

**Solid Dynamics**  
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**Lecture - 57**  
**Isolation of Vibration (Part 2)**

Hello friends. So, we have already started to discuss Isolation of Vibration. So, today is the second class on this topic. Already we have discussed different types of isolation. So, in last class if you recall we have talked about force isolation. So, let us solve one numerical problem to clear our understanding on force isolation and how we will design force isolation.

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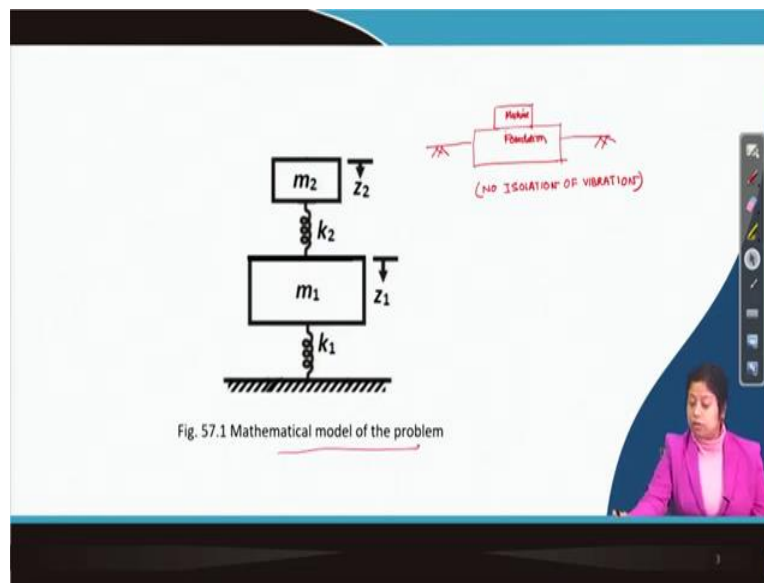
**Numerical Problem**

Design a suitable isolation system for keeping the amplitude of the foundation of a reciprocating machine less than 0.025 mm. The weight of the machine is 25 kN and it produces sinusoidally varying unbalanced force of 4 kN in the vertical direction. The operating speed of the machine is 600 rpm. The dynamic shear modulus of the soil is  $2.5 \times 10^4 \text{ kN/m}^2$ . Assume suitably any data not given.

So, this is the problem which we are interested to solve today. Design a suitable isolation system for keeping the amplitude of the foundation of a reciprocating machine less than 0.025 millimeter. That means, permissible amplitude of vibration of the foundation is already provided. The weight of the machine is 25 kilo Newton and it produces sinusoidally varying unbalanced force of 4 kilo Newton in the vertical direction.

The operating speed of the machine is 600 rpm, that means  $F$  is given. The dynamic shear modulus of the soil is also provided that is 2.5 into 10 to the power 4 kilo Newton per square meter. Asked to assume suitably any data which are not given. So, for this what we need to do, we need to find out the stiffness of the, mainly the stiffness of the isolator, which we use.

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So, if I go to the next page, here you can see, basically this is the mathematical model of the problem with the vibration isolator. Actually, how machine is resting, this is the machine, it is resting on the foundation. Now, there is a possibility of excessive settlement, so this is machine, this is foundation, so no isolation for this case, isolation of vibration.

Now, if the amplitude of displacement of the ground or the system, soil foundation system does not exceed the permissible limit in this case it is 0.025 millimeter. If it is less than this permissible limit, then we do not need to think about any kind of vibration isolation. So, first, we will check what is the magnitude of the or amplitude of ground displacement when there is no isolation.

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Size of foundation :  $3\text{ m} \times 2\text{ m} \times 2\text{ m (H)} \Rightarrow \rho_{10} = \sqrt{\frac{3 \times 2}{\pi}} \text{ m} = 1.38 \text{ m}$

Mass of foundation ( $m_1$ ) =  $\frac{(3)(2)(2) \times 10^3}{9.81} \text{ kg} = 29357.8 \text{ kg}$

Mass of machine ( $m_2$ ) =  $\frac{25 \times 10^3}{9.81} \text{ kg} = 2548.42 \text{ kg}$

$k_1 = \frac{(1)(2.5 \times 10^4) \left( \frac{\text{kN/m}^2}{\text{m}} \right) (1.38)}{1 - 0.35} \text{ kN/m} = 212307.7 \text{ kN/m}$

$\omega_{n2} = \sqrt{\frac{k_1}{(m_1 + m_2)}} = \sqrt{\frac{212307.7 \times 10^3}{31906.22}} = 81.57 \text{ rad/s}$

$A_z = \frac{F_0}{m(\omega_{n2}^2 - \omega^2)} = \frac{4 \times 10^3}{31906.22(81.57^2 - 62.83^2)} \text{ m}$

$A_z = 4.633 \times 10^{-5} \text{ m} = 0.046 \text{ mm} > 0.025 \text{ mm}$

$\eta_m = \frac{m_2}{m_1} = 0.087$

$\xi = \frac{A_{z1}}{A_z} = \frac{0.025}{0.046} = 0.54$

$\xi = \frac{a_w^2 (1 + \eta_m)(a_z^2 - 1)}{[1 - (1 + \eta_m)(a_z^2 + a_w^2 - a_z^2 a_w^2)]}$

$a_w = \frac{\omega_{n2}}{\omega} \quad a_z = \frac{\omega_{n2}}{\omega} = 1.3$

## Numerical Problem

Design a suitable isolation system for keeping the amplitude of the foundation of a reciprocating machine less than 0.025 mm. The weight of the machine is 25 kN and it produces sinusoidally varying unbalanced force of 4 kN in the vertical direction. The operating speed of the machine is 600 rpm. The dynamic shear modulus of the soil is  $2.5 \times 10^4 \text{ kN/m}^2$ . Assume suitably any data not given.

$$\omega = 2\pi f = 2\pi \left( \frac{600}{60} \right) = 62.83 \text{ rad/s}$$

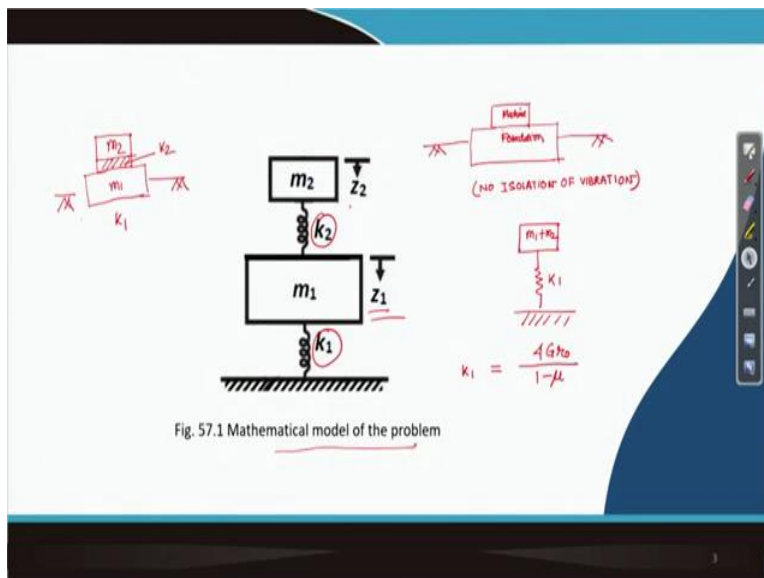


Fig. 57.1 Mathematical model of the problem

So, let us go to the white board. We can assume data for the size of the foundation because it is not given here. So, let us take size of foundation that is 2 meter by 3 meter by 2 meter. So, this, I can write it 3 meter by 2 meter that looks, 3 meter by 2 meter by 2 meter. So, last one is height. That means what is the volume that we can calculate from this information?

If the foundation block is made of concrete and if we consider the unit weight of the concrete is 24 kilo Newton per meter cube, then from that we can calculate the mass of the foundation. So, mass of foundation, this is, let us take  $m_1$  is equal to 3 times 2 times 2 times 24 into 10 to the power 3, this is now in Newton divided by 9.81, then we are getting the thing in kg. So, how much I am getting 3 into 2 into 2 into 24, 10 to the power 3 divided by 9.81.

So, it is coming approximately 29357, that means 29357.8, actually it is coming 0.79817, etcetera, so I am taking the approximate value that means 29357.8 in kg. Similarly, we can write the mass of machine. Let us take it is  $m_2$ . So, if you go back to the problem here, what is the weight of the machine, 25 kilo Newton, so with this we can find out mass of the machine. So, 25 into 10 to the power 3 divided by 9.81, it is in kg.

So, 25 into 10 to the power 3 divided by 9.81, it is coming 2548.42 in kg. Now, go to the previous, go to here, so when there is no vibration isolator that time how we can represent this by a mathematical model. There is a mass, this mass is equal to  $m_1$  plus  $m_2$  attached to a spring  $k_1$  because of the soil. Now, how we can find out  $k_1$ ?

For  $k_1$  we can use this expression, we have already learned for when we consider elastic half space soil as elastic half space we can use this equation, if no other data is provided, I mean, if only  $G$  and  $\mu$  is mentioned, so what is  $K_1$  then in this case, 4 times  $G r_0^{2.5}$  into 10 to the power 4 this is  $G$ , okay  $r_0$  that I have not calculated. What is  $r_0$  here?

$r_0$  is the equivalent radius because in this case I have taken a rectangular cross-sectional area for the foundation block, so I need to find out from these what is  $r_0$ . So,  $r_0$  is equal to 3 into 2 divided by  $\pi$  in meter. And how much you are getting? It is coming 1.382 meter. I can write just 1.38. So, here then I will write 1.38, so 4  $G r_0$  divided by 1 minus  $\mu$ , let us take  $\mu$  is equal to 0.35, then it is coming in kilo Newton per square meter.

Sorry, this is kilo Newton per square meter, this is meter, so finally what we are getting is kilo Newton per meter. So, how much it is coming? Let me calculate. We are getting 212307.7, approximately I am writing kilo Newton per meter. Now, then for this condition what is the natural frequency  $\omega_{nz}$  that is equal to  $K_1$  divided by  $m_1$  plus  $m_2$  that is the total mass attached to the spring  $K_1$ , this is equal to 81.57, 81.57 in radian per second.

Then what is the amplitude of vertical vibration of the ground? Let me calculate. So, that is, if that is  $A_z$ , then that is equal to  $F_0$  divided by  $m$ ,  $m$  means here  $m_1$  plus  $m_2$  times  $\omega_{nz}$  square minus  $\omega$  square. So, far we have not calculated  $\omega$ , so let us do it from the problem, so  $F$  is given then  $\omega$  is equal to  $2 \pi f$ ,  $2 \pi 600$  means divided by 60, then it is cycle per second.

So, what we are getting is  $2 \pi$  times 10, which is 62.83 in radian per second. So, this value we can use here now and  $F_0$  you have already seen, it is 4 kilo Newton, so 4 into 10 to the

power 3 divided by m, which is the total mass here, so total mass here, if you will add these two it is coming 31906.22 that is the total mass times  $\omega z$  square minus  $\omega$  square, that means 62.83 and this is in meter.

So, finally we are getting  $A_z$  is equal to, it is coming 4.633, 4.633 into 10 to the power minus 5 meter, so I can write it in millimeter, so 0.046 millimeter, which is greater than 0.025 millimeter, which is permissible here. That means, we now need to reduce the amplitudes of vibration of the ground. How do we do that? For that we will use one isolator that means if I will go here, now I will use one isolator.

So, this is machine, let us take this is the isolator, then the foundation which is resting on the soil, so  $m_2$ ,  $m_1$ , for this  $K_2$  and for soil  $K_1$ . So, this  $K_1$  represents the stiffness of the soil and this  $K_2$  represents stiffness of the vibration isolator. And you can see  $z_1$  and  $z_2$  are the displacement of the, dynamic displacement of the machine and the foundation,  $z_1$  for the foundation,  $z_2$  for the machine.

Now, that means we need to now solve the 2 degrees of freedom system. So, how to solve it that is already discussed in previous class? Here we will do first, we know the values for the several things, so with that we will try to get the solution for this kind of problem. So, first what we will do, first we will calculate  $\eta$ , which is the ratio of  $m_2$  divided by  $m_1$ . Already we have calculated  $m_2$ , this one and this is  $m_1$ .

So, it is coming approximately equal to,  $m_2$  divided by  $m_1$  is coming approximately equal to 0.087. Now, we need to find out efficiency  $\xi$ . So, this is the ratio of the  $A_{z1}$  which is the permissible amplitude to the  $A_z$ , which is the amplitude of the foundation, amplitude of the displacement of the foundation when there is no isolator used. So, that means 0.025 divided by  $A_z$  we have calculated that is 0.046, so it is coming 0.025 divided by 0.046.

It is coming approximately equal to 0.54. So, these two parameters are calculated. Now, from this  $\xi$ , we can find out the term  $a$ , what is  $a$  that we have already discussed,  $a$  is equal to  $\omega_n$  divided by  $\omega$ . So, now from these we can find out, we will try to find out this  $a$ . Now, how we will do this?

First, we will write the expression for  $\xi$ . I am writing the expression for  $\xi$  that is  $a$  square times  $1 + \eta$  times  $A_z$  square minus 1, this divided by  $1 - 1 + \eta$  times  $A_z$  square plus  $a$  square minus  $A_z$  square times  $a$  square. Here you can

see this is equal to 0.5. I think I will not get enough space to write this expression. I will write it in the next page.

Another thing what is eta m that I have already mentioned, now the question what is Az. So, let us find it out. Here Az is equal to omega nz divided by omega. So, omega nz is 81.57 and omega is already calculated which is, I can show here 62.83, so 81.57 divided by 62.83, so we are getting approximately 1.3 for Az. So, now in this expression I will write xi is equal to 0.54 and for Az I will write is it is equal to 1.3.

So, now I will write the expression for xi using the value of, on the left-hand side I will use the value of xi and on the right-hand side I will use the value of Az that I have already calculated. And for a omega that is unknown, which we are trying to find out.

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$$0.54 = \frac{-a_w^2(1+0.087)(1.3^2-1)}{[1 - (1.087)(1.3^2 + a_w^2 - 1.3^2 a_w^2)]}$$

$$\Rightarrow a_w^2 = 0.391 \Rightarrow a_w = 0.625$$

$$\omega_{nz} = \omega a_w = (62.83)(0.625) \text{ rad/s} = 39.27 \text{ rad/s}$$

$$\omega_{nz} = \sqrt{\frac{k_2}{m_2}} \Rightarrow k_2 = m_2 \omega_{nz}^2$$

$$= (25.48 \cdot 10^{-2}) (39.27)^2 \text{ N/m}$$

$$= 393002.3 \text{ N/m}$$

$$\approx 3.93 \times 10^5 \text{ N/m}$$

$$A_z = 4.633 \times 10^{-5} \text{ m} = 0.046 \text{ mm} > 0.025 \text{ mm}$$

$$\eta_m = \frac{m_2}{m_1} = 0.087$$

$$\xi = \frac{A_{z1}}{A_z} = \frac{0.025}{0.046} = \underline{0.54}$$

$$\xi = \frac{a_\omega^2 (1 + \eta_m) (a_2^2 - 1)}{[1 - (1 + \eta_m) (a_2^2 + a_\omega^2 - a_2^2 a_\omega^2)]}$$

$$a_\omega = \frac{\omega_{na}}{\omega} \quad a_z = \frac{\omega_{nz}}{\omega} = 1.3$$

Size of foundation :  $3 \text{ m} \times 2 \text{ m} \times 2 \text{ m (H)} \Rightarrow \rho_{10} = \sqrt{\frac{3 \times 2}{\pi}} \text{ m} = 1.38 \text{ m}$

$$\text{Mass of foundation (m)} = \frac{(3)(2)(2)(24) \times 10^3}{9.81} \text{ kg} = 29357.8 \text{ kg}$$

$$\text{Mass of machine (m}_2) = \frac{25 \times 10^3}{9.81} \text{ kg} = \underline{2548.42 \text{ kg}}$$

$$k_1 = \frac{(4)(2.5 \times 10^9) \text{ kN/m}^2}{1 - 0.35} = 212307.7 \text{ kN/m}$$

$$\omega_{nz} = \sqrt{\frac{k_1}{(m_1 + m_2)}}$$

$$= 81.57 \text{ rad/s}$$

$$A_z = \frac{F_0}{m (\omega_{nz}^2 - \omega^2)} = \frac{4 \times 10^3}{31906.22 (81.57^2 - 62.83^2)} \text{ m}$$

So, xi means, sorry, xi means 0.54 which is equal to a omega square times 1 plus 0.087, this is eta m times 1.3 square minus 1, 1.3 square means Az square divided by 1 minus 1 plus eta m which is 1.087 times Az square, Az square means 1.3 square plus a omega square minus 1.3 square times a omega square. This is the expression now using the value of xi and Az and also eta m. So, with this now we can find out a omega square.

Here one thing I would like to mention, in this case actually the value of Az1, which we will get that is for the, should be with a negative sign, so I am placing negative sign here. If you recall, we have done the similar exercise, I mean, we have found out the amplitudes of displacement for impact machine foundation and some others problem related to 2 degrees of freedom system, so same thing basically we are doing here also.



But in this case, we are not just using  $A_{z1}$ , we are using the ratio of  $A_{z1}$  to  $A_z$ .  $A_z$  is the amplitude of the displacement of soil foundation system, when there is no vibration absorber. So, with this, first we will try to find out the values of  $\omega$ . So,  $\omega$  from this, I will get a  $\omega$  or a  $\omega$  square. So, a  $\omega$  square is equal to 0.391. I am just trying to say sometimes, so I am directly writing the value.

You can write the full expression and then you will get this a  $\omega$  square is equal to 0.391. And from this you can calculate  $\omega$ , which is equal to 0.625. Now, what is  $\omega$ ? If I will go this page, you can see it is the ratio of  $\omega_{na}$  divided by  $\omega$ . So, from these then we can find out  $\omega_{na}$ , which is  $\omega$  times  $\omega$ . So,  $\omega_{na}$  means 62.83, a  $\omega$  means 0.625.

So, we are getting  $\omega_{na}$ , it is coming approximately 39.27 radian per second. Now, we know, sorry, we know now  $\omega_{na}$ . What is  $\omega_{na}$  actually in terms of stiffness and mass? It is  $K_2$  divided by  $m_2$ . Then from this we can find out  $K_2$ , which is the stiffness of the vibration isolator. So, in this is  $m_2$  times  $\omega_{na}$  square. So,  $m_2$  is how much? I can show you  $m_2$  is, this is  $m_2$ , 2548.42. This is  $m_2$  times  $\omega_{na}$ ,  $\omega_{na}$  is 39.27 whole square.

So, you will get  $K_2$ , so  $K_2$  is coming to 548.42 into 39.27 that is good. So,  $K_2$  is 3930002.3 in Newton per meter or we can write it also as in 3.93 into 10 to the power 3 in kilo Newton per meter. So, in this way we can write  $K_2$ . Now, next step is to find out  $A_{z2}$ . There is no space here, so I should go to the next page.

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The image shows a handwritten derivation on a whiteboard. The first equation is:

$$A_{z2} = \frac{[(1+0.087)1.3^2 + 0.087(0.625)^2] (4 \times 10^3)}{(2548.42)(62.83)^2 [1 - (1.087)(1.3^2 + 0.625^2 - 1.3^2 \times 0.625^2)]}$$

The second equation is:

$$= -6.38 \times 10^{-4} \text{ m}$$

The third equation is:

$$F_2 = K_2 A_{z2} = (3.93 \times 10^3)(6.38 \times 10^{-4}) \text{ kN}$$

The final result is:

$$= 2.5 \text{ kN}$$

A small video inset in the bottom right corner shows a person in a pink shirt speaking.

$$0.54 = \frac{-a_w^2 (1+0.087)(1.3^2-1)}{[1 - (1.087)(1.3^2 + a_w^2 - 1.3^2 a_w^2)]}$$

$$\Rightarrow a_w^2 = 0.391 \Rightarrow a_w = 0.625$$

$$\omega_{na} = \omega a_w = (62.83)(0.625) \text{ rad/s} = 39.27 \text{ rad/s}$$

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$$\approx \underline{3.93 \times 10^3 \text{ N/m}}$$

## Numerical Problem

Design a suitable isolation system for keeping the amplitude of the foundation of a reciprocating machine less than 0.025 mm. The weight of the machine is 25 kN and it produces sinusoidally varying unbalanced force of 4 kN in the vertical direction. The operating speed of the machine is 600 rpm. The dynamic shear modulus of the soil is  $2.5 \times 10^4 \text{ kN/m}^2$ . Assume suitably any data not given.

$$\omega = 2\pi f = 2\pi \left(\frac{600}{60}\right) = 62.83 \text{ rad/s}$$

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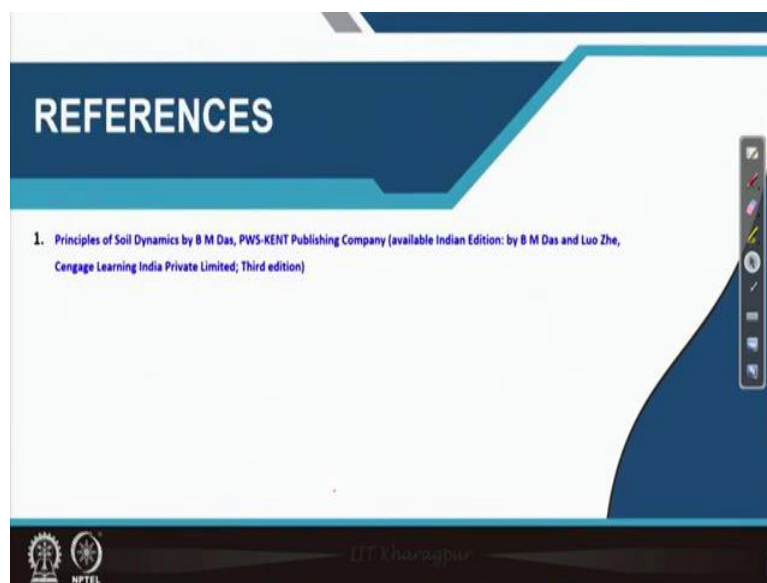
So, Az2, I am first writing the expression for Az2, denominator, there should, there is a change in denominator also. So, I am just writing the expression 1 plus eta m, so already we get eta value, which is 0.087 times Az square, Az means 1.3 square plus eta m 0.087 times a omega square, a omega means 0.625 square, minus 1, this into the force, in this case it is 4 into 10 to the power 3 unbalanced force of the vertical vibration.

This divided by m2 2548.42 times omega square, 62.83 square, this is multiplying with 1 minus 1 plus eta m, so 1 plus eta m means 1.087, this multiplying with Az square plus a; Az square plus a omega square minus Az square times a omega square. So, here we already know a omega, which is 0.625, so with this we can calculate Az2. Az2 is equal to minus 6.38 into ten to the power minus 4 in meter.

Then we can find out the total force for the spring, so total compressive force for the spring, I can write it  $F_2$  is equal to  $K_2$  times  $Az^2$ ,  $K_2$  is how much,  $K_2$  we have already calculated that is  $3.93 \times 10^3$ , so  $3.93 \times 10^3$ , this is  $K_2$ . And  $Az^2$ , of course, we will take the positive magnitude, so  $6.38 \times 10^{-4}$ . So, this is in kilo Newton per meter, this is in meter.

So, finally, we are getting the  $F_2$  in kilo Newton,  $3.93 \times 10^3$  into  $6.38 \times 10^{-4}$  divided, so it is coming approximately 2.5 in kilo Newton. So, this is the property of the absorber that we want to use, so that the permissible, so that the amplitude of displacement due to the vibration should not exceed the permissible limit, which is mentioned as 0.025 millimeter. So, in this way we can calculate, we can design a suitable isolation system which are, which is asked in this numerical problem.

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So, for these this is the textbook which the mathematical concept for that in single degrees of freedom system and two degrees of freedom system are discussed, so referring that I have solved the numerical problem. I am stopping here, so we will meet once again in next class. Thank you.