NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 03 : Surface Waves and Dispersion
Lecture 03: Love wave and its characteristics, Modes of Dispersion

CONCEPTS COVERED

- **Love waves and its dispersion**
- **Summary**

Love Waves

Love waves:

1. **Love waves consist of SH motion.**
2. **A positive velocity jump is required for Love waves to exist.**
3. **They are dispersive in nature.**

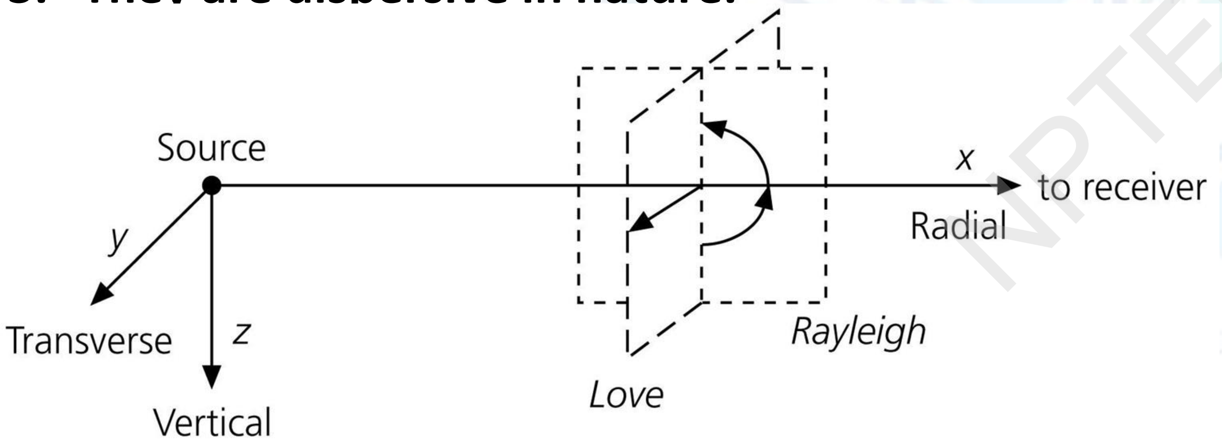


Fig 2.7-2 Geometry for surface waves propagating in a vertical plane containing the source and receiver. Rayleigh(P+SV) waves appear on the vertical and radial components. Love(SH) waves appear on the transverse component.

Love Waves

- **In order to understand Love waves, let's consider a lower velocity layer over an infinite (and higher velocity) half space.**

- **Consider an SH wave travelling with multiple bounces in this layer. Since it is harmonic (i.e. a sine wave), there are multiple places on the wave path with the same phase.**

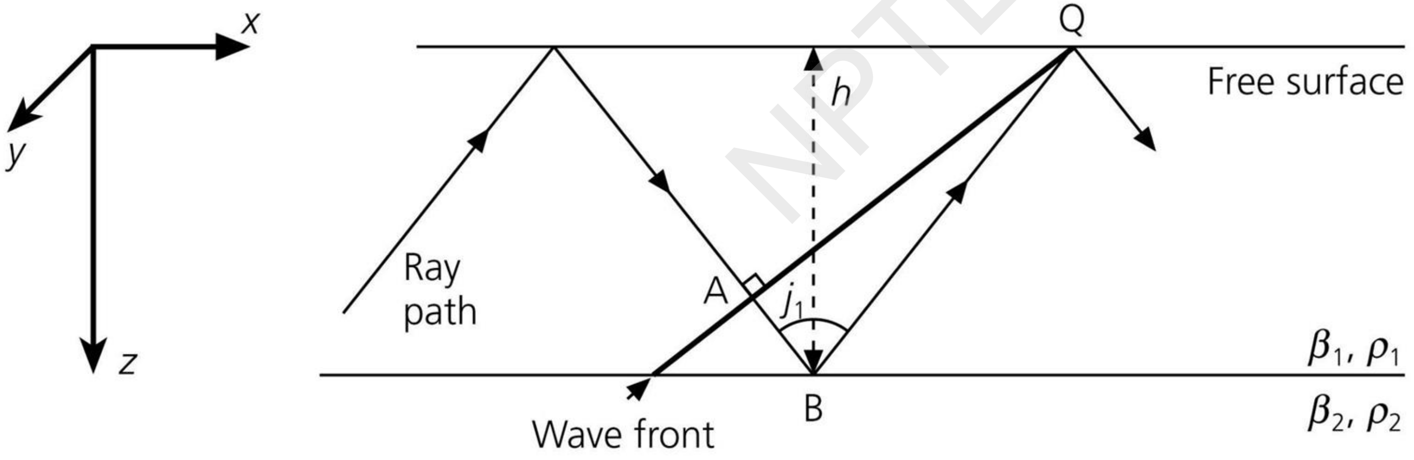


Fig. 2.7-7 Geometry of the Love wave in a layer over a half space

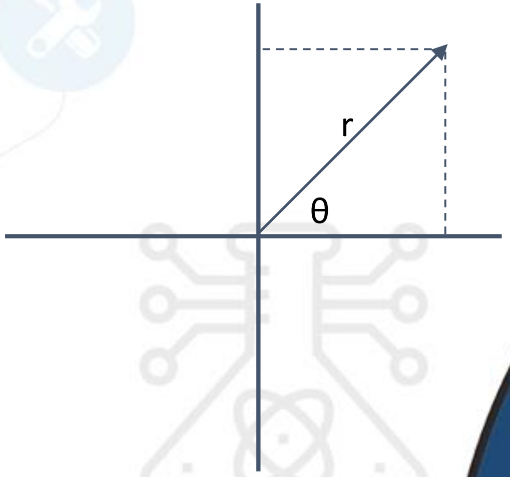
Love Waves

Let's consider case where the wave is such that it undergoes constructive interference at the surface: we'll look for conditions where the phase at point A is the same as point Q.

Before we do so, let's recall how we designate phase. A complex number, $z = re^{i\theta}$, can be represented as such in the complex plane:

So, for a typical SH wave, in which:

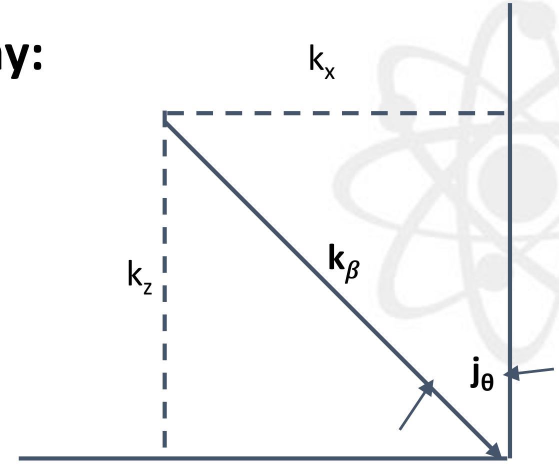
$$u = Be^{i(\omega t - k_x x - k_z z)}$$



The phase is:

$$\phi = \omega t - k_x x - k_z z$$

Rotating the spatial component into the direction of propagation of the ray:



$$u = Be^{i(\omega t - k_\beta d)}$$

where

- **k_β is the wave vector component in the direction of the ray, and**
- **'d' is the distance in the direction of the ray.**

In this case we'll define distance as distance from point A. Some relationships that will come in handy in a bit are

$$\frac{k_z}{k_\beta} = \cos(j)$$

$$k_\beta = \frac{k_z}{\cos(j)} = \frac{\omega}{\beta}$$

The phase for the ray is given by $\phi = \omega t - k_\beta d$

We want to examine the phase difference between the wave at point A and point Q.

If it is some multiples of 2π, then the waves constructively interfere. We'll do this at time t=0.

$$\phi(A) = (\omega t - k_\beta d) = (\omega.0 - k_\beta \cdot 0) = 0$$

$$At\, Q,$$

$$\phi(Q) = (\omega t - k_\beta d) = (\omega \cdot 0 - k_\beta \cdot (AB + BQ))$$

But there is one other factor that needs to be considered.

That is, a phase change will occur when the wave is reflected at the interface between the layer and the half space.

The magnitude of the phase angle is given by:

$$\Delta\phi = 2\tan^{-1}\left(\frac{\mu_2 r^{\star}_{\beta_2}}{\mu_1 r_{\beta_{12}}}\right)$$

Hint:

AB+BQ = BQcos2j + h/cosj
= (cos2j + 1)(h/cosj)
= 2hcosj

So, the condition for constructive interference is thus that the total phase change

$$k_\beta(AB + BQ) + 2\tan^{-1}\left(\frac{\mu_2 r^{\star}_{\beta_2}}{\mu_1 r_{\beta_{12}}}\right) = 2n\pi$$

Using some trigonometric substitutions for AB and BQ, the equation turns into

$$k_\beta(-2h\cos j) + 2\tan^{-1}\left(\frac{\mu_2 r^{\star}_{\beta_2}}{\mu_1 r_{\beta_{12}}}\right) = 2n\pi$$

And since $\frac{k_z}{k_\beta} = \cos(j)\ and\ r_\beta = \frac{k_z}{k_x}$

$$-k_x r_\beta h + \tan^{-1}\left(\frac{\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_{12}}}\right) = n\pi$$

We can take the tangent of both sides to get:

$$-\tan(k_x r_\beta h) + \left(\frac{\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_{12}}}\right) = \tan(n\pi)$$

since tan(nπ) = 0, then

$$\left(\frac{\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_{12}}}\right) = \tan(k_x r_\beta h)$$

We will write in terms of r_β and k_x

As we know,

$$r_\beta = \left(\frac{c_x^2}{\beta^2} - 1\right)^{1/2},\ k_x = \frac{\omega}{c_x}$$

$$\tan\left[\frac{\omega h}{c_x}\left(\frac{c_x^2}{\beta_1^2}-1\right)^{1/2}\right]=\frac{\mu_2\left(1-\frac{c_x^2}{\beta_2^2}\right)^{1/2}}{\mu_1\left(\frac{c_x^2}{\beta_1^2}-1\right)^{1/2}}$$

Because the tangent function is defined only for real values, the square roots above must be real for

$$\frac{c_x^2}{\beta_1^2}>1 \implies \beta_1<c_x$$

$$\frac{c_x^2}{\beta_2^2}>1 \implies \beta_2>c_x$$

So, $\beta_1<c_x<\beta_2$

A graphical solution can be derived by defining a new variable,

$$\zeta = \left(\frac{h}{c_x}\right)\left(\frac{c_x^2}{\beta_1^2} - 1\right)^{1/2} \qquad \beta_1 < c_x < \beta_2$$

Note that

$$\zeta_{\min} = 0 \textbf{ when } c_x = \beta_1$$

$$\zeta_{\max} = h\left(\frac{1}{\beta_1^2} - \frac{1}{\beta_2^2}\right) \textbf{ at } c_x = \beta_2$$

ζ can only vary between these two values, because the conditions necessary for the Love wave require. With ζ, our equation becomes

$$\tan(\omega\zeta) = \left(\frac{\mu_2\left(1 - \frac{c_x^2}{\beta_2^2}\right)^{1/2}}{\mu_1}\right)\left(\frac{h}{c_x\zeta}\right)$$

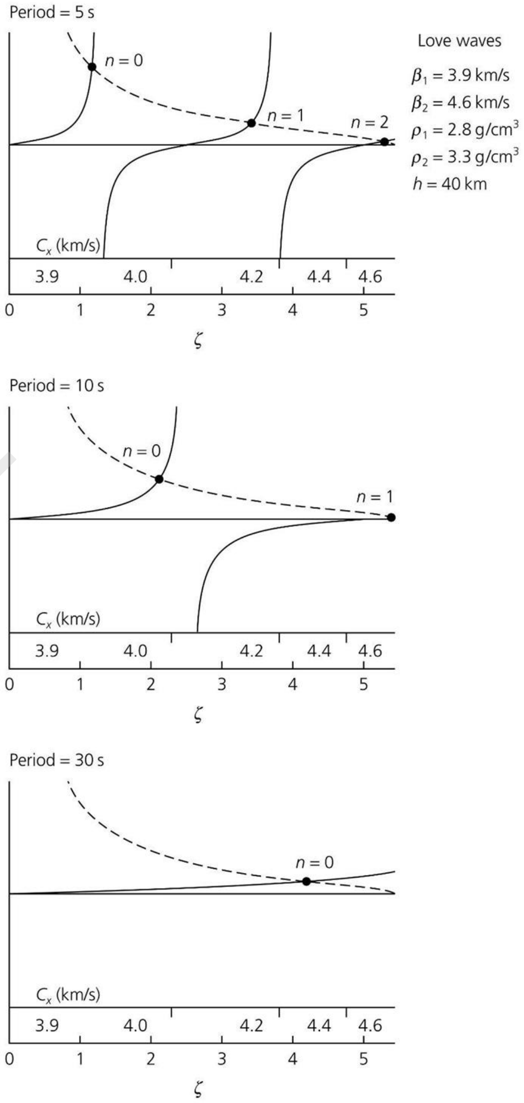
The LHS of the equation is 0 at $\zeta = \frac{n\pi}{\omega}$ and goes to +/- infinity at $\zeta = \frac{m\pi}{2\omega}$ for m = 1,3,5……

The RHS is hyperbolic because of 1/ζ dependence, is infinite for $c_x = \beta_1$, where ζ=0, and decays monotonically to zero at $c_x = \beta_2$, where $\zeta = \zeta_{max}$.

Where the two plots meet are the values of ζ that solve the equation

The lowest c_x (i.e. n=0) for which there is a solution is called "fundamental mode". Higher values of c_x solutions are called higher nodes or overtones.

Figure 2.7-8: Solution of the dispersion relation for Love waves in a layer over a halfspace.



Let us consider the effect of wave period on the tanh (ζ) term. Since tan is 0 at integral multiples of π, then zeros of the tan(ζ) term will occur when,

$$\left(\frac{2\pi}{T}\right)\zeta = n\pi$$

This occurs when

$$\zeta = \frac{nT}{2}$$

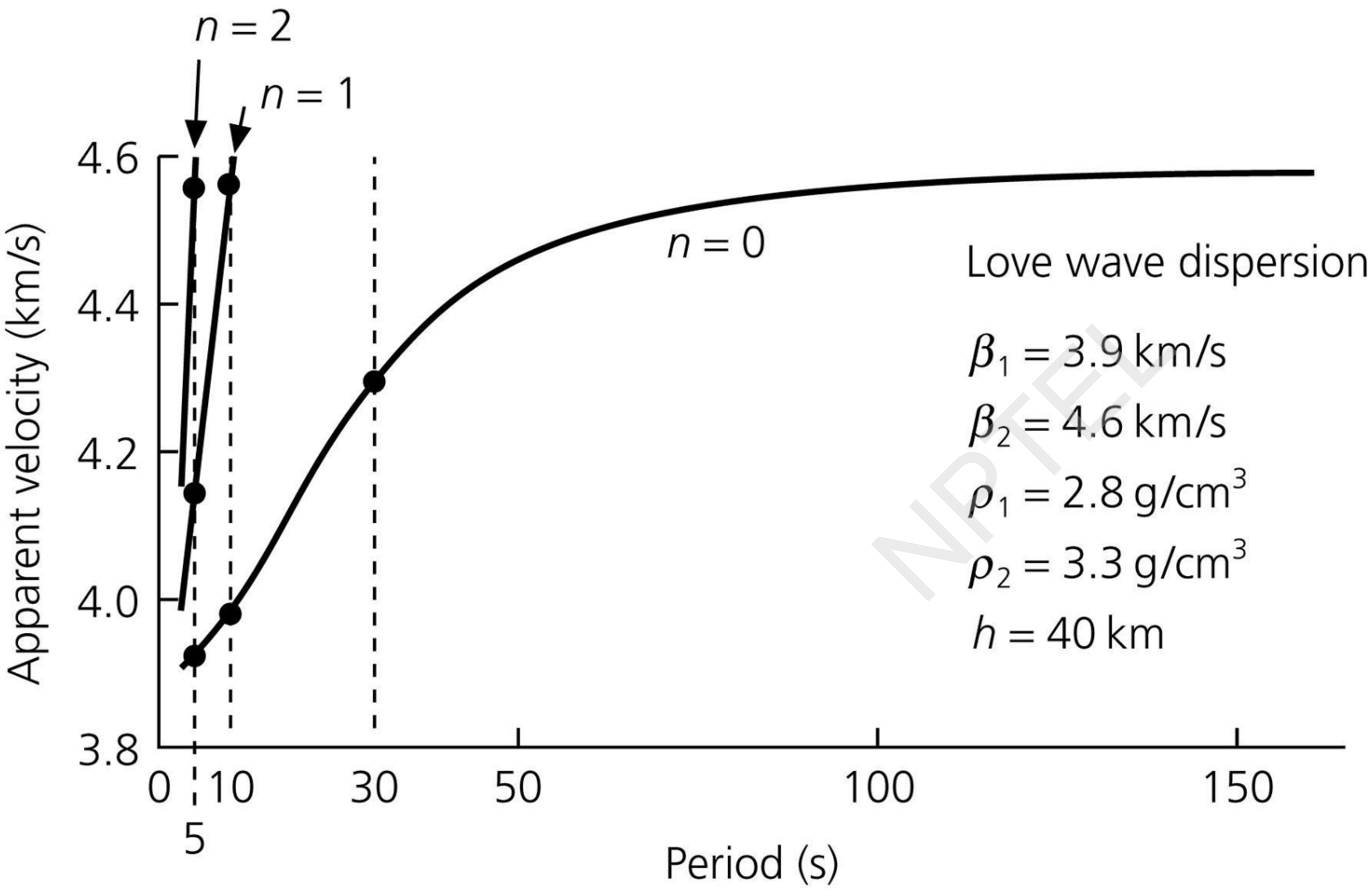
So, when wave period goes up, the amount that ζ has to change to get from one zero to the next increases.

$$\zeta_{n+1} - \zeta_n = \frac{T}{2}$$

This causes the normal modes to be further apart.

We can plot mode velocity (c_x) verses wave period to get.

Figure 2.7-9: Dispersion curves for Love waves in a layer over a halfspace.



Note that fundamental mode velocity increases with increasing wave period. Thus Love waves are inherently dispersive.

Summary

- Love waves have SH motion , need positive velocity jump or gradient to exist, are inherently dispersive.

- The condition for the constructive interference at surface is given by

$$k_\beta(AB + BQ) + 2\tan^{-1}\left(\frac{\mu_2 r^\star_{\beta_2}}{\mu_1 r_{\beta_{12}}}\right) = 2n\pi \qquad \tan(\omega\zeta) = \left(\frac{\mu_2\left(1 - \frac{c_x^2}{\beta_2^2}\right)^{1/2}}{\mu_1}\right)\left(\frac{h}{c_x\zeta}\right)$$

where

$$\zeta = \left(\frac{h}{c_x}\right)\left(\frac{c_x^2}{\beta_1^2} - 1\right)^{1/2}$$

- The lowest c_x for which there is a solution is called "fundamental mode". Higher values of c_x solutions are called higher nodes or overtones.
- To make love wave we need either "velocity gradient" or curved surface.

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THANK YOU !