Soil Dynamics
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Lecture 4
Single Degree of Freedom System (SDOF) Part 3

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Hello friends, today, we will continue our last class.

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Last class, we have discussed about general solution of the damped system.

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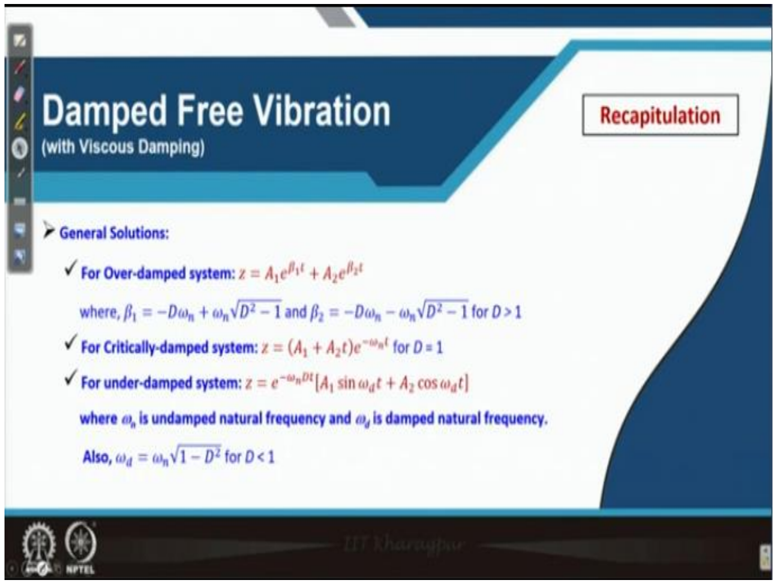

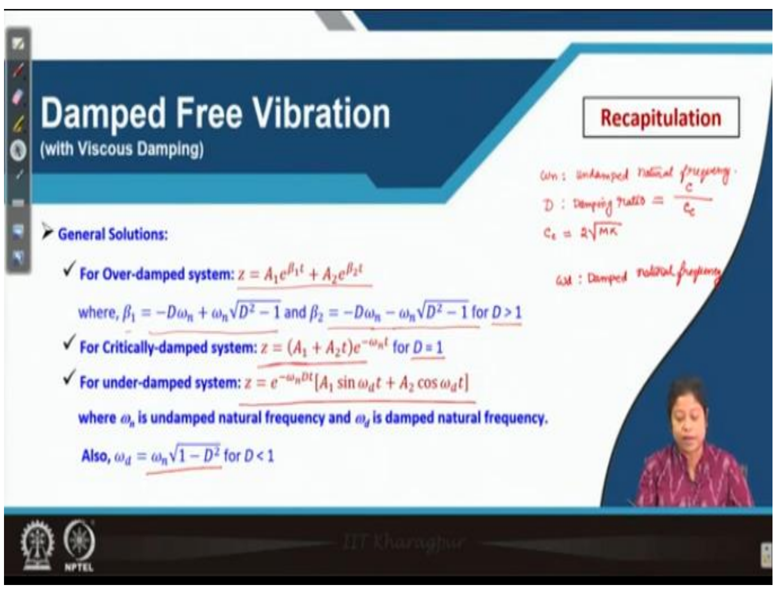

We have seen three types of damped system with viscous damping. First one was the over-damped system and the general solution for over-damped system is expressed by z is equal to A1 times e to the power beta 1t plus A2 times e to the power beta 2t. What was beta 1 and beta 2? Beta 1 was we have seen beta 1 was minus D1 times omega n plus omega n times square root of D square minus 1. In this case D is greater than 1 that is the reason I have written here as square root of D square minus 1.

At the second root was beta 2 which is minus D times omega n, minus omega n times square root of D square minus 1. So, what is omega n and D here? Omega n is natural or I can write it as undamped natural frequency, so omega n is undamped natural frequency of the system, d is the damping ratio or damping factor which is expressed by the ratio of the coefficient of damping to the critical damping and critical damping we can calculate by 2 times square root of mass times (speed) stiffness. What is D (er) here? That is now clear.

Next to us critically damped system that we have discussed in the last class, so the general solution for critically damped system was found out Z is equal to A1 plus A2 times t whole thing under bracket is multiplying by the omega e to the power minus omega n times t. In this case since D is equal to 1 that is the reason here when writing e to the power minus omega n times t I have not written D because D is equal to 1 here.

The third case was under-damped system and the general solution for under-damped system is Z which can be written by e to the power minus omega n Dt times A1 sine omega dt plus cosine of omega dt. What is omega d here? Omega d is damped natural frequency, and these damped natural frequency can be calculated by undamped natural frequency omega n times square root of 1 minus d square. In this case, d is less than 1 that is the reason I have written here 1 minus d square instead of writing d square minus 1 in case of over-damped system.

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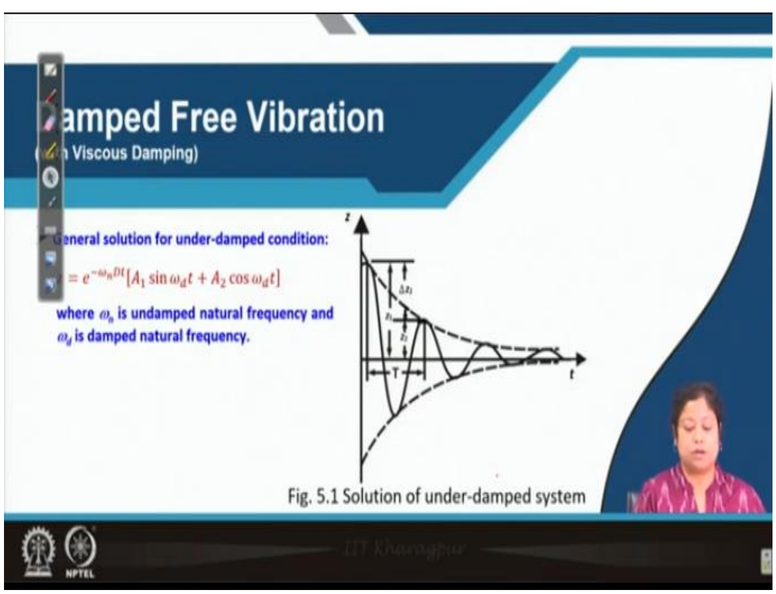

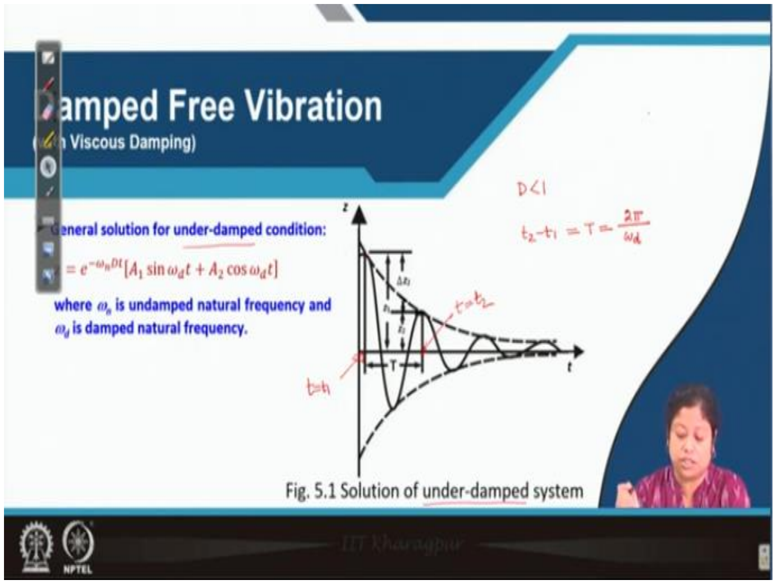

So, now, today we will focus on the under-damped condition that means, damping ratio is less than 1 here you can see the general solution for under-damped system. What we can see over the time the amplitude of vibration reduces, let us take this amplitude that means the first peak in this diagram that is z1 at time t equals to t1 likewise the second peak which is your z2 (that is nothing) that is occurred when t equals to t2.

Another interesting thing which we can note here is the difference between time t2 and time t1 that means t2 minus t1 is the time period for this type of system which is equal to 2 pi divided by omega d, omega d is nothing but the natural frequency of the damped system.

So, let us do some exercise now, what will be the expression for z1 and z2 from these that we will try to find out. So, now, we will see the expression for the z1 and z2 in this under-damped condition. So, let us go to the whiteboard. So, now, we will see (how we can write the magnitude?) how we can write the expression for z one and z two? So, z1 is the vibration at time t equals to t1 and z2 is the response at time t equals to t2.

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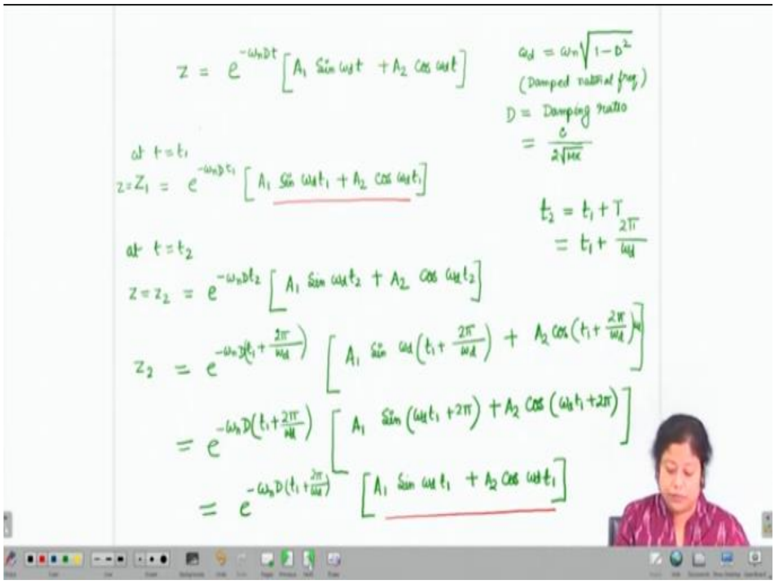

So, let us see we have the expression z is equal to e to the power minus omega n Dt times A1 sine of omega dt plus A2 times cosine of omega dt. Now, here what is omega d? And what is d? omega d is omega n times square root of 1 minus D square which is called as damped natural frequency likewise, D is called as damping ratio which is c divided by 2 times of capital MK, how it is coming that we have already seen in the last class.

Now, the Z1 will be at t equals to t1, z is equal to Z1 and that can be written as e to the power minus omega n D t1 times A1 sine of omega dt 1 plus A2 times cosine of omega dt 1. Now, at time t equals to t2 displacement Z is equal to Z2 and that can be written as e to the power minus omega in DT 2 times A1 times sine omega dt 2 plus A2 times cosine omega dt 2. Now, here we know the relationship between t1 and t2 what is it?

We know t2 is equal to t1 plus capital T, capital T is the time period. So, under under-damped condition capital T is 2 pi divided by omega d. Therefore, Z2 can be rewritten as e to the power minus omega D in place of t2. Now, we can write t1 plus 2 pi divided by omega d this thing will be multiplied by A1 sine of omega d in place of t2 now we can write t1 plus 2 pi by omega d.

So, t1 plus 2 pi by omega d plus A2 times cosine of t1 plus 2 pi by omega d, we can rewrite this expression now, as first term will remain as it is. Now, the second term within bracket can be written as A1 sine of omega dt 1 plus 2 pi plus A2 times cosine of in the previous expression one omega d is missed,, so please write it here. So, now, here we can write A2 times cosine of omega dt 1 plus 2 pi.

So, now, we know sine of 2 pi plus theta is nothing but sine theta likewise cosine of 2 pi plus theta is also cosine theta. So, here then we can write e to the power minus omega in nD t1 plus 2 pi by omega d times A1 sine omega dt 1 plus A2 times cosine of omega dt 1 please note here that the term within bracket is same as the term within bracket for Z1.

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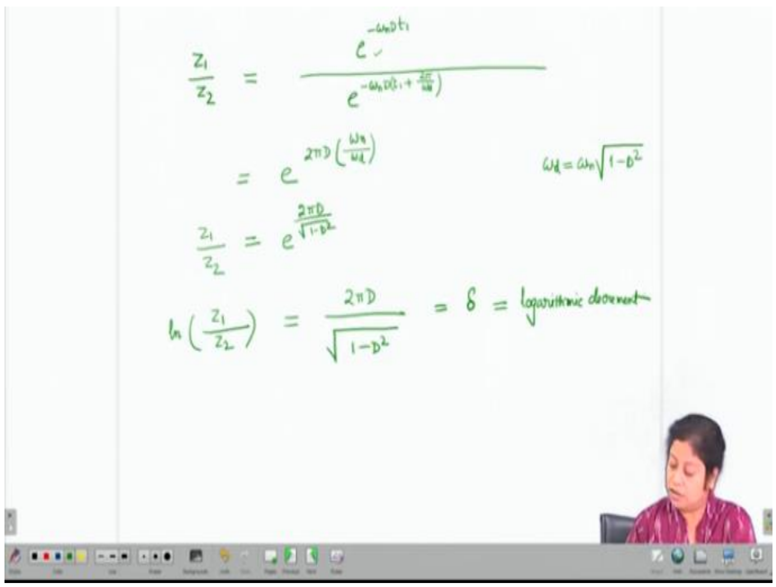

Then we can write the ratio of Z1 by Z2 as e to the power minus omega nDt1 since the term within bracket which is multiplied with this term for both numerator and denominator same,, so I am not writing that term and that also is non-zero term. So, I can write it here as numerator already I have written and in the denominator, I can write e to the power minus omega nDt 1 plus 2 pi by omega d finally, what we can write from this?

We can write from these 2 pi D omega n times omega d. Now, we know omega d is equal to omega sorry! Now, we know omega n or omega d actually omega d is equal to omega n times square root of 1 minus D square. So, omega n by omega d will be one divided by square root of 1 minus D square. So, we can write then Z1 divided by Z2 is equal to 2 pi D divided by square root of 1 minus D square.

Now, if we will take log of Z1 divided by Z2, it will be 2 pi capital D divided by 1 minus D square and these sometimes called as logarithmic decrement it is called as logarithmic decrement.

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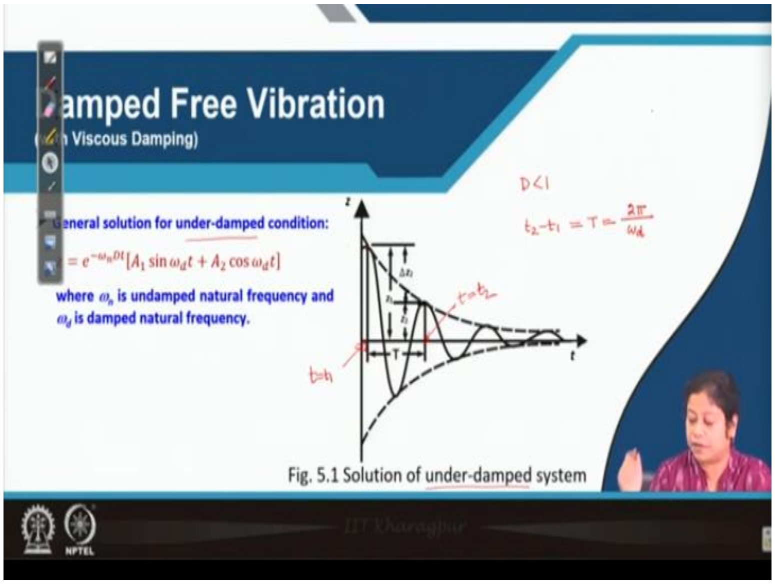

Now, another interesting thing we can see in this figure,, so here Z1 and Z2 are two consecutive amplitudes what will be happen, if instead of taking two consecutive peaks, we are taking second peak after n number of cycles, let us see.

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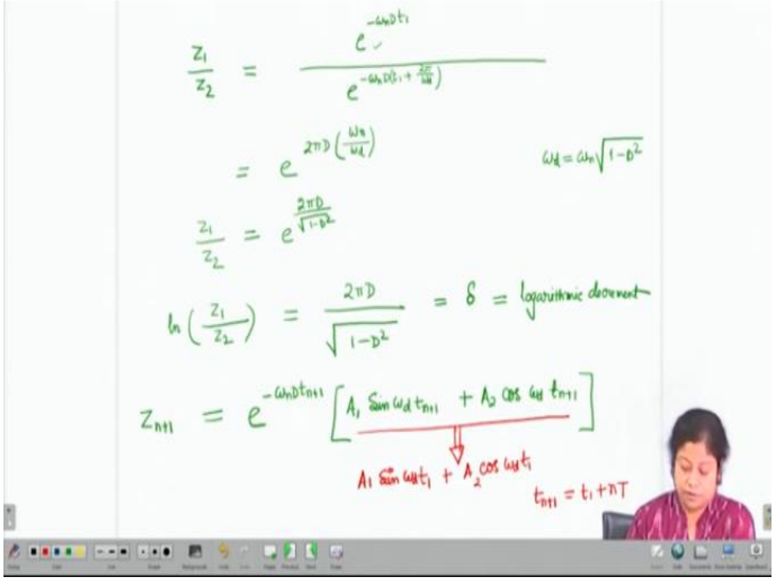

So, in place of Z2, now, we will write the term Z n plus 1 because, we are now interested for the amplitude of peak after a number of cycles. So, what will be our equation? e to the power minus omega n D times t times at n plus 1 (it) cycle then rest of the thing within bracket can be written like sine omega dtn plus 1 plus A2 cosine of omega dtn plus 1 and these term within bracket can be rewritten as…

I am just writing the term within bracket as again the same way A1 plus sine omega dt 1 because in this case the what is the relationship between t one and tn plus one? In this case tn plus 1 is equal to t1 plus n times of capital T, so we can write it this way only for the term within bracket.

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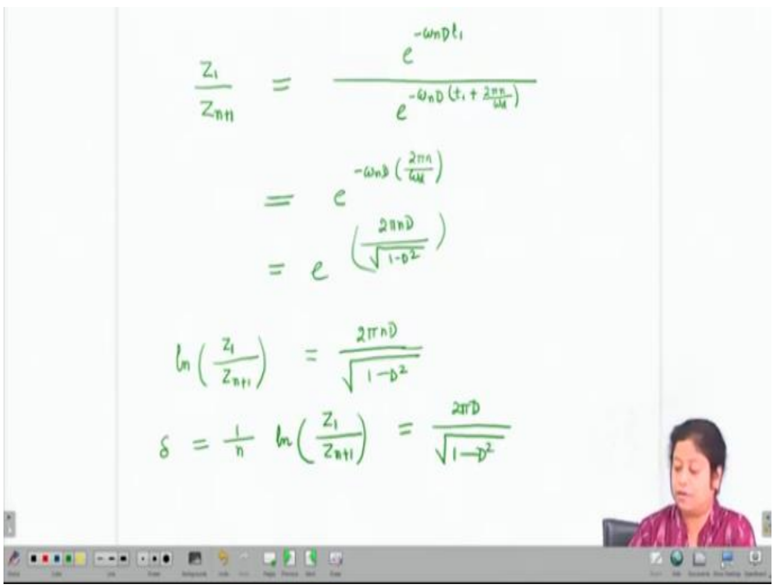

Then what we can write for Z1 divided by Zn plus 1 numerator it will be e to the power omega n d t 1 whereas, in the denominator we can write e to the power minus omega n times d times t1 plus n times capital T or we can write it as 2 pi N divided by omega d. So, finally, what we are getting is e to the power sorry! I am making mistake in writing.

So, finally, we can write it as e to the power minus omega nD and 2 pi n divided by omega d or we can write it also as e to the power minus 2 pi n capital D divided by square root of 1 minus D square this is as power of e with the negative sign.

Then what will be log of Z1 divided by Zn plus 1? It will be sorry! Here there will be no negative sign. So, the next step log of Z1 divided by Zn plus 1 can be written as 2 pi n capital D divided by 1 minus D square. So, in this case if we are interested to find out delta which is the logarithmic decrement then delta is equal to 1 upon n times log of Z1 divided by Zn plus 1 which is 2 pi D divided by 1 minus D square.

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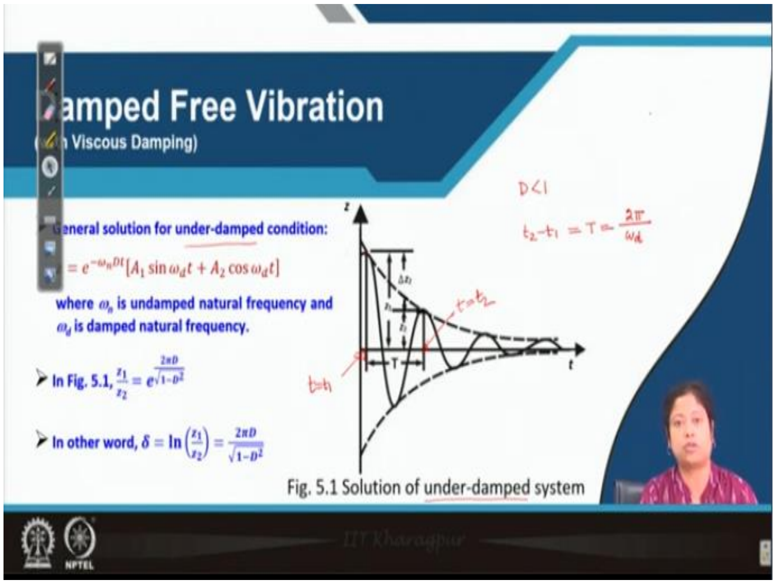

So, now here we can see what I say that is written in short what is the logarithmic decrement delta? And what is the ratio of two consecutive peak, amplitudes of two consecutive peak?

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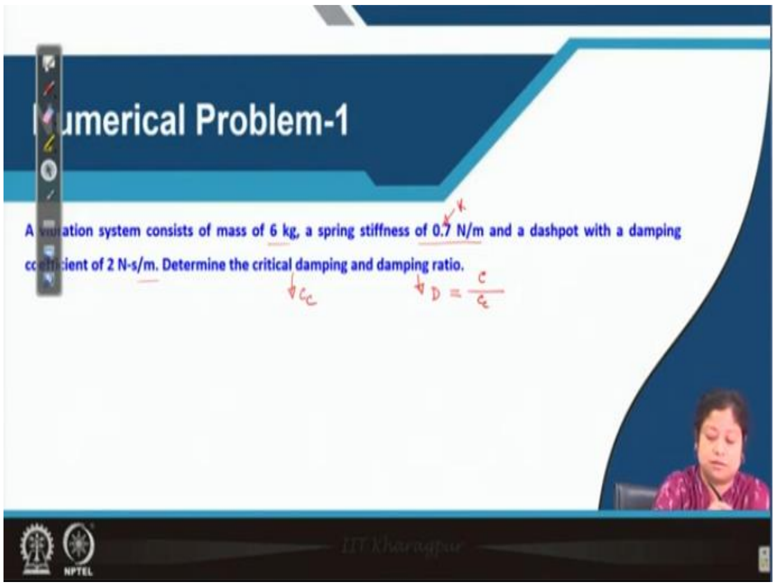

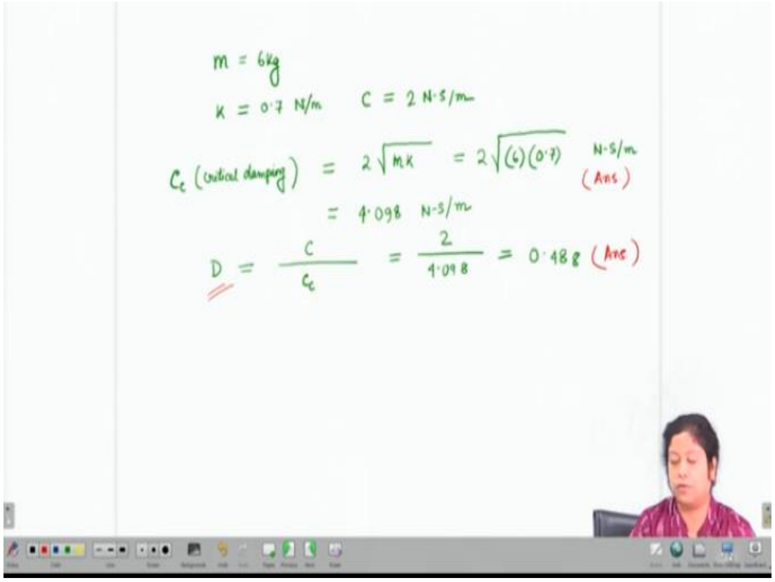

Now, with this we can take one numerical problem to clear our understanding on under-damped condition and how to calculate logarithmic decrement damped natural frequency of a freely vibrating system with viscous damping etc. So, let us take this numerical problem. So, in this problem it is said that (vibration) a vibration system consists of mass of 6 kg sorry! Mass of 6 kg we can use the pen, a spring having stiffness 0.9 in Newton per meter and dashpot with a damping coefficient to Newton second per meter.

We are asked to determine the critical damping and damping ratio. So, critical damping means CC and damping ratio means capital D which is the ratio of C to CC. So, let us solve this problem, mass is 6 kg, K is this is K which is 0.7 Newton per meter and damping is 2 Newton second per meter, mass is 6 kg, then stiffness is 0.7 Newton per meter and damping coefficient of damping is how much 2 Newton second per meter.

We are asked to determine CC and capital D. So, critical damping means Sorry! Critical damping is CC not this one I am just rewriting the thing once again mass is 6 kg, then K is 0.7 Newton per meter and coefficient of damping is I think it was 2 Newton second per meter. So, CC which is the critical damping that is equal to 2 times of square root of capital in this case it is small m that means mass times stiffness.

So, mass is how much in this case? It is 6 kg unit is in SI system, K is 0.7 Newton per meter its unit is also in SI system so the unit for critical damping in this case will be in SI system which is

Newton second per meter. So, let us see what will be the final answer for this question? So, let me calculate it 2 times square root of 0.7 times 6, so it is coming something 4.098 Or we can take 9 also Newton second per meter this is critical damping.

Then damping ratio will be the ratio of the coefficient of damping to the critical damping. So, it will be 2 divided by 4.098 therefore, the final value of these is let me calculate 2 divided by 4.098 which is coming 0.488. So, the final answer for this problem is, this is answer one that critical damping and the second one is second answer which is for the damping ratio.

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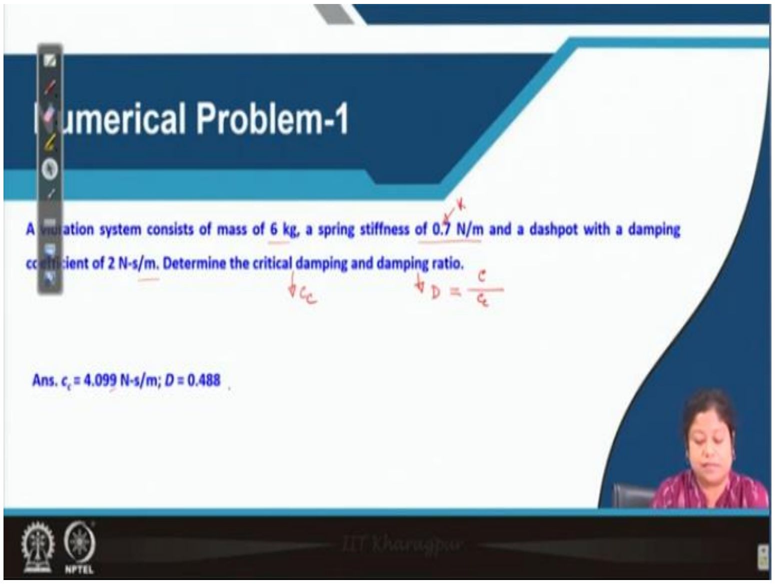

So, here I have written the final answer of these numerical problem, you can see CC is 4.099 actually the value which I got was 4.0987, so they have done the approximation at three decimal place. So, 4.099 Newton second per meter and capital D is 0.488.

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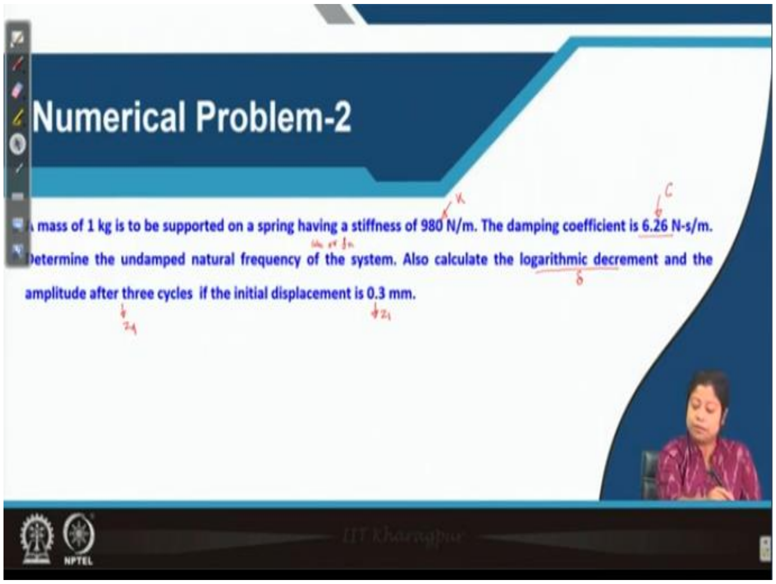

Now, let us see another numerical problem. So, in this numerical problem, it is said that a mass of 1 kg is to be supported on a spring having a stiffness 980 Newton per meter. The damping coefficient is also given which is 6.26 Newton second per meter. So, damping coefficient means it is C and stiffness means it is K. So, we are asked to determine the undamped natural frequency of the system.

So, we can calculate either omega a or fn or also we are asked to calculate the logarithmic decrement which is delta here and the amplitude up after three cycles if the initial displacement is 0.3 millimeter,, so initial displacement I can write as Z0. So, we are asked to find out it for Z3 I can write it this way or I can write it is as Z1 and it is as Z4 I can write it as Z1 and this one as Z4.

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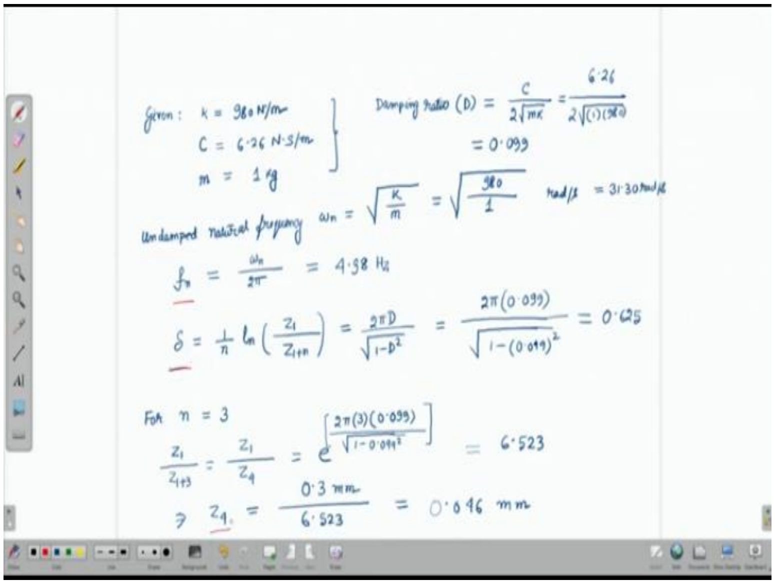

So, let us solve this numerical problem then, first I am writing what is given to us, given data K is given which is 980 Newton per meter, also coefficient of damping which is c is given that is 6.26 in Newton second per meter, the mass of the system m is 1 kg. Now, first thing we need to calculate net undamped natural frequency. So, undamped natural frequency here is that is omega n is equal to square root of K by m.

So, K is 980 divided by 1 that is in a radian per second. So, how much it is coming? Let me see, it is coming 31.30 in radian per second. So, we can calculate fn also which is omega n by 2 pi, omega n means here circular frequency,, so this divided by 2 pi that is coming 4.98 in hertz. The next thing which is asked is logarithmic decrement. So, logarithmic decrement is delta that can be written as 1 upon n times natural log of Z1 divided by Z1 plus n.

And we have already seen that, that is equal to 2 pi D divided by square root of 1 minus D square. Now, here D is unknown,, so we need to first calculate D. So, the data which is given to us from that we can calculate damping ratio D that is equal to C divided by 2 times mK. So, C is (9) sorry! 6.26 divided by two times square root of mK means this quantity and it is a unique place number, so let me calculate,, so it is coming 0.099 this is the damping ratio.

Then we can find out logarithmic decrement here 2 pi 0.099 divided by 1 minus 0.099 square which is coming 2 pi times 0.099 divided by square root of 1 minus 0.099 square,, so we are getting 0.625 this is logarithmic decrement. Now, in the next step we need to calculate the Z1 plus n,, so in this case n is 3. So, Z1 divided by Z1 plus 3 or you can write it as Z1 divided by Z4 is equal to e to the power 2 pi n is 3 here,, so 2 pi nd, d is 0.099 here divided by square root of 1 minus 0.099 square.

So, how much will be this? This is nothing but let me calculate first e to the power 2 pi n (divide) into 0.099 divided by square root of 1 minus 0.099 square, so we are getting 6.522 or 523. Already Z1 was given I think yeah Z1 is 0.3 in millimeter,, so Z4 is Z1 which is 0.3 in millimeter divided by 6.523 which is coming 0.3 divided by 6.523 it is comings just one minute, so 0.3 divided by this is coming.

Let me check once again whether I have calculated this part correctly or not, then it will be easy for me to proceed further. I think it is calculated correctly 6.523 I am just taking and it is coming 0.45 sorry! 0.045. There is another term here which is coming in my case more than five so I am writing it approximately, I am writing it as 0 sorry! 0.046 in millimeter. So, in this way we can calculate first the undamped natural frequency then logarithmic decrement then the Z4.

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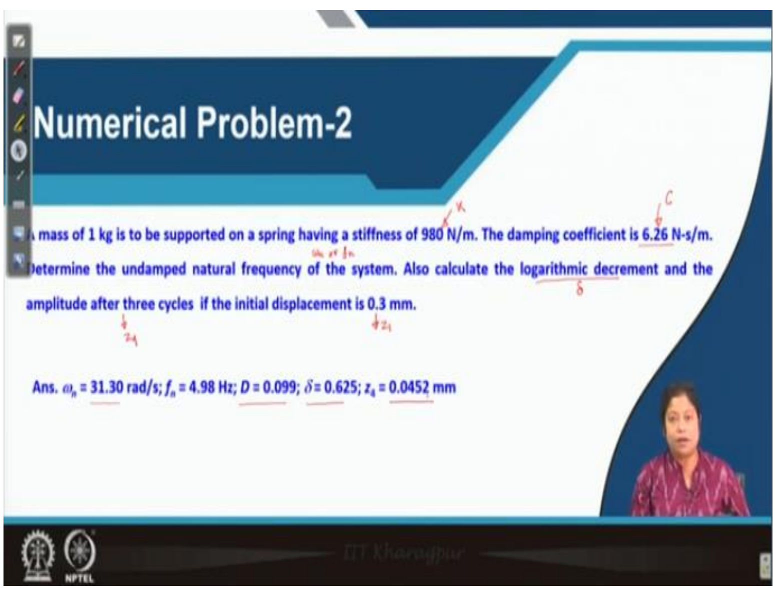

Now let us see what is the final solution for this problem? So, final solution is given omega n which is undamped natural frequency 31. 30 radian per second or fn 4.98 hertz, it matches to our solution. You can see damping ratio also matches to our solution. Logarithmic decrement also matches only Z4 very small difference, it depends up to which decimal place we have approximated our result depending upon that this much difference only we can see. Alright!