Soil Dynamics
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Lecture 50
Analysis of Machine Foundations
(For Rotary Machines - Part III) (Contd.)

Hello everyone, welcome to the course, Soil Dynamics.

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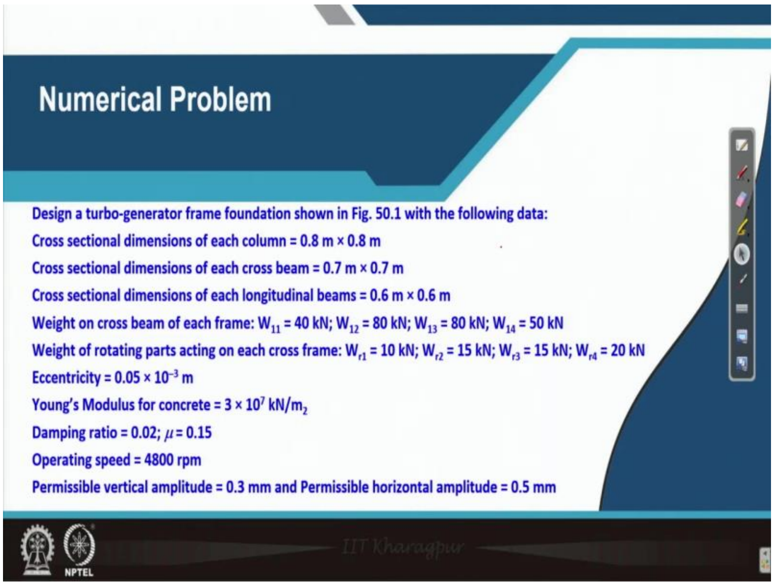

So, last class we were discussing a numerical problem, you can see here the same numerical problem which we have started to solve.

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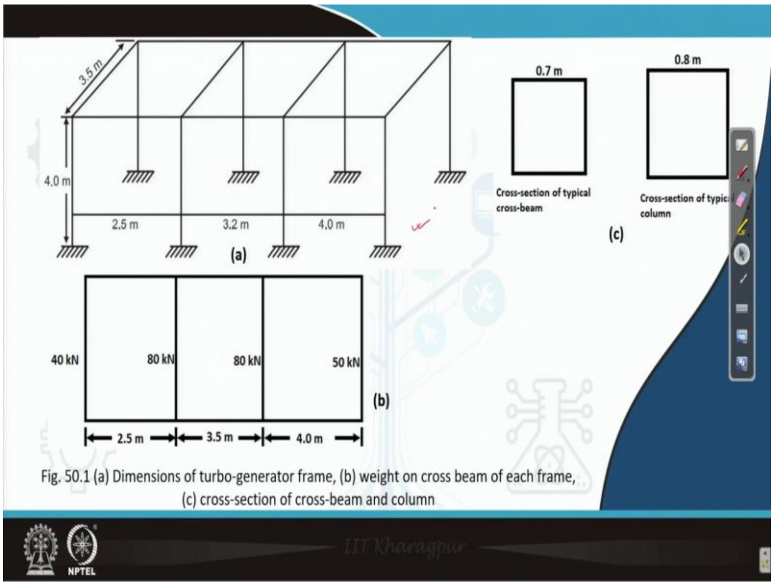

Fig. 50.1 (a) Dimensions of turbo-generator frame, (b) weight on cross beam of each frame, (c) cross-section of cross-beam and column

So, in this numerical problem here you can see the diagram also, we are interested to solve this frame foundation. This frame foundation is for a turbo generator. So, what we have done last class, we have calculated the average natural frequency of the system when it is subjected to vertical vibration.

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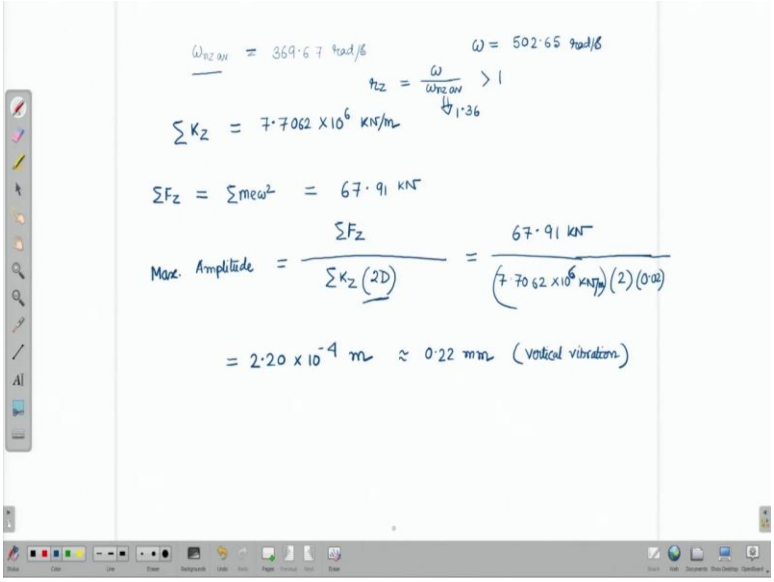

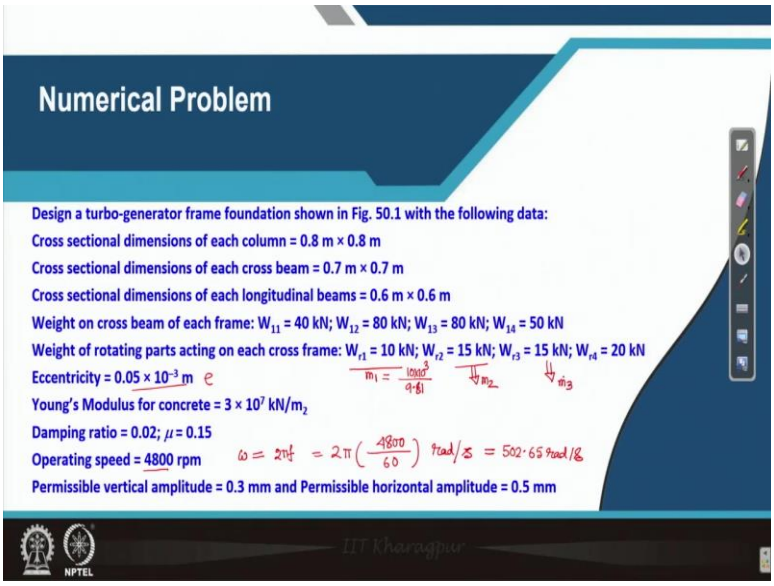

So, I am using the data which we have already discussed in last class. So, we what we got from our discussion first I am writing that part. So, we got omega n z average, it is the average natural frequency average means first we have calculated the natural frequency for the 4 frames frame. If you see this figure, so, this is frame 1, 2, 3 and 4. So, for these 4 different frames we have first calculated the natural frequency, then we have averaged that natural frequency and got the value which was 369.67 radian per second.

Now, today what we will do we will find out the amplitudes of vibration for vertical vibration and then we will find out the natural frequency and the amplitude of horizontal vibration using the same method, that means, resonance method.

So, if you recall the values which we got last class, from that first what we need to do, we need to calculate the summation of k z. That means, we have k z value for the 4 different frames frame number 1 I can write it 1, 2, 3 and 4. So, what we will do we will sum up these four k z values and then what you will get I am directly writing here it will it is 7.7062 into 10 to the power 6, unit is kilo newton per meter.

Now, we need to also find out the sum of the vertical forces or I can say the sum of the magnitudes or amplitudes of the vertical forces. So, summation of f z we also call it as horizontal unbalanced sorry vertical unbalanced force. So, that is equal to summation of M e omega square. Now, if you see the data which provided to us here, we know the mass M for each cross frame, also we know the value of omega this is actually f. So, from f we can calculate omega, we know omega is equal to 2 pi f provided f is in cycle per second.

So, in this case we can calculate it as this way and from this what we will get is the 502.65 radian per second. Also we know the value for m1 from this we can calculate m1 that is 10 times 10 to the power 3 divided by 9.81. Now, this is e, from this we can calculate actually m2, from this we can calculate M3 and so on. So, if we will do this exercise, then finally, what we will get that is important here. So, we will get actually I am writing the value directly you can check it.

So, when we will for 4 different frames we will get f z value and that when we will sum it the final value is 67.91 in kilo Newton. Now, we need to calculate the maximum amplitude, maximum amplitude of vibration for in vertical direction. So, how we can get it? We can get it using this equation. Now, here what we can note if you see I have taken 2 D instead of writing square root of 1 minus r z square whole square minus 2 D times r z whole square, where r z is the frequency ratio, why I have done so?

If you see here, already we have omega n z average, also we have calculated omega which is 502.65 in radiant per second. So, r z which is frequency ratio if you find it out, it is omega divided by omega n z average and that is greater than 1, not equal to 1 or very close to 1. If you see it is coming I can tell you the value, 502.65 divided by 369.67, that means, it is actually this ratio is 1.36 which is reasonably higher than 1. So, that is the reason we will, so, basically in this case we can see the under tuned system that is the reason we will take directly here 2 D instead of taking a big expression.

So, what we can get from this? Let me write here, this is in kilo Newton, summation of k z is 7.7062 into 10 to the power 6 in kilo Newton this is 10 to the power 6 times 2 times D. So, here you can see damping ratio is 0.02. So, it has unit kilo Newton per meter. So, finally, the amplitude which we will get that is in meter then and it is 2.20 into 10 to the power minus 4 in meter or we can write it 0.22 in millimeter. So, this is our final answer for maximum amplitude for vertical vibration.

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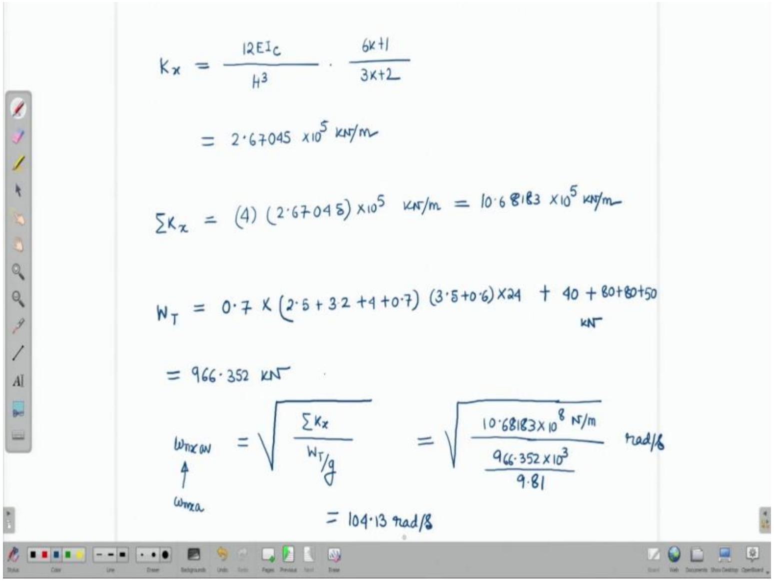

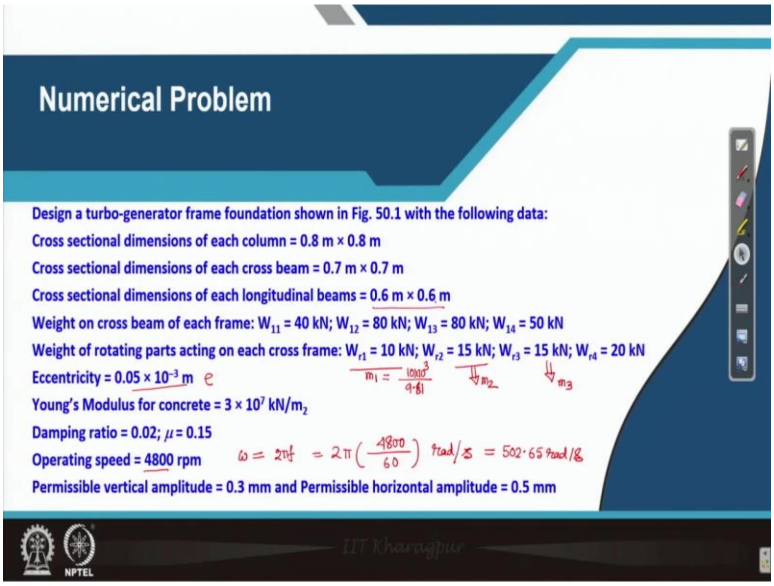

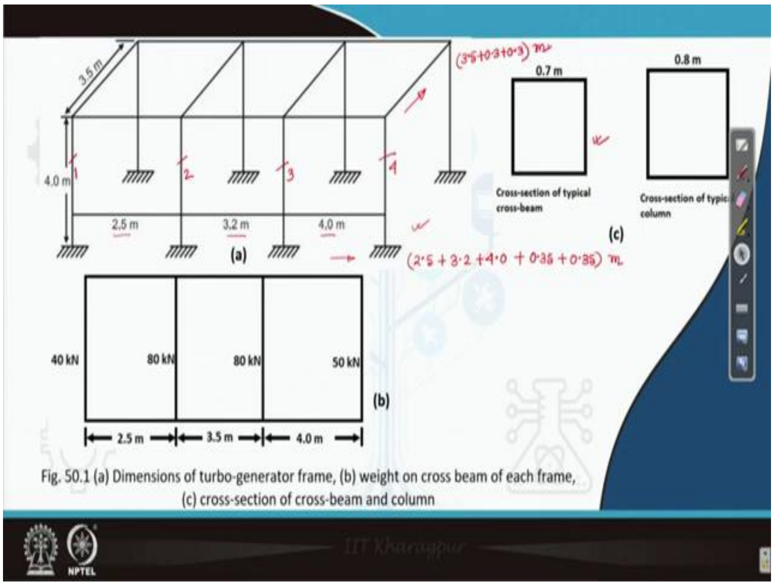

Fig. 50.1 (a) Dimensions of turbo-generator frame, (b) weight on cross beam of each frame, (c) cross-section of cross-beam and column

Now the next step is to solve the problem for horizontal vibration. So, go to the next page. So, first thing which we will do here we will calculate k x which is the stiffness coefficient for horizontal vibration. So, k x we can calculate by using this equation 12 E times I c divided by h cube times 6 k plus 1 divided by 3 k plus 2. So, here already last class we know the values for E, E is already given actually that is 3 into 10 to the power 7 in kilo Newton per square meter. I c is moment of inertia of the column which is coming 0.03413 in meter to the power 4, h is 3.867 that we have calculated in last class.

Now, k, k is equal to 0.677. So, using these values, I am writing it directly. I have already told the values of different parameters here. So, we are getting 2.67045 into 10 to the power 5 in kilo Newton per meter. Now, this is for single frame. So, we have total 4 frames and for every frame the value of h and I c remain same that is the reason k x for each of the 4 frames are same. So, I can directly then get summation of k x and it is coming 10.68183 into 10 to the power 5 in kilo Newton per meter. So, in this way we can calculate summation of k x; next is to calculate W t. So, how we will calculate W t?

If you see here, I can go to the next slide. So, if you see here we need to now find out the self weight of this deck slab. Now, what is the thickness of this deck slab? If you see its thickness is equal to the depth of this cross beam. So, it is 0.7 meter. What is the length or I can say perimeter?

So, for perimeter what we need to do? We have center to center distance, we know it, also with this we need to add half of the width of the beam on right hand side and on left hand side. So, the total length in this direction is I am just writing for your understanding 2.5 plus

3.2 plus 4 plus 0.35 for left hand side and 0.35 for the right hand side. Likewise, in this direction its what will be its length? 3.5 center to center distance.

Now, if I will go back to previous slide you can see here cross sectional dimension of each longitudinal beam is also given. So, here you can see we need to add 0.6 meter. So, here I can add that and the distance or length is 3.5 plus 0.3 plus 0.3. So, total we are getting 4.1. So, from this now we can calculate the self weight of the deck slab. So, I am not writing the full thing. I can write also. 24 kilo Newton per meter cube is the unit weight of the concrete. Now, on this frame what are the loads acting that we need to add which is 40 for frame 1, 80 for frame 2 and frame 3 and frame 4 it is 50.

So, total it is in kilo Newton. So, we are getting finally, 966.352 in kilo Newton. From this we can find out the natural frequency, average natural frequency which is equal to summation of k x divided by w t by g. I think when we have discussed the theory we represent it by omega n x a. So, you can write it omega n x a also. Then it is coming summation of k x already known to us. Now, it is in Newton per meter divided by 966.352 into 10 to the power 3, it is in Newton divided by g which is 9.81.

So, we are getting 104.13 in radian per second. So, this is the natural frequency of the frame foundation average natural frequency when vibrating in horizontal direction. Now, we need to find out the amplitude.

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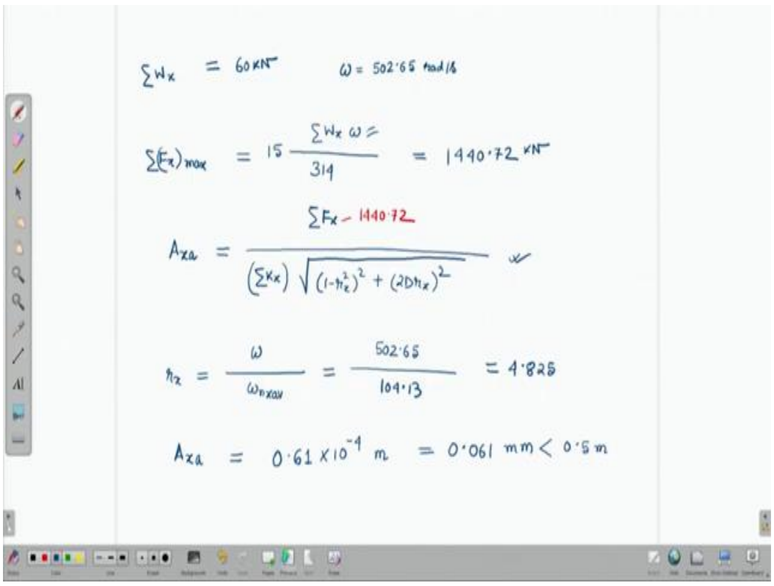

$\omega_{nz\,av} = 369.67 \text{ rad/s}$ $\quad\omega = 502.65 \text{ rad/s}$

$r_z = \frac{\omega}{\omega_{nz\,av}} > 1$

$\Downarrow 1.36$

$\sum K_z = 7.7062 \times 10^6 \text{ kN/m}$

$\sum F_z = \sum m e \omega^2 = 67.91 \text{ kN}$

$$\text{Max. Amplitude} = \frac{\sum F_z}{\sum K_z (2D)} = \frac{67.91 \text{ kN}}{(7.7062 \times 10^6 \text{ kN/m})(2)(0.02)}$$

$$= 2.20 \times 10^{-4} \text{ m} \approx 0.22 \text{ mm} \quad (\text{vertical vibration})$$

$< 0.3 \text{ mm}$

$$K_x = \frac{12EI_c}{H^3} \cdot \frac{6k+1}{3k+2}$$

$$= 2.67045 \times 10^5 \text{ kN/m}$$

$$\sum K_x = (4)(2.67045) \times 10^5 \text{ kN/m} = 10.68183 \times 10^5 \text{ kN/m}$$

$$W_T = 0.7 \times (2.5 + 3.2 + 4 + 0.7)(3.5 + 0.6) \times 24 + 40 + 80 + 80 + 50 \text{ kN}$$

$$= 966.352 \text{ kN}$$

$$\omega_{nx\,av} = \sqrt{\frac{\sum K_x}{W_T/g}} = \sqrt{\frac{10.68183 \times 10^8 \text{ N/m}}{\frac{966.352 \times 10^3}{9.81}}} \text{ rad/s}$$

$\uparrow$ ω_{nxa}

$$= 104.13 \text{ rad/s}$$

Numerical Problem

Design a turbo-generator frame foundation shown in Fig. 50.1 with the following data:

Cross sectional dimensions of each column = 0.8 m × 0.8 m

Cross sectional dimensions of each cross beam = 0.7 m × 0.7 m

Cross sectional dimensions of each longitudinal beams = 0.6 m × 0.6 m

Weight on cross beam of each frame: W_{11} = 40 kN; W_{12} = 80 kN; W_{13} = 80 kN; W_{14} = 50 kN

Weight of rotating parts acting on each cross frame: W_{r1} = 10 kN; W_{r2} = 15 kN; W_{r3} = 15 kN; W_{r4} = 20 kN

$m_1 = \frac{10 \times 10^3}{9.81}$ $\quad\Downarrow m_2$ $\quad\Downarrow m_3$

Eccentricity = 0.05×10^{-3} m e

Young's Modulus for concrete = 3×10^7 kN/m_2

Damping ratio = 0.02; μ = 0.15

Operating speed = 4800 rpm $\quad \omega = 2\pi f = 2\pi\left(\frac{4800}{60}\right) \text{ rad/s} = 502.65 \text{ rad/s}$

Permissible vertical amplitude = 0.3 mm and Permissible horizontal amplitude = 0.5 mm

So, for that what we will do? We will find out summation of w x which is if you see here it is coming I can show you here itself. So, summation of w x 10 plus 15 10 plus 15 plus 15 plus 20 in kilo Newton. So, total it is coming then 60 kilo Newton and already omega which is the operating frequency this is known to us. So, the operating frequency is 502.65 in radian per second. So, with this now we can calculate summation of f x its amplitude basically we are calculating. So, it is for that this is the expression. So, how much we are getting we are getting. So, omega is known summation of w x is known.

So, it is 1350.688 in kilo Newton. So, this is the total amplitude of the total horizontal force. Now, A x a which is the amplitude of vibration displacement in horizontal direction, that we can calculate using this expression summation of f x divided by summation of k x times square root of 1 minus r x square whole square plus 2 d times r x whole square. So, here we know the value of d, only thing r x is unknown. So, calculate it, omega divided by omega n x a or average which is 502.65 divided by let me check, 104.13 is the natural, average natural frequency 104.13.

So, we are getting how much? Let me calculate, 502.65 divided by 104.13. So, we are getting 4.827 or you can take it 4.83 also. So, finally, then we are getting A x a, this is equal to I am not showing all the values you can put these values in this expression and finally, what we are getting that I am trying to write, just one thing let me check, I think I have made some mistake here. So, let me check this value. I think it is not exactly correct, let me check. 1 4, yes I made a mistake here, please correct this value, it is actually 1440.72 in kilo Newton.

Now, with so, here I need to write 1440.72, summation of k x already we have calculated in the previous page you can see this one and also we can calculate r x. So, from r x finally, what we will get that I am trying to write it is coming approximately equal to 0 point, sorry I am writing approximate value. So, it is coming 0.6, you can take 6 1 also into 10 to the power it is coming approximately 0.61 into 10 to the power minus 4 in meter or in millimeter we can write it 0.061 in millimeter.

So, what we can see? Let us see the permissible amplitude. So, for horizontal displacement horizontal vibration permissible horizontal amplitude is 0.5 millimeter. If you see our answer it is lower than the permissible value.

Likewise you can see for vertical vibration we are getting amplitude 0.22 millimeter, whereas, the permissible value if you see it is 0.33 millimeter. So, I can write it, this one is also lower than the permissible value. So, with this you can see how we can solve problem using resonance method.

Now, we have also discussed another method which is amplitude method. So, in next few minutes what I will do I will just tell you the steps particularly how to calculate the load for the frame foundation while using amplitude method.

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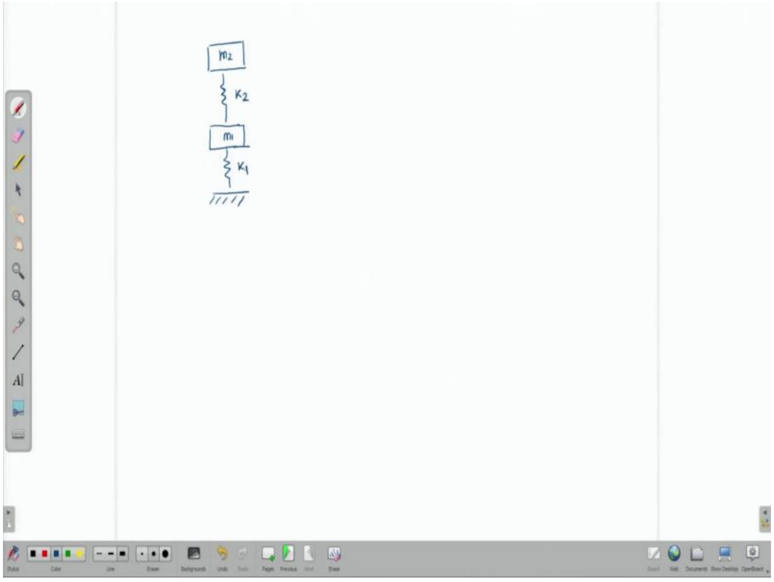

So, in amplitude method we already learned that the frame foundation will be represented by a 2 degree of freedom system, this is mass m2, this is mass m1, k2 and this is k1. So, here we need to know what is how to calculate k1, k2, m1, m2, etcetera. So, let us take the frame 1.

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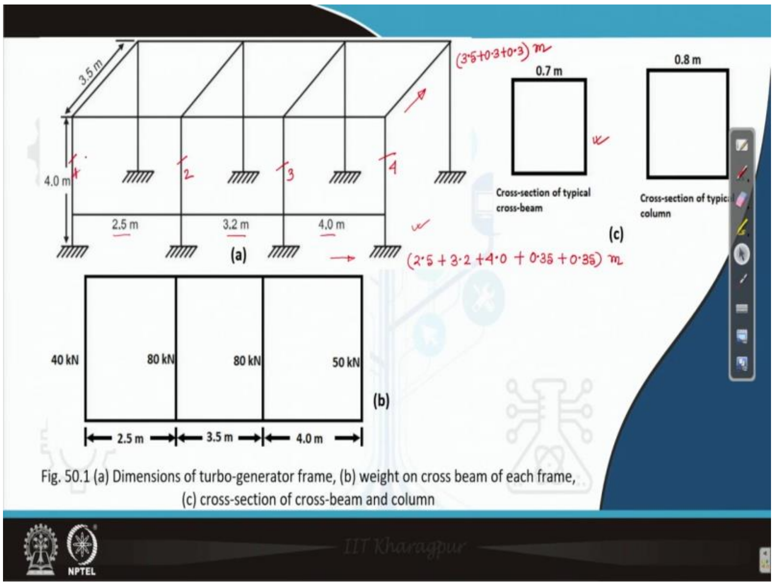

Fig. 50.1 (a) Dimensions of turbo-generator frame, (b) weight on cross beam of each frame, (c) cross-section of cross-beam and column

So, frame 1 means for this frame.

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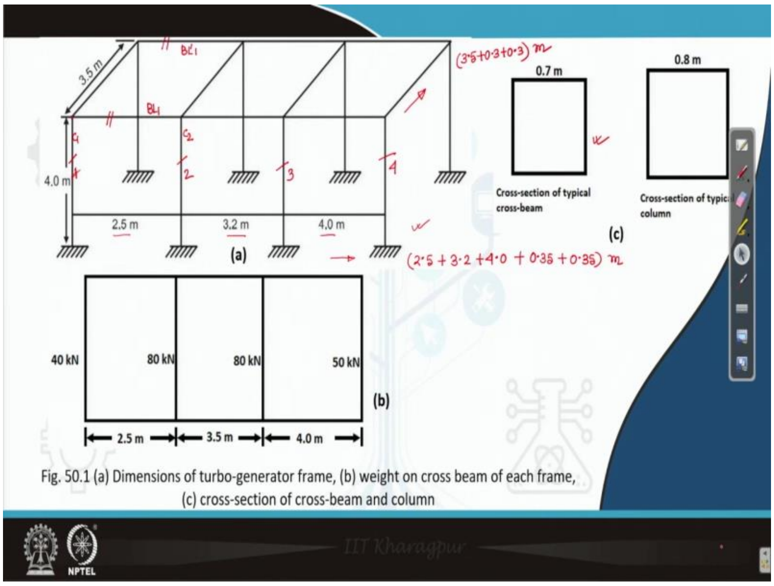

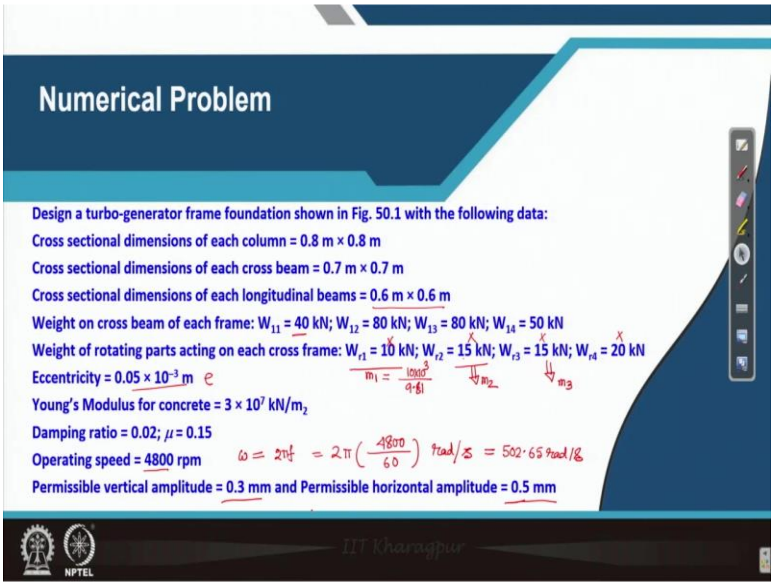

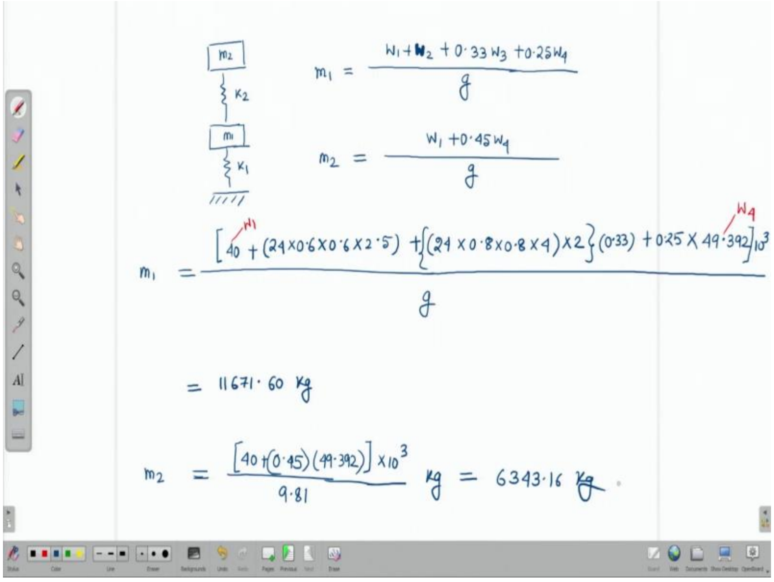

So, for this frame, for any frame actually, m1 is equal to w1 plus w2 plus 0.33 w3 plus 0.25 into w4 and this is divided by g. Likewise we can calculate m2, which is equal to w1 plus 0.45 w4 divided by g. Now, the question how we will calculate w1, w2, w3 and w4? So, w1 we already, we can get the information of w1, from here you can see w1 is 40 kilo Newton which is the weight on the cross beam of each frame. Now, what is w2? So, w2 is basically the weight of, the self-weight of this beam.

Now, if you see there are two when we represent it there are two longitudinal beams. So, we have to consider the total weight of these two longitudinal beams here. And also we need to note here that these two longitudinal beams are simply supported by the column C1 and C2. So, whatever is the total self-weight for let us take b l 1 and this is b l dashed 1.

So, we will calculate first the self-weight of b l 1 and b l 1 dashed and half of that total load will be taken by this frame and that is our w2. So, I am just calculating m1, 40, then you will get this value of w2 which is I am just writing here if 24 is the unit weight of the concrete 24 in kilo Newton per meter cube then we get it as 2.5.

So, this is also in kilo Newton plus, now w3. So, w3 is to again we need to consider the unit weight of the concrete. Now, this time we are considering the weight of the column. So, 0.8, height is 4 meter and there are two columns so 2. So, this is w2 now 0.33 will be multiplied to w3 plus 0.25 into w4. So, w4 here is how much? 24 times cross sectional area of the beam times its length. So, again I am referring this figure. Now, here if we see what is 3.5 meter is center to center distance.

So, we need to also add half of the width of both the longitudinal beams. So, that if we will add we will get the weight of this beam itself. So, it is coming I am directly writing because space is this I have already calculated this. So, it is coming approximately I hope it is understood by all. So, it is coming 49.392 in kilo Newton. So, we are considering only 25 percent of this value divided by g. So, g is instead of g you can directly write 9.81 also. So, finally, what we are getting?

For m1 is 11671.60 in kg. Likewise now we can calculate m2 also, we know already the value of w1, this is w1 and this is w4. So, with these two values we can calculate m2. In the previous expression also we need to convert it in Newton. So, let me do this thing by multiplying it by 10 to the power 3. Here also we need to convert it in Newton, then divided by 9.81 to get it in. So, it is coming 6343.16 kg. So, the same way you can calculate m1 and m2 for the other three frames. I hope this exercise will be all right for all of you. Now, next what we need to do?

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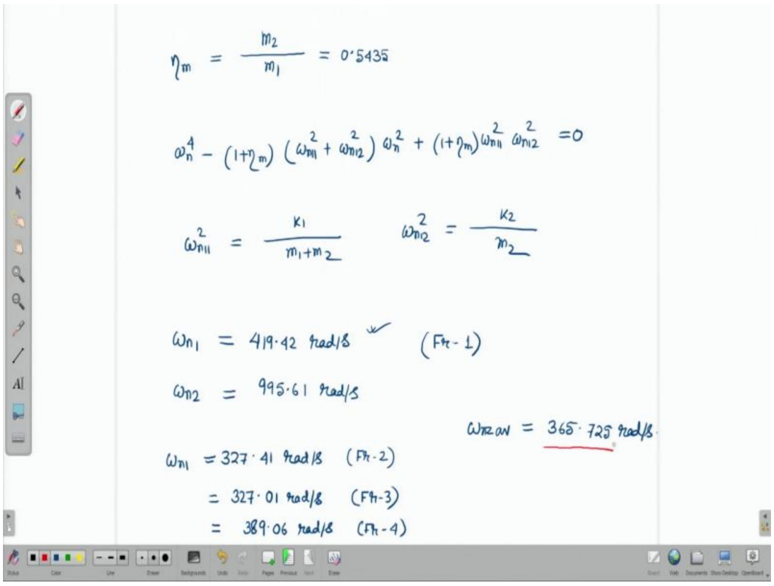

Next step is to find out eta m. So, we already have seen eta m means m2 divided by m1. So, m2 for frame 1, it is coming how much? I think we are getting 0.5435. Please check the value may change a little bit depending upon approximation, where you are approximating in which decimal place.

So, after this we need to find out the natural frequency using this equation which I am writing here. So, omega n 1 1 square is equal to k1 divided by m1 plus m2 whereas, omega n 1 2 square is equal to k2 divided by m1 plus, sorry only m2. With this if you will solve the equation finally, what you will get that I am directly writing for frame 1.

So, for frame 1, you will get the natural frequency omega n 1 is equal to 419.42 in radian per second; for second natural frequency for frame 1 is 995.61 in radian per second. So, we will right now we are interested for the lower mode of or first mode of vibration. So, this is this value.

Now, the same way we can find out the omega n 1 and omega n 2 for other frames. So, I have already done this exercise. So, I am just directly writing what you will get for other frames. So, for other frames, just give me 1 minute time, yes I am writing this is for frame 1.

So, I am just writing omega n 1 value for other frames that is 327.41 in radian per second for frame 2. For frame 3, it is little bit low, 327.01, this is for frame 3 and for frame 4 it is 389.06. So, if you will take average value, that means, omega n z average, you will get 365.725 in radian per second.

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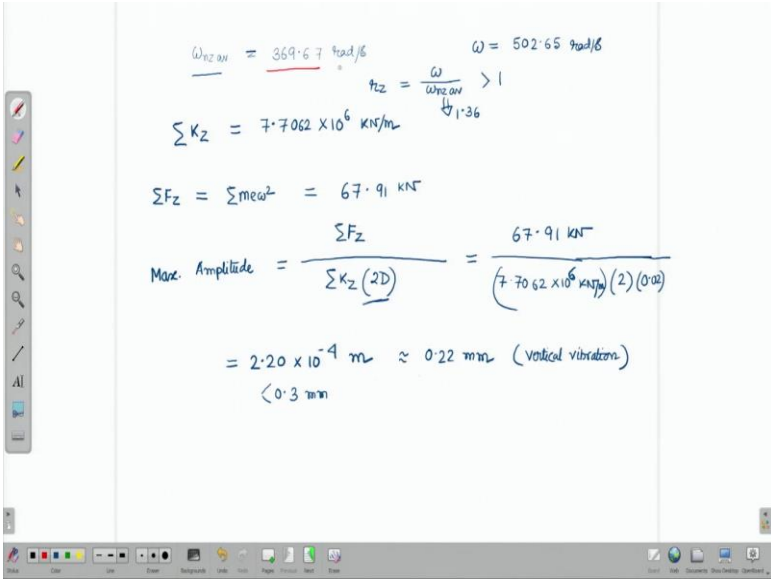

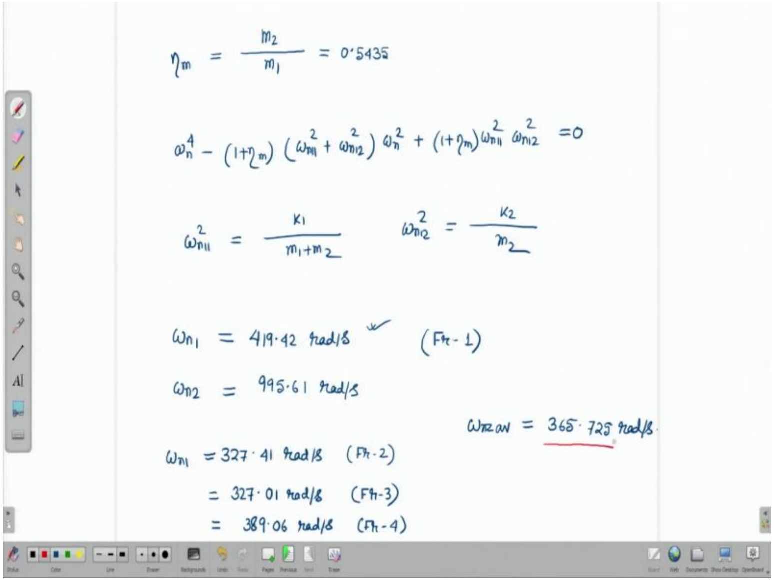

So, here what you can note is that I am just going back to previous pages. So, here if you see earlier when we solve the problem using resonance method we get omega n z average is equal to 369.67 radian per second. Right now when we solve the same problem using amplitude method what we get is 365.725. That means, this time we are getting slightly lower value.

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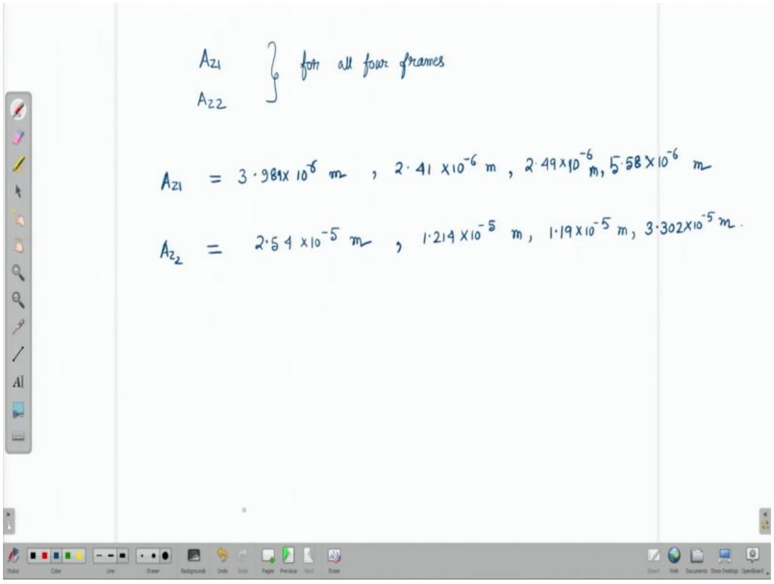

So, after calculating this next task is to find out for each frame you need to find out the value of after finding out omega n z average. So, next step is to find out a z 1 and a z 2 for all 4 frames. I hope you will be able to find it out I am just writing the final value of a z 1. For frame 1 it is 3.989 into 10 to the power minus 6 meter, likewise a z 2 for frame 1 is 2.54 into

10 to the power minus 5 in meter. Similarly, for other frame you can find out a z 1, I am just writing the value. For frame 2 it is 2.41 into 10 to the power minus 6.

For frame 3 it is 2.49 into 10 to the power minus 6 and for the last frame it is, that means, fourth frame it is coming 5.58 into 10 to the power minus 6, all are in meter, this is also in meter. Likewise a z 2 if you will calculate, for frame 2 it is 1.214 into 10 to the power minus 5 meter; for frame 3 it is 1.19 into 10 to the power minus 5 meter; for frame 4 that means, fourth one it is coming 3.302 into 10 to the power minus 5 meter. So, this is for the vertical vibration. Now, the next thing is horizontal vibration.

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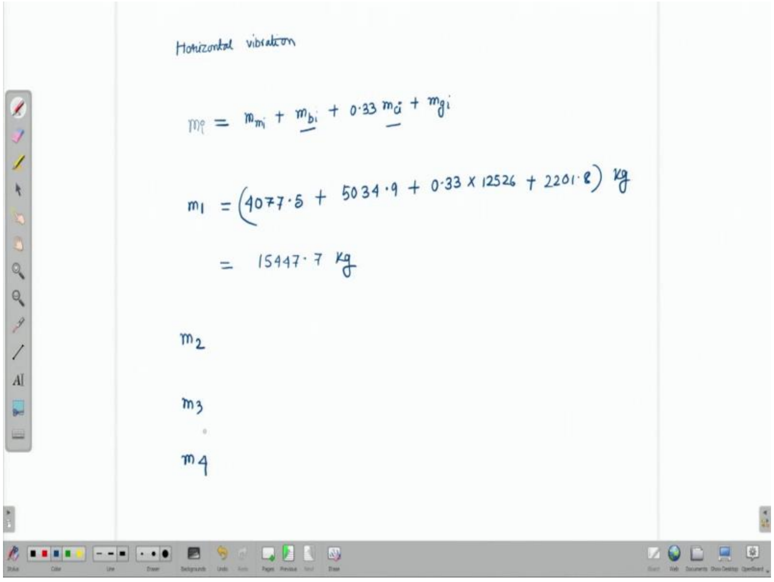

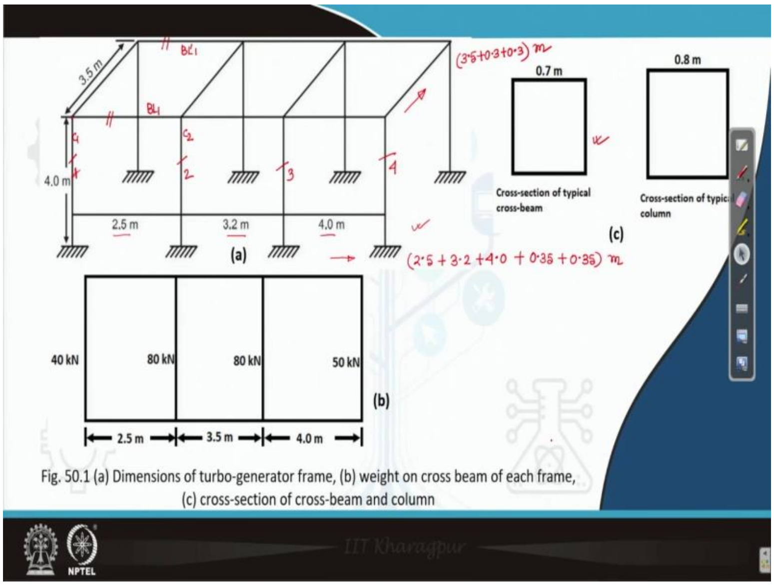

So, for horizontal vibration we need to do a little bit calculation. So, that I am showing. So, we know that we need to find out, that means, total mass acting on that frame is equal to mmi plus mbi plus 0.33 mci plus mgi all. Now, how to calculate this mmi? We have this data because if you see the problem here itself it is given; from this weight we can calculate mmi. Now, we need to calculate mhi we need to sorry, mbi, then mci and mgi. So, for that what we will do?

So, this is for the beam I hope you can calculate it. For first case I am just writing what will be the value. So, let me write it, for the first frame these values 4077.5 plus 5034.9 plus 0.33 times 12526 plus mgi, this is 2201.8, this is in kg. So, you can calculate the total mass which is coming approximately 15447.7 in kg. So, now here what is important for us that I am just showing. So, in this way for all the 4 frame we can calculate mi or I can call it as m1 then m2 then m3 and m4. Now, so, m2 for frame 2, m3 for frame 3, and m4 for frame 4.

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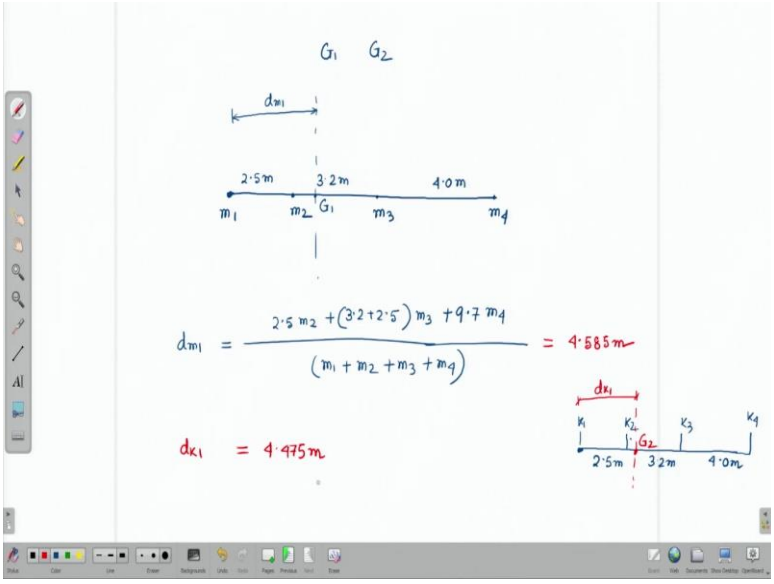

Then what we need to do? This is the original position of the deck. So, this is m1, this is m2 this is m3 and this is m4. So, this distance is 2.5 meter, this distance is 3.2 meter and this distance is 4 meter.

Now, we need to find out g1 and g2. What is g1? g1 is the center of masses whereas, g2 is the center of stiffness's. So, for g1 if this distance is let us take this is the position of g1 and this distance from m1 is d m1, then we can calculate d m1 as 2.5 times m2 plus 3.2 plus 2.5 which is coming 5.7 times m3 plus 4. So, total distance now from m1 to m4 this distance is 9.7 times m4 divided by total mass which is m1 plus m2 plus m3 plus m4.

So, in this way you can get d m1, the same way you can find out d k1. So, for that you need to refer this diagram where this is one spring point, this is the second spring point, this is the third and this is the fourth. Now, k1, the values of k1, k2, k3 and k4 are same, that we have already seen. So, in this case what we need to do? This distance once again, 2.5 meter, 3.2 meter and this is 4 meter span length. So, now, what we need to do is that we will again take moment about this point and we will find out the position of g2.

So, let us take g2 is somewhere here. So, this distance which is called d k1 we will calculate. So, if you will do this exercise the same way as I have shown for d m1, then what you will get that I am just writing for d m1 you will get the value 4.585 in meter and d k1 is 4.475 meter. Thereafter rest of the things already we have explained in the theory, the same way we solve the problem using resonance method, here also we can solve this problem. Rest of the things actually is very simple, you just need to follow a few steps.

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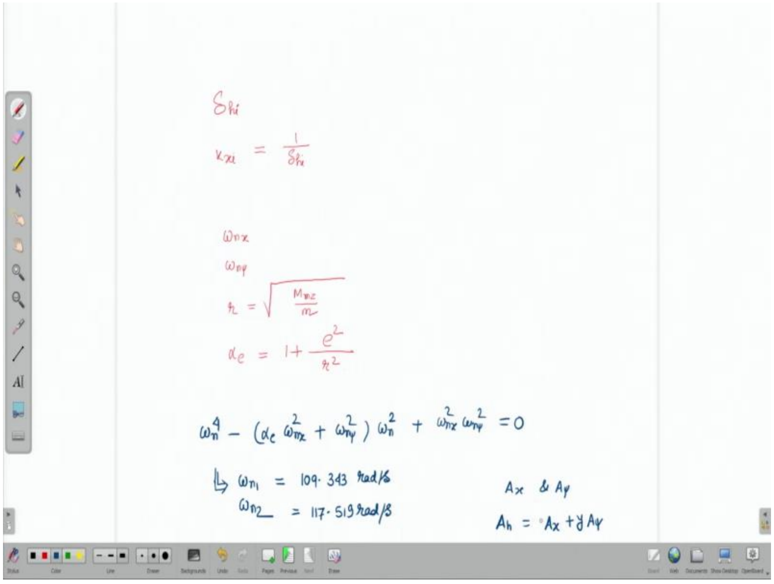

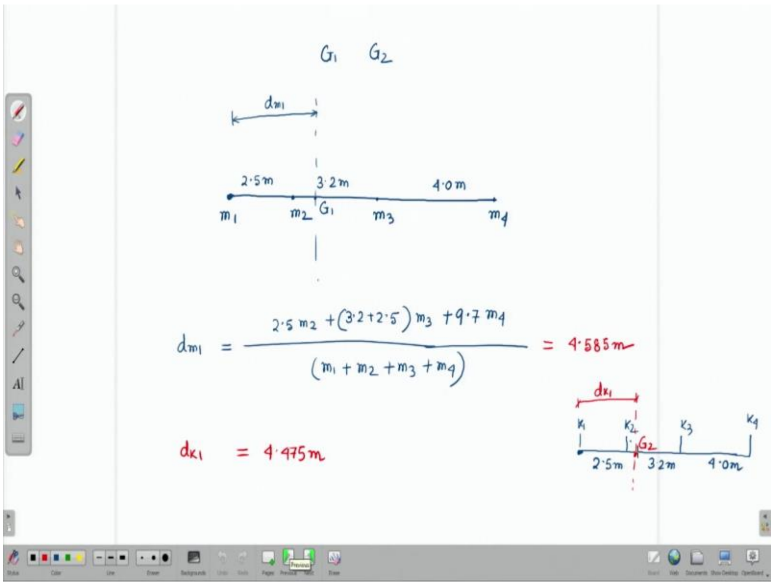

First, after this you will find out delta h i, then you will find out k x i which is 1 by delta h i and then you will solve, you will find out the natural frequency and that natural frequency for in horizontal direction which is omega n x and omega n psi. After this you will find out r value which is the ratio of m m z divided by small m also you will find out alpha e which is equal to 1 plus e square divided by r square where e is the distance between g1 and g2. So, in this diagram from this figure and this figure you can find out the location of g1 and g2 with respect to this point.

Now, from that you will find out the distance between g1 and g2 and that will be used here. So, after that you need to solve the equation I am just writing for you the equation this is the equation which you will solve omega n x square plus omega n psi square. So, solving this equation what you will get? You will get the value of omega n 1 and omega n 2 and from, I am just writing for you what is the value of omega n 1. So, this is the value of omega n 1, which you will get and for omega n 2, you will get this value.

And then finally, from this you need to find out you know already how to calculate m z which is the unbalanced moment and then f x and from that you will find out a x and a psi. And then finally, you will find out total amplitude of the total horizontal displacement which is by using the equation which I am writing right now. A h is the total amplitude of the total horizontal displacement which is equal to a x plus y times a psi, already the meaning of the symbol used here are explained in previous classes. So, you can refer the class notes and from that you can calculate the values for a x, a psi and then a h.

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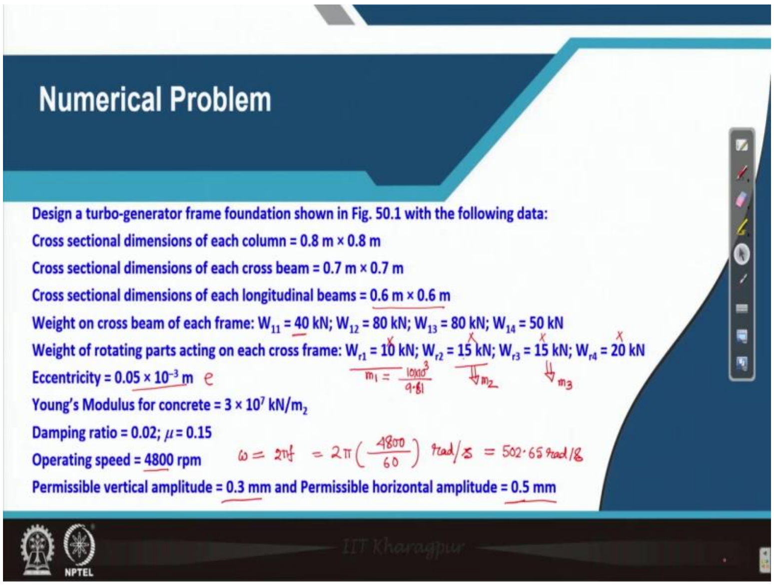

I think you will be able to find it out and then the problem solution will be ready. So, in this way I am requesting all of you to complete the exercise. If you have any doubt you can send your queries to us.

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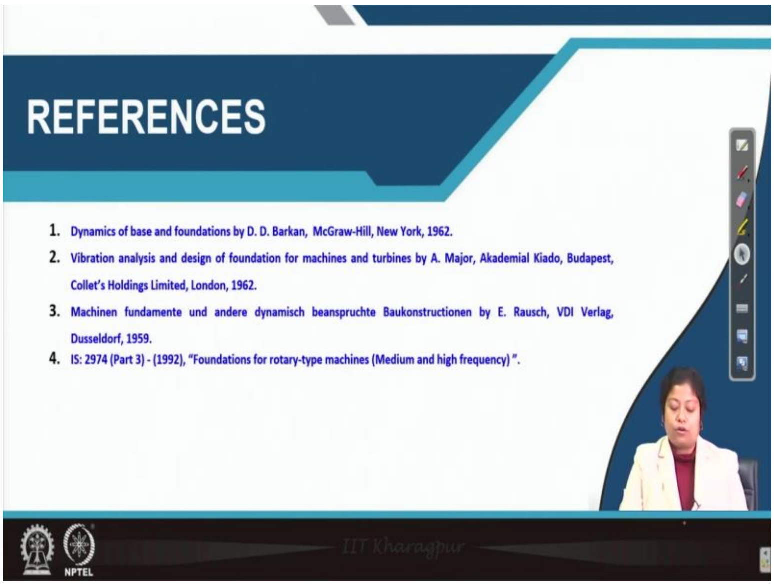

So, these are the references which I have used for today's class. If possible what I will say to all, please try to get this textbook if you can get. It is available in online also. So, most of the discussions related to the analysis of machine foundation for rotary machine was taken, I have taken from that textbook. Thank you.