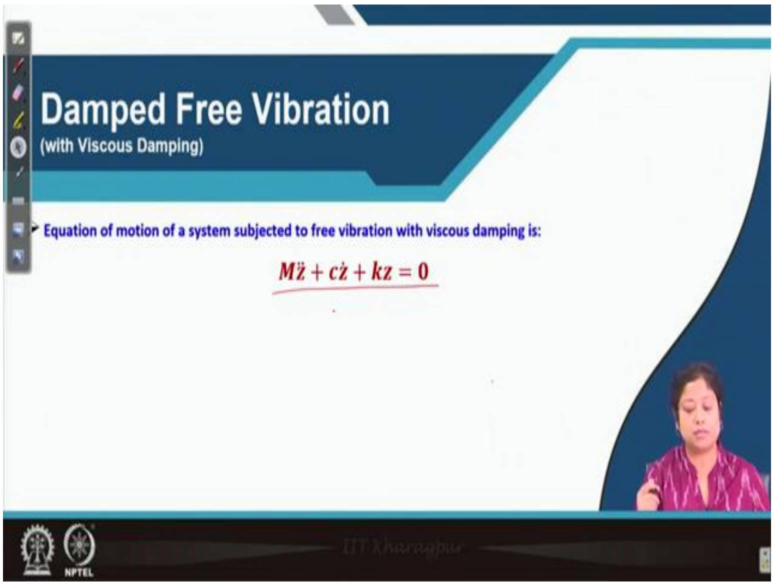

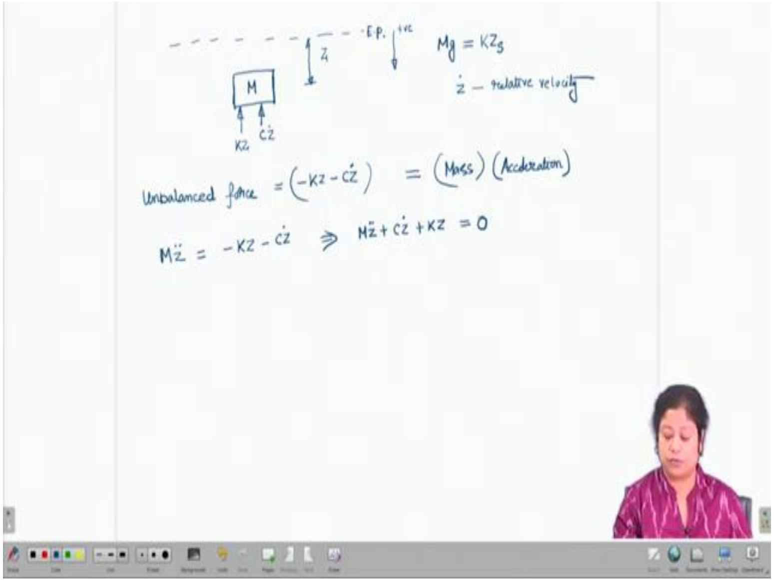

So, in this case we can derive the equation of motion of this system which is subjected to free vibration and connect it to viscous damping, let us see. So, first thing what we need to do is to draw the free body diagram of the mass M. So, this is our mass M, we have already seen that when external forces are not present, we can write M g is equal to K times of Z S, where Z S is the static displacement of the mass M under the action of the load M g.

Now, that is the reason in the free body diagram I will neither show M g nor K Z S, since this mass is moving in downward direction so I am taking the current position of the mass is at a distance Z from the equilibrium position under static condition. Now, what are the different forces other than this M g and K Z S are acting on this mass M.

When it is connected to the spring, spring exerts a force which is equal to K times Z, when it is connected to the dashboard also what will be happened at the two ends of the dashboard, it will be subjected to a kind of relative velocity which is equal to Z dot, so Z dot is the relative velocity in this case for the dashboard, two veins of the dashboard.

So, the force, damping force which is exerted by the dashboard is equal to C times Z dot. Then under, then what is the amount of the unbalanced force present in this system? Minus K Z which is the spring force minus C Z dot which is damping force. Why minus? Because in this case sin convention is Z is positive in downward direction and this unbalanced force should be equal to the mass times acceleration which is the inertia force.

So, we can write then M Z two dot which is inertia force is equal to minus of K Z minus of C Z dot, if we take all the parameters, all the forces on the left hand side then it will become M Z two dot plus C Z dot plus K Z is equal to 0. Why 0? Because we are considering freely vibrating system in this analysis.

Now, we can go back to the slide. So, here the final form you can see once M Z two dot plus C Z dot plus K Z is equal to 0 where C Z dot is damping force because of the viscous damping, K Z is the spring force because of the spring.

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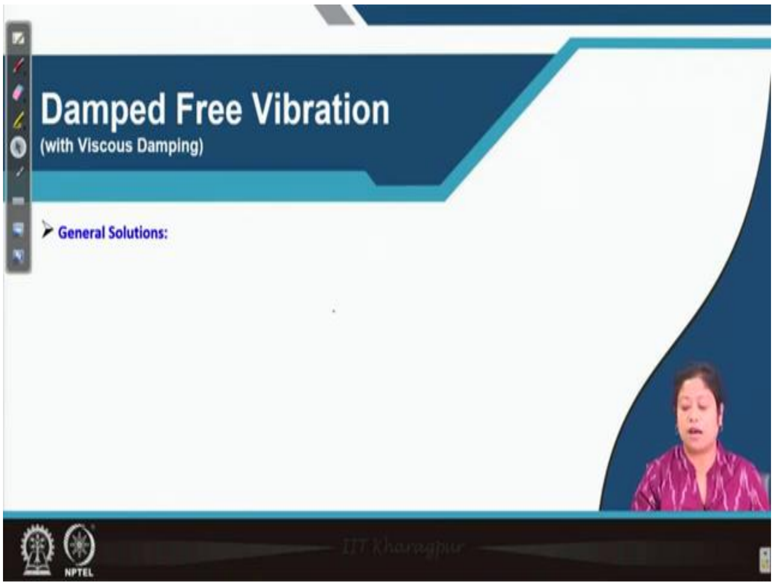

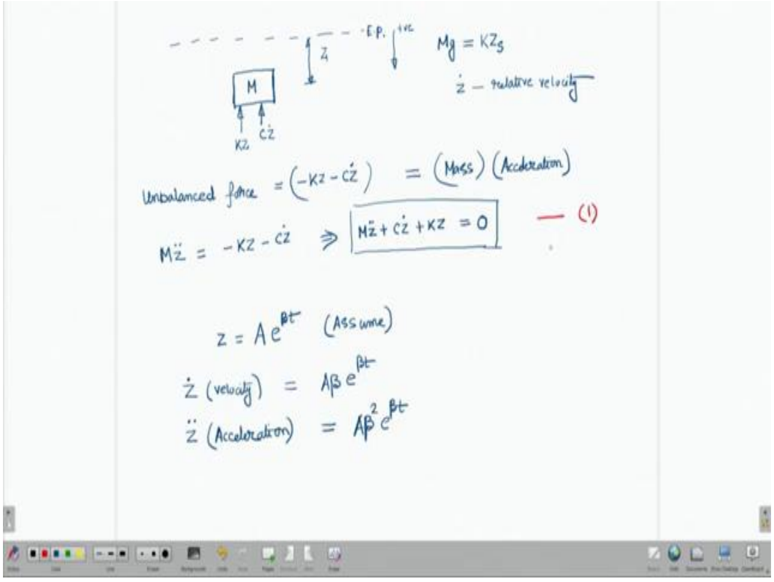

Now, we are interested to find out the general solution for this equation. For this purpose what do we do? We will assume that Z, it is better to write it as small z, z is equal to A times e to the power beta t, this is solution which we assume right now and this kind of trial solution we can take for differential, homogeneous differential equation having form this.

Now, from these trial solution what will be the value of the Z dot which is the velocity? Velocity will be A times beta times e to the power beta, likewise we can write Z two dot which is acceleration in this case will be equal to A times beta square e to the power beta t. Now, let us give this equation one number. Now, if we will write the value of Z, Z Dot and Z double dot to this equation 1 what we will get?

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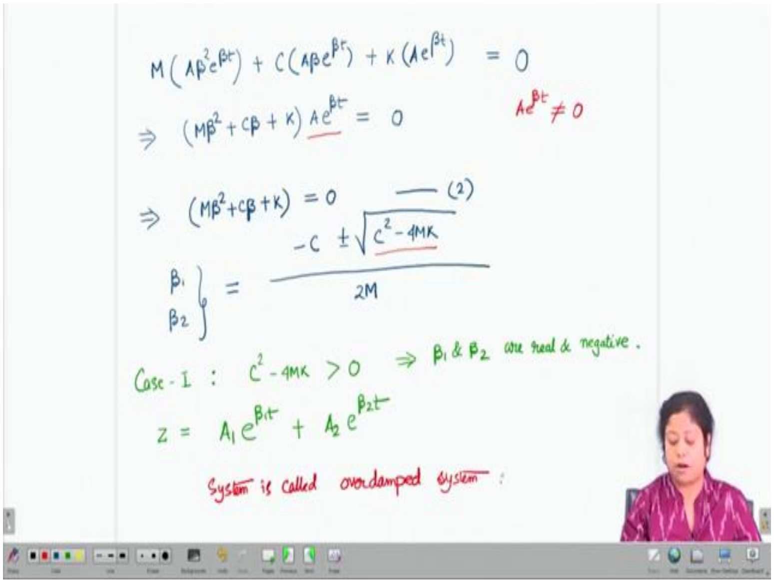

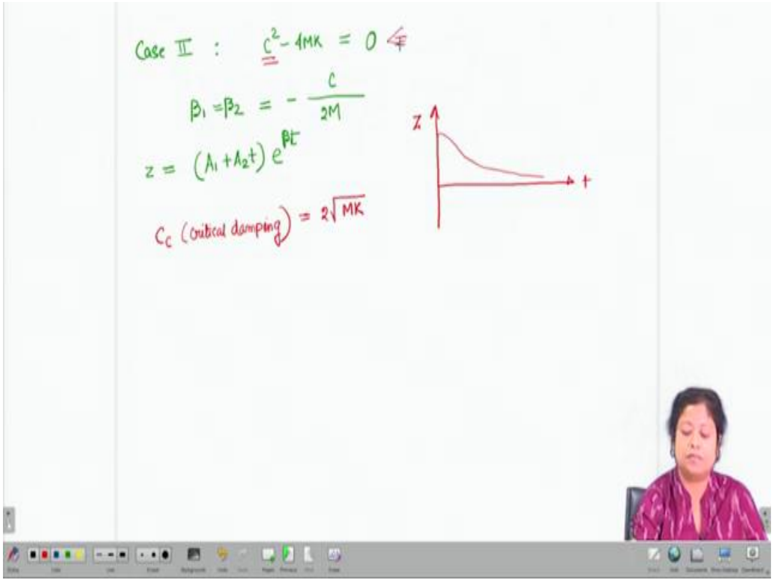

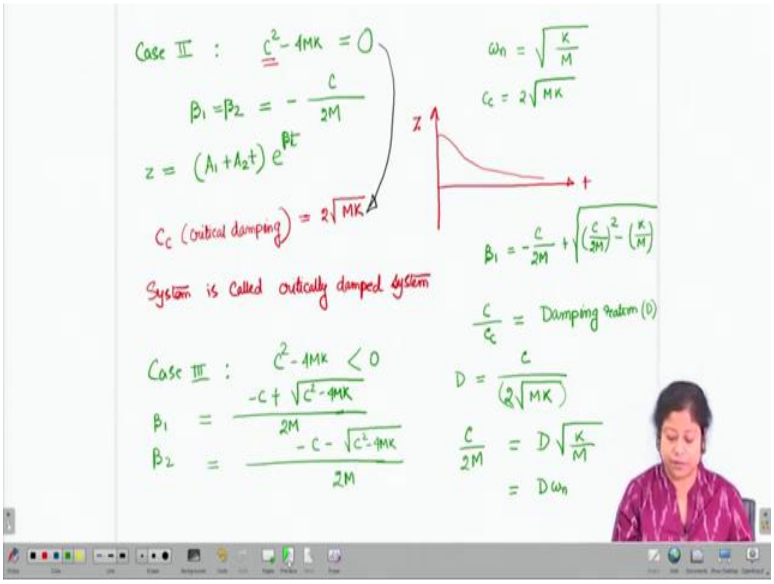

We will get M times Z double dot means A times beta square e to the power beta t plus C Z dot which is nothing but A times beta times e to the power beta t plus K Z that means K times A times e to the power beta t which is equal to 0. Now, it can be rewritten as M beta square plus C times beta plus K times, plus K whole thing can be multiplied by A times e to the power beta t and that should be equal to 0.

Now, this A times e to the power beta t is non zero quantity. So, which implies that whatever is written within the bracket should be 0. So, in this way now we can find out the two roots of the equation 2, it is a second order differential, it is a second order equation of beta, so beta has two roots. What are these two roots? Beta 1 and beta 2 together I am writing, this is minus C plus minus square root of C square minus 4 times M K divided by 2 M.

Now, you can see here depending upon the value of C square minus 4 times M K the nature of the roots of equation 2 may change. There are three possibilities in this case. So, let me write all these three cases. So, our case 1, which is first possibility is C square minus 4 M K is greater than 0 that means the quantity under square root is positive quantity.

Then what will be, what can we say about the two roots beta 1 and beta 2? Obviously then we can say beta 1 and beta 2 both are real and negative because there is a negative sign with C and obviously whatever we will get under this square root of C square minus 4 M K that will always be less than C. So, we can write from here that the two roots beta 1 and beta 2 are real and negative. Now, then what will be the form of this solution for case 1?

Then we can write Z is equal to A 1times e to the power minus beta 1 t plus A 2 times e to the power beta 2 t. Now, come to the next condition which is our case 2. What is case 2? There is second possibility that C square minus four times of M K is equal to 0. Then what will be happened? The quantity I can go back to the previous page, the quantity under square root will be 0 in this case.

As a consequence the two roots beta 1 and beta 2 will be same 2 minus C divided by 2 M. Therefore what will be the general solution for this case? Z is equal to A 1 plus A two times t times e to the power beta 1 or beta 2 or you can write beta t. Now, in this case we can introduce a new thing, what is that?

See in this case we have seen that see, if I will draw the response of the system let us take this is t or I can write omega t also and this is Z, then the response will look like something like this. So, what we can note here the damping finally because of this damping the system will not be subjected to periodic motion. So, finally we can say this damping is something which is maximum in magnitude, so this damping at this stage is called as critical damping which is represented by C C critical damping.

And that is equal to if you see here 2 times square root of M K which we can get from this or I can show it other way. So we are getting this from actually this relationship, so this damping is called critical damping. And the system is called critically damped system. I have forgot to mention one thing the previous case when C square is greater than 4 times of M K under that condition the system is called over damped.

Now there is the next case third case when we can see C square minus four times of M K will be less than 0. If C square minus 4 times of M K that means the quantity under root is less than 0 then what will be happened? The quantity will be a negative quantity under this square root.

So, we can write it as I am writing the general solution, sorry the roots of the equation 2, these are beta 1 and beta 2, so beta 1 will be equal to, here I will not write plus minus but I will write it as only plus or only minus, so let us take only plus, likewise for beta 2 I can write minus C minus square root of C square minus 4 M K divided by 2 M.

Now, we can rewrite these two roots in other form also, how, let me do that. So, for beta 1 what we can write? Minus C divided by 2 M plus square root of C by 2 M whole square minus K by M, we are already introduced to the term K by M, we are familiar with the term natural damping of the undamped system which is equal to square root of K by M. So, in this case K by M is omega n square.

Now, how we can write C divided by 2 M? We have already introduced, we have, we are already introduced to the term critical damping which is C C and that can be expressed as 2 times square root of capital M times K. Now, if we will find out the ratio of the damping of any system to the critical damping, this ratio is called damping ratio, damping ratio and symbolically it is represented by D.

So, now I can write then D is equal to C divided by C C, C C means 2 times of square root of M K. So, finally with this if capital D is equal to C divided by 2 times square root of M K then what I can write for C divided by 2 M? C divided by 2 M means we can write D times K, square root of K by M which is equal to D times omega n.

So, in the expression of beta 1 and beta 2 now we can write in place of C divided by 2 M the term D times omega n where D is the damping ratio and omega n is the natural frequency of undamped system. So, let us do this work.

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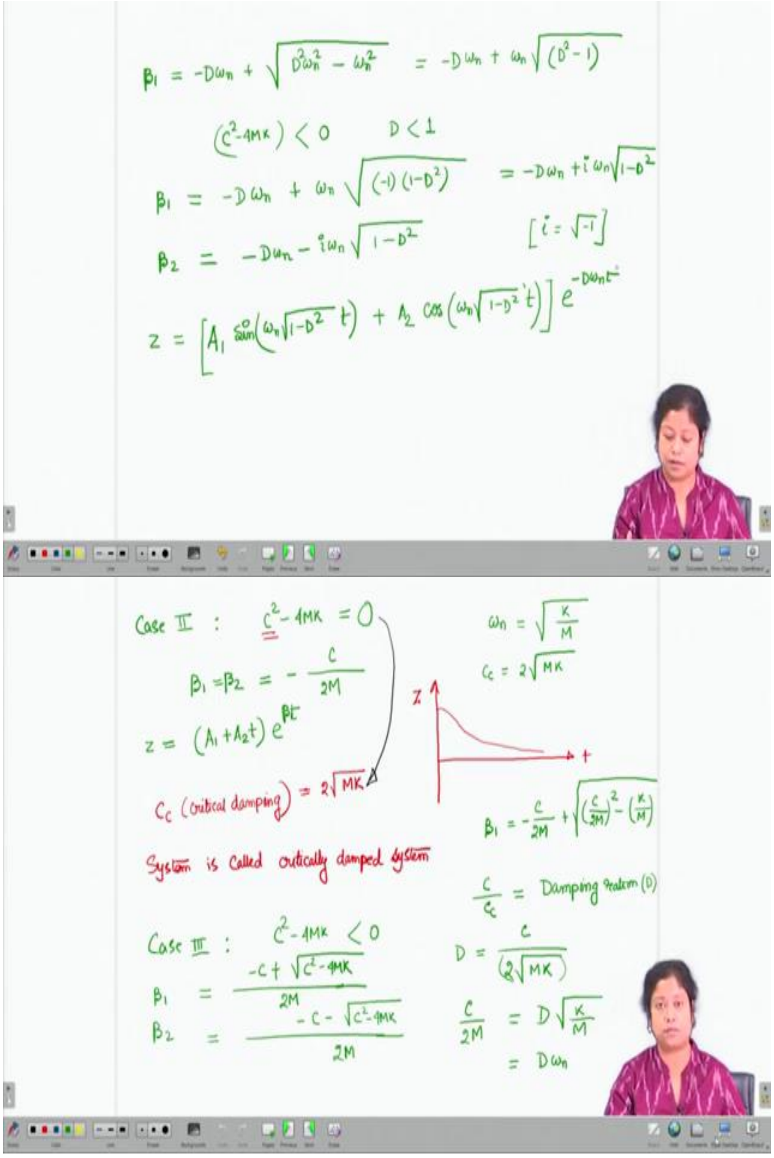

Then beta 1 is equal to, sorry beta 1 is equal to minus D omega n plus square root of D square omega n square minus omega n square or I can write it as D times omega n plus I can take omega n outside the square root and it will be D square minus 1. Now, under underdamped condition when C square minus 4 M K this is less than 0 that means negative that time obviously D is less than 1.

So, because of that reason I can write beta 1 once again as 1 minus D square times minus 1 or minus D omega n plus i omega n times square root of 1 minus D square likewise beta 2 will be minus D times omega n minus i times omega n times square root of 1 minus D square. So, then what will be the general solution for this type of problem?

Z is in this case I can write it as A 1 sin of this imaginary, coefficient of imaginary part I need to write here, so sin of omega n times 1 minus D square t plus A 2 times cosine of omega n times 1 minus D square t and the thing which is written within bracket should be multiplied by e to the power minus D omega n t.

So, here we have studied, in today's lecture what we have studied? We have studied how to get the general equation for free vibrating system with viscous damping then what are the, what are the possible solutions for that kind of system.

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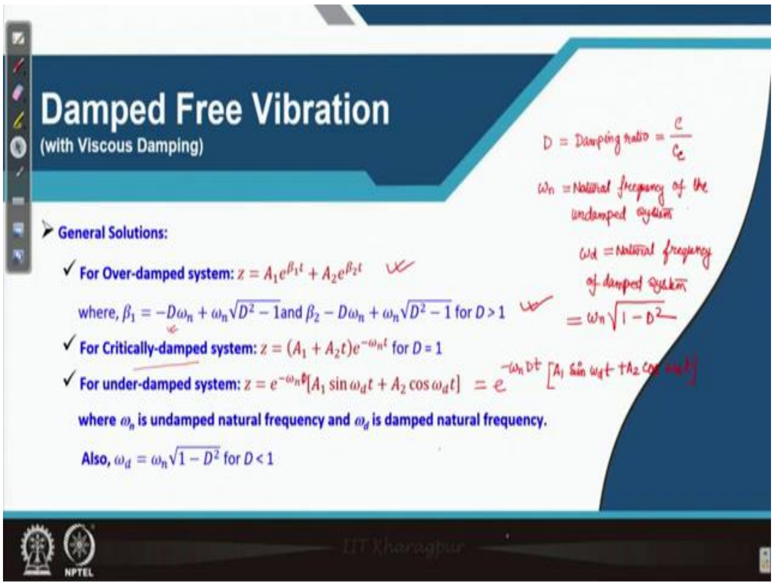

And I can show now what we have done here. So, the general solutions for the three cases which are discussed today can be summarized here, for the over damped system we get Z is equal to A 1 times e to the power beta 1 t plus A 2 times e to the power of beta 2 t. We can write beta 1 and beta 2 in terms of D which is damping ratio and the natural frequency of the undamped system which is omega n.

So, here D is damping ratio which is the ratio of the coefficient of damper to the critical damping and omega n is the natural frequency of the undamped system. Now, for critically damped system the general solution can be written as A 1 plus A 2 times t everything within bracket being multiplied by e to the power minus omega n t. In this case actually D is equal to 1 that is the reason when writing e to the power minus omega n t D is not written.

Third case is under-damped system and the general solution for under-damped system is Z equals to e to the power omega n, one D is missing here, so let me write it, so there will be a

D also, so I can write once again e to the power omega n D t times A 1 sin of omega D t plus A 2 times cosine of omega D t. Now, in this case omega D is called the natural frequency of the damped system which is equal to omega n times square root of 1 minus D square.

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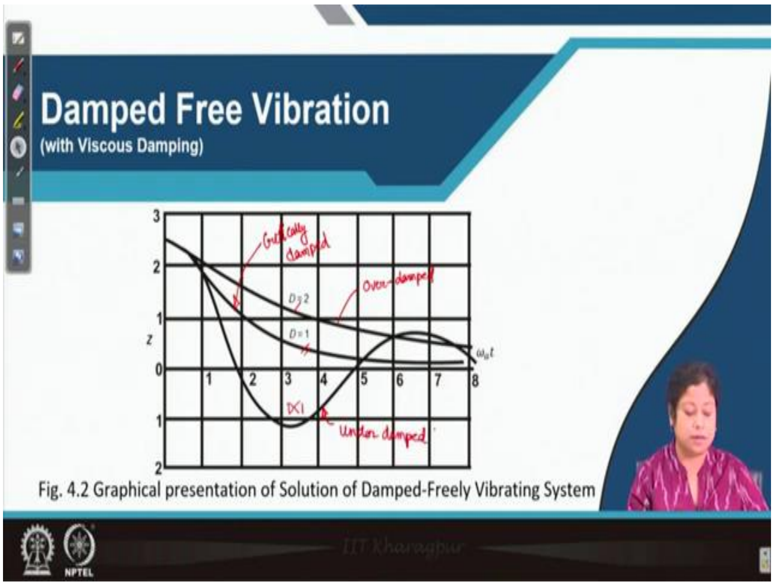

You can see here the graphical presentation of the solution for damped freely vibrating system for three different cases, this one for D when D is equal to D is less than 1, this curve for D is equal to 1 and the third curve is for D equals to 2. That means this one for over damped, middle one is for critically damped and the third one is for under damped system.

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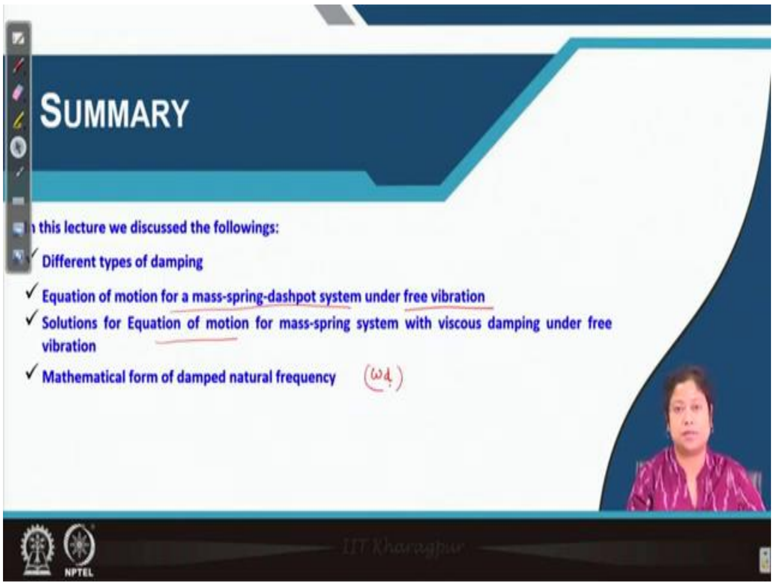

So, finally come to the summary. Today we have discussed different types of damping, say viscous damping, coulomb damping, material damping and non-viscous damping. Then we discussed different, then we discussed about the equation of motion how to form the equation of motion for a mass spring dashpot system under free vibration. And then we determine the solution of equation of motion. Finally we have seen what will, what is the mathematical form of damped natural frequency which is omega D. Thank you.