NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 02 : Snell’s law, Plane wave reflection and transmission

Lecture 06: Precritical, Critical, and Postcritical waves, reflection and transmission coefficients

CONCEPTS COVERED

- Recap
- Precritical waves
- Critical waves
- Evanescent Wave
- Reflection and transmission coefficients for Evanescent Wave

Recap from previous lecture

-Snell's law, for a wave moving from one medium to another $\dfrac{\sin\theta}{v_1} = \text{constant}$

-Take off angle is the angle at which ray leaves from the source measured in the plane containing source and receiver

-In order to derive the reflection and transmission coefficients, two boundary conditions has been applied: 1. The wave is continuous; 2. Tractions are equal

$$R = \frac{\rho_1\beta_1\cos j_1 - \rho_2\beta_2\cos j_2}{\rho_1\beta_1\cos j_1 + \rho_2\beta_2\cos j_2}$$

$$T = \frac{2\rho_1\beta_1\cos j_2}{\rho_1\beta_1\cos j_1 + \rho_2\beta_2\cos j_2}$$

-Fermat's Principle, it states that the ray paths between two points are those for which travel time is an extremum, which is a minimum or maximum travel time.

Evanescent Wave

It is also known as inhomogeneous, head, and postcritical waves

When the incident angle is greater than the critical angle, the transmitted wave is trapped just below the interface. The reflection coefficient is 1, but there is a phase shift.

Let $j_i = \text{incident angle}$

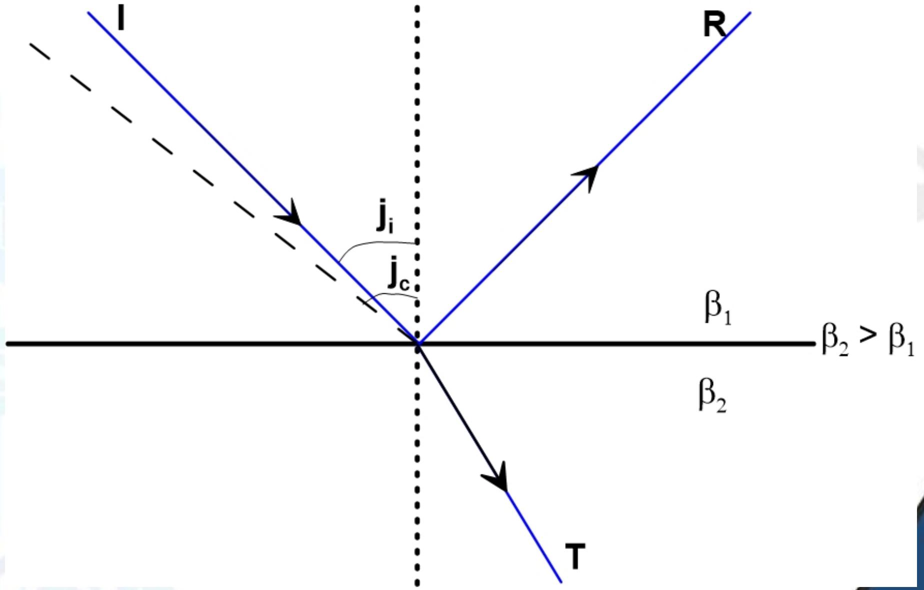
$j_c = \text{critical angle}$

Precritical rays ($j_i < j_c$)

Follow Normal Snell's law

$$R = \frac{\rho_1\beta_1\cos j_1 - \rho_2\beta_2\cos j_2}{\rho_1\beta_1\cos j_1 + \rho_2\beta_2\cos j_2}$$

$$T = \frac{2\rho_1\beta_1\cos j_1}{\rho_1\beta_1\cos j_1 + \rho_2\beta_2\cos j_2}$$

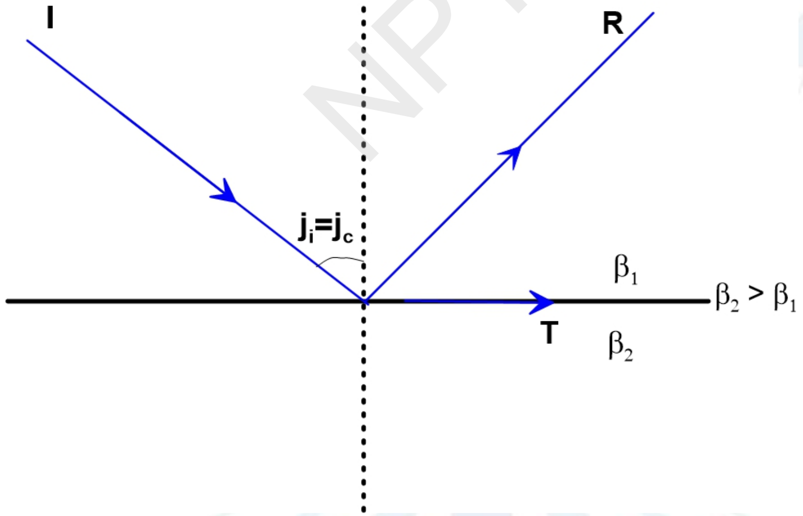


Critical Rays ($j_i = j_c$)

Perfectly Refracted ray

R=1

T=2



Postcritical Rays ($j_i > j_c$)

Wave will be phase shifted after reflection and transmitted wave will be an evanescent wave that decays exponentially with depth.

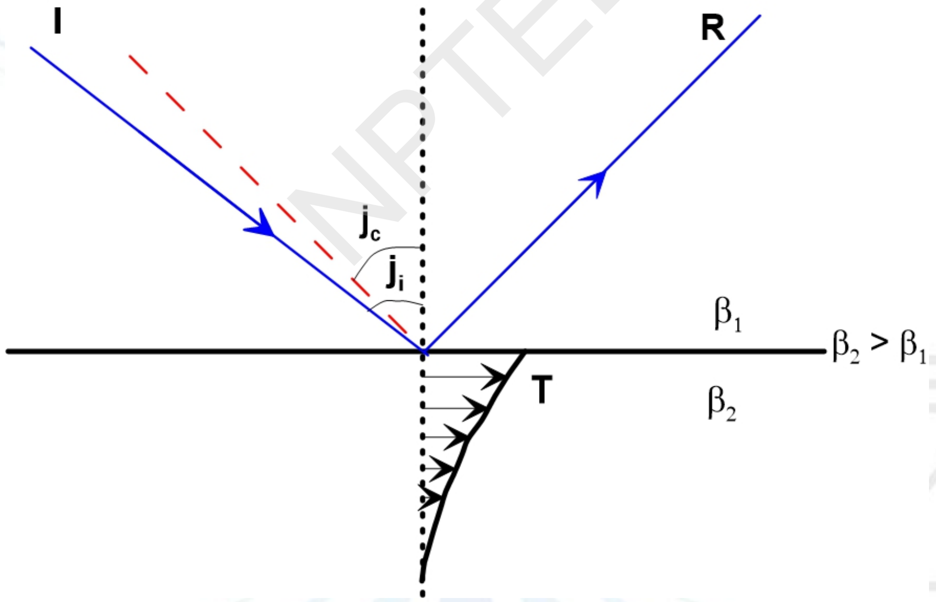
$$R = e^{2i\theta}, \; \theta = \tan^{-1}\left(\frac{\mu_2 r_2^*}{\mu_1 r_1}\right) \qquad \exp\left[-k_2\left(1 - \frac{c_x^2}{\beta_2^2}\right)^{1/2} z\right]$$

As we know,

$$r_\beta = \frac{k_z}{k_x}$$

$$p = \frac{1}{c_x} = \frac{\sin j_i}{\beta} = \frac{k_x}{\omega}$$

$$\eta = \frac{1}{c_z} = \frac{\cos j_i}{\beta} = \frac{k_z}{\omega}$$



As per Snell's law, apparent horizontal velocity across the interface should be equal

$$p = \frac{1}{c_x} = \frac{\sin j_1}{\beta_1} = \frac{\sin j_2}{\beta_2}$$

when, $j_1 > j_c$ then

$$\sin j_1 > \sin j_c \left(= \frac{\beta_1}{\beta_2} \right)$$

$$\implies \sin j_1 > \frac{\beta_1}{\beta_2}$$

$$\implies \beta_2 > \frac{\beta_1}{\sin j_1} (= c_x)$$

So, $j_1 > j_c$ implies $\boldsymbol{\beta_2} > c_x$

From our previous lecture, the displacement of a transmitted SH-wave

$$u_y^T(x, y, z) = B' \, e^{[i(\omega t - k_x x - k_x r_2 z)]}$$

Recall that :

$$\eta_\beta = \sqrt{\left(\frac{1}{\beta^2} - p^2\right)} \quad -------(1)$$

and

$$r_\beta = \frac{k_z}{k_x} \textbf{ then } \eta = \frac{k_z}{\omega} = \left(\frac{k_x}{\omega}\right) \cdot \left(\frac{k_z}{k_x}\right) = p r_\beta = \frac{r_\beta}{c_x}$$

so the equation (1) will become

$$\frac{r_2}{c_x} = \sqrt{\left[\frac{1}{\beta^2} - p^2\right]} \textbf{ and we know } p = \frac{1}{c_x}$$

$$\implies r_2 = \sqrt{\left(\frac{c_x^2}{\beta_2^2} - 1\right)}$$

"r_2" will be imaginary if $c_x < \beta_2$

So, for $c_x < \beta_2$, $$r_2 = \pm i\left(1 - \frac{c_x^2}{\beta_2^2}\right)^{1/2}$$

Let us focus on z- component of transmitted SH- wave.

i.e.,

$$ik_x r_2 = ik_x(\pm i)\left(1 - \frac{c_x^2}{\beta_2^2}\right)^{1/2}$$

$$= \pm k_x\left(1 - \frac{c_x^2}{\beta_2^2}\right)^{1/2}$$ **;which is real**

so, this wave either decays or grow exponentially away from the interface. We can eliminate the exponentially growing wave because conservation of energy will not hold true. So, we are left with a wave that decays: it is trapped at the interface.

The displacement of the SH-wave can be rewritten as:

$$u_y^T(x,z,t) = B' e^{i(\omega t - k_x x - k_z z)} = B' e^{i(\omega t - k_x x - k_x r_2 z)}$$
$$= B' e^{i\omega t} e^{-ik_x x} e^{-ik_x r_2 z}$$

if, $j_1 < j_c$ then z-component is real

$$e^{-k_x r_2 z} = e^{-k_x \left(1 - \frac{c_x^2}{\beta_2^2}\right)^{1/2} z}$$

This gives rise to evanescent, inhomogeneous or head wave, which is similar to P_n wave that travels along Moho. Moho is the discontinuity which separates the low velocity crust from high velocity mantle.

We can also define a new exponential term as:

$$r_2^* = +i\left(1 - \frac{c_x^2}{\beta_2^2}\right)^{1/2}$$

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So, $r_2^* = -ir_2$ or equivalently, $r_{\beta_2}^* = -ir_{\beta_2}$, for the reflected wave, the reflection coefficient rather being

$$R_{pre} = \frac{\mu_1 r_{\beta_1} - \mu_2 r_{\beta_2}}{\mu_1 r_{\beta_1} + \mu_2 r_{\beta_2}} \qquad \textbf{becomes} \qquad R_{post} = \frac{\mu_1 r_{\beta_1} - i\mu_2 r_{\beta_2}^*}{\mu_1 r_{\beta_1} + i\mu_2 r_{\beta_2}^*}$$

R_{post} in polar form can be rewritten as:

$$R_{post} = e^{i2\theta} \qquad \textbf{where} \quad \theta = \tan^{-1}\left(\frac{\mu_2 r_2^*}{\mu_1 r_1}\right)$$

Its magnitude is 1 and phase shift is 2θ

This phase shift depends upon the angle of incidence. Since,

$$r = \frac{k_z}{k_x}$$

k_z varies with angle

So,

- There is phase shift in reflected ray

$$R = e^{i2\theta} \qquad \textbf{where} \quad \theta = \tan^{-1}\left(\frac{\mu_2 r_2^*}{\mu_1 r_1}\right)$$

- The transmitted ray is trapped

$$u_z = -k_x\left(1 - \frac{c_x^2}{\beta_2^2}\right)^{1/2} z$$

Summary

- For pre-critical waves ($j_i < j_c$), reflection and transmission coefficients will be as derived in the previous lecture.

- For perfectly refracted waves ($j_i = j_c$), R = 1, and T = 2

- For the post-critical waves or evanescent waves ($j_i > j_c$), the transmitted wave is trapped just below the interface. The reflection coefficient is 1, but there is a phase shift.

- Wave will be phase shifted after reflection and transmitted wave will be an evanescent wave that decays exponentially with depth.

$$R = e^{2i\theta}, \ \theta = \tan^{-1}\left(\frac{\mu_2 r_2^{*}}{\mu_1 r_1}\right) \qquad \exp\left[-k_2\left(1 - \frac{c_x^2}{\beta_2^2}\right)^{1/2} z\right]$$

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THANK YOU !