NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 04 : Normal modes of the earth
Lecture 02: Torsional and Spheroidal Modes, Splitting, Normal Modes Synthetic Seismogram

CONCEPTS COVERED

- Modes Separation
- Torsional Modes
- Spheroidal Modes
- Normal Modes Synthetic Seismogram
- Animations of different modes

Recap

- Normal modes are "Oscillation of the system" and is sum of the standing waves. Example, whole earth vibration after a large earthquake, free oscillation of sun.

  For displacement in 3D:

$$\mathbf{u}(r,\theta,\phi) = \sum_n \sum_l \sum_m {}_nA_l^m \, {}_ny_l(r)\mathbf{x}_l^m(\theta,\phi)e^{i_n\omega_l^m t}$$ Displacement

n,l, m-radial, angular and azimuthal order

${}_ny_l(r)$-scaler radial eigenfunction

$\mathbf{x}_l^m(\theta,\phi)$ - vector surface eigenfunction

${}_nA^m{}_l$- excitation amplitudes (weights for eigenfunctions) that depend on the seismic source

Recap

- Two types of Normal modes: Spheroidal modes; analogous to P-SV motion.
  Toroidal Modes; analogous to love of SH motion.
- Boundary condition on the bounded string (at x=0 and x=L) produces normal modes, and solution is given by

$$Y(x,t)=\sum_{n=0}^{\infty} A_n Y_n(x,\omega_n)\cos(\omega_n t) \qquad Y_n(x,\omega_n) = \sin(\omega_n x/v)$$

- Similarly, fully normalized and orthogonal spherical harmonics for earth is given by

$$Y_l^m(\theta,\phi) = (-1)^m \left[\left(\frac{2l+1}{4\pi}\right)\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m(\cos\theta)e^{im\phi}$$

Where n,l and m are radial, angular and azimuthal order.

Modes Separation

As we have seen in previous module that P-SV and SH behaviour can be separated out (like with Rayleigh and Love waves). We can perform same with normal modes.

Spheroidal mode (${}_nS^m_l$)	Toroidal mode (${}_nT^m_l$)
Have horizontal (tangential) and vertical (radial) components to motion.	Have no radial component to motion-motion is confined to the surface of n concentric spheres within the Earth.
No simple relationship between n and nodal spheres.	Do not have volume changes
${}_0S^0_2$ is the fundamental mode.	Do not exist in a fluid, since fluids can not have shear.
Affect the whole Earth, from the surface to the core.	

(From Derek Schutt, CSU)

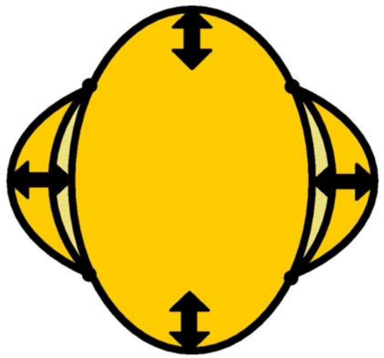
Physical effects of n, l, m

Spheroidal:

n: no direct relationship with nodes with depth

l: number of nodal planes in latitude. *“Singlet”*

m: number of nodal planes in longitude m = -l to +l. “*multiplet*”



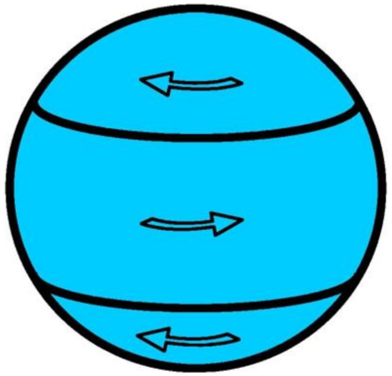
$_0S^0_2$

Toroidal:

n: nodal planes with depth

l: l-1 = number of nodal planes in latitude. *“Singlet”*

m: number of planes in longitude m=-l to +l. *“multiplet”*



$_0T^0_3$

Torsional or (Toroidal) Modes (Analogous to SH waves)

Surface eigenfunctions given by a vector spherical harmonics:

$$\mathrm{T}_l^m(r,\theta,\,\phi) = \left(0,\ \frac{1}{\sin\theta}\frac{\partial Y_l^m(\theta,\,\phi)}{\partial\phi},\ -\frac{\partial Y_l^m(\theta,\,\phi)}{\partial\theta}\right)$$

The displacements are given by

$$\mathbf{u}^T(r,\theta,\phi) = \sum_n \sum_l \sum_{m=-l}^{l} \underbrace{{}_nA_l^m}_{amplitude}\ \underbrace{{}_nW_l(r)}_{radial\ eigenfunction}\ \underbrace{{}_n\mathbf{T}_l^m(\theta,\phi)}_{\theta,\phi\ motion}\, e^{i\,{}_n\omega_l^m t}$$

where u is a vector (u_r, u_θ, u_ϕ)

${}_nW_l(r)$ – **The radial eigenfunction (varies with depth)**

Note: Radial eigenfunction involves complex calculations so we will keep it out from discussion.

For $_nT^m$

$\mathrm{n} = \textbf{radial order}, \mathrm{l} = \textbf{angular order}, \mathrm{m} = \textbf{azimuthal order}$

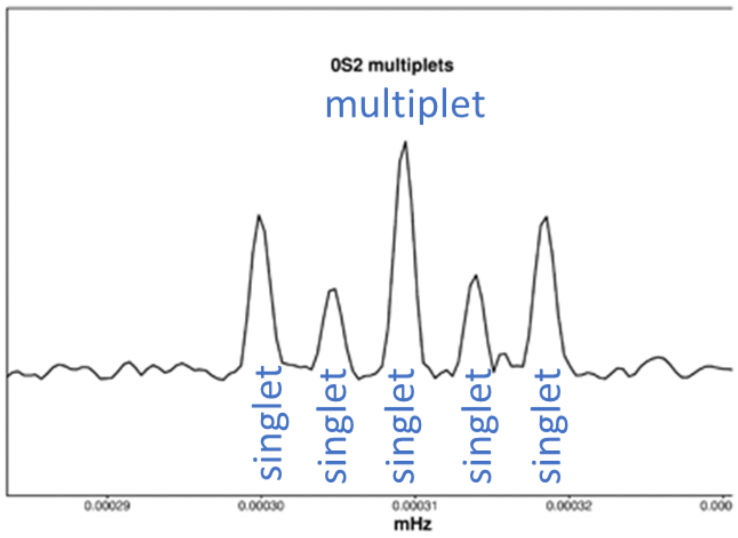
The $2l+1$ modes of different azimuthal orders $-l \leq m \leq l$ are called singlets and group of singlets is called a multiplet.

For SNREI (Solid Non-Rotating Earth that is Isotropic), all singlets in a multiplet (for constant l and n, but m varies) would have the same eigenfrequency. This is called "Degeneracy". No degeneracy if no spherical symmetry.

For each m = one singlet
The 2l+1 group of singlets = multiplet

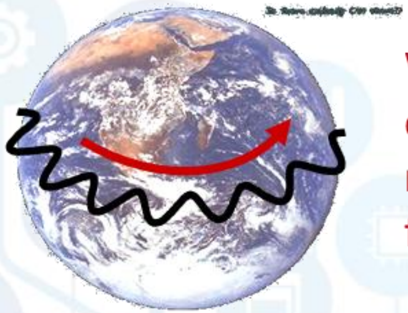
For example, the period of $_nT^0_l$ would be same for $_nT_l^{\pm1}$,$_nT_l^{\pm2}$,$_nT_l^{\pm3}$, etc. In the real earth, singlet frequencies vary (called splitting).

The splitting is usually small enough to ignore, so we drop the m superscript and refer to entire $_nT_l^m$ multiplet as $_nT_l$, with eigenfrequency $_n\omega_l$



From Derek Schutt, CSU

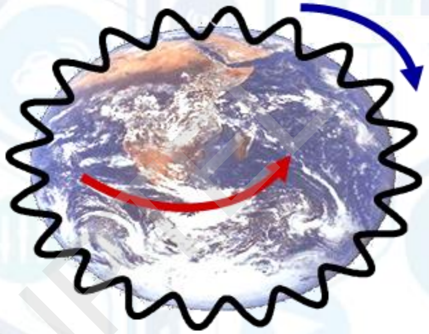
⇨ Rotation (Coriolis)



Waves in the direction of rotation travel faster

(From Derek Schutt, CSU)

⇨ Ellipticity

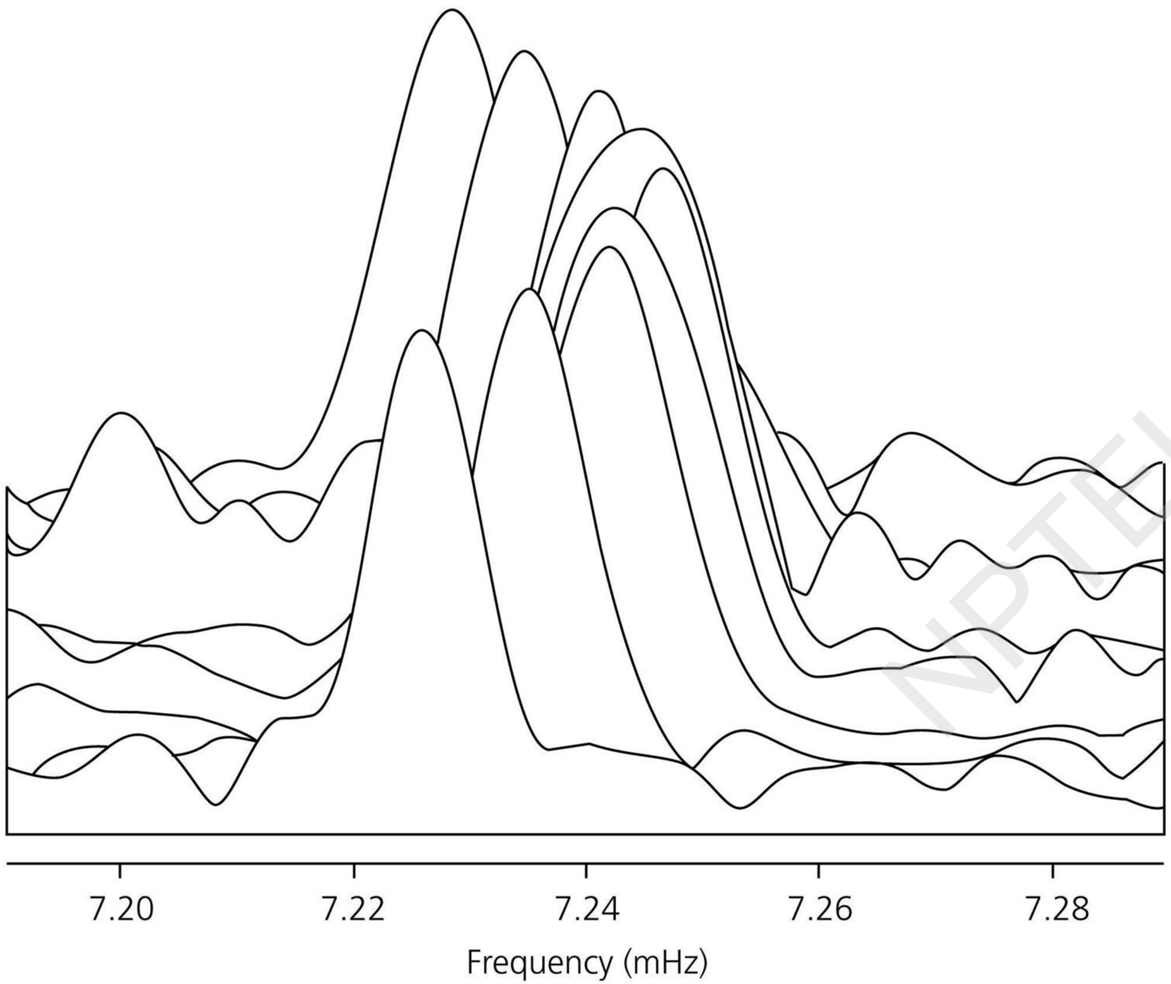


Waves from pole to pole run a shorter path (67 km) than along the equator

⇨ 3D

Waves slowed down (or accelerated) by heterogeneities

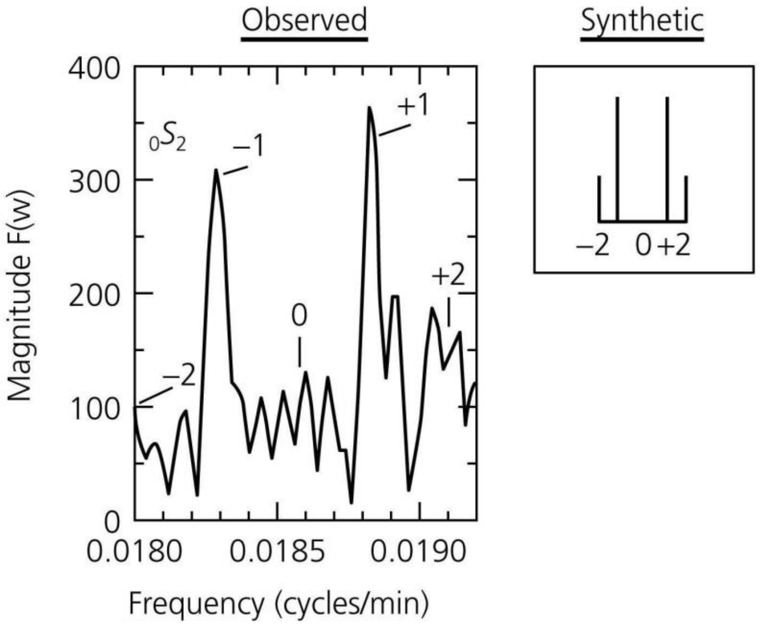
Figure 2.9-15: Example of singlets for a split spheroidal mode multiplet.

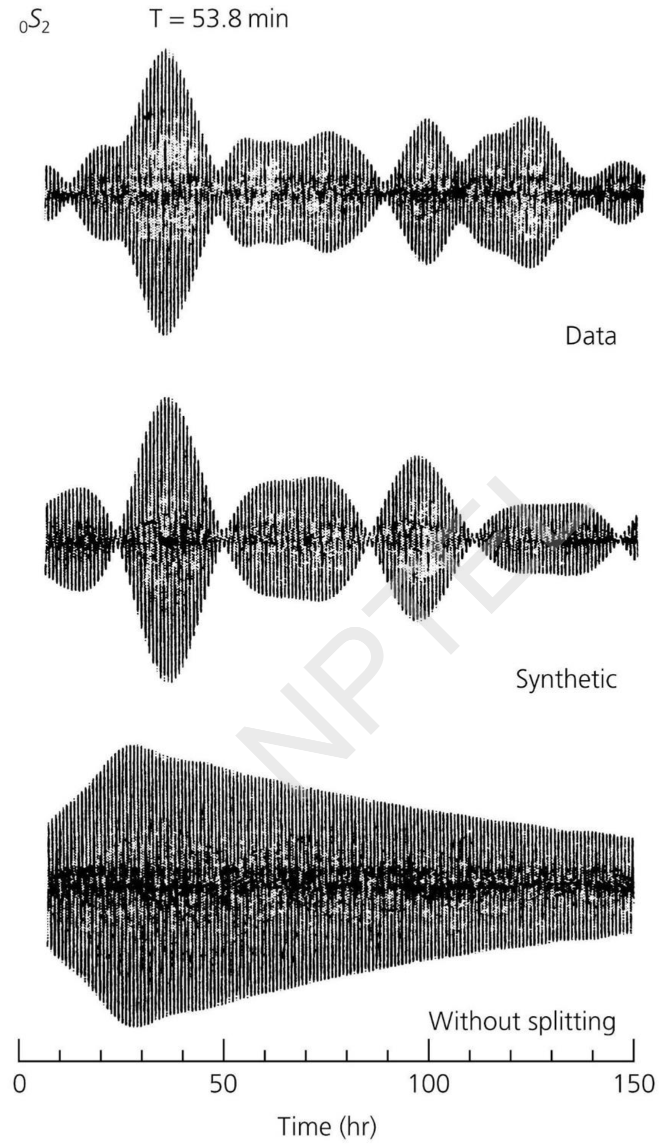


Amplitude spectra of the nine singlets of the split spheroidal mode multiplet $_{18}S_4$. The m=-4 singlet is in front, and the m=4 singlet is in back.

Splitting results from Earth's asphericity, inhomogeneity, anisotropy, and rotation.

Figure 2.9-16: Splitting of the $_0S_2$ mode for wave from the 1960 Chile earthquake.





- Splitting observations for the football mode $_0S_2$ from the great 1960 Chilean earthquake.

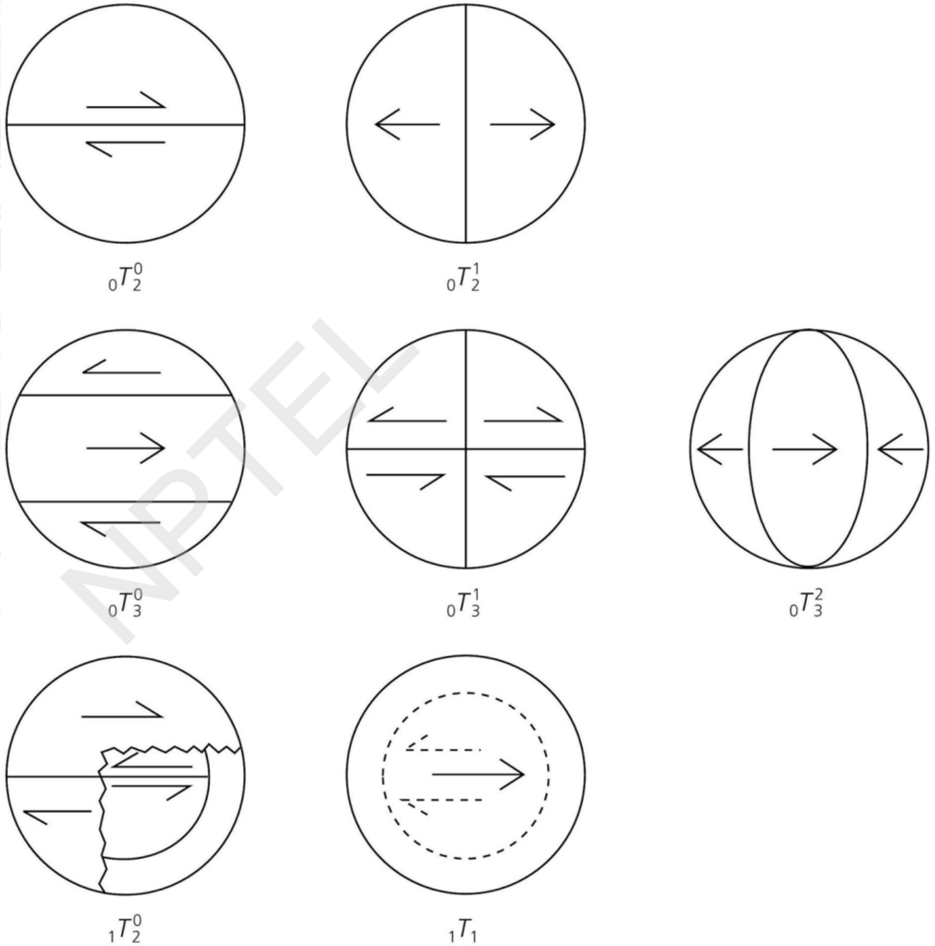
- Splitting causes the singlets to stand out as distinct peaks in the spectrum and the time series.

- A synthetic seismogram, computed by predicting the singlet amplitudes and combining them in the time domain with the effect of attenuation and finite seismogram length matches the data better than a similar synthetic seismogram without splitting.

Torsional modes with n=0(${}_0T^m_l$) are called fundamental modes. (motions at depth in the same direction as at the surface)

Modes with n>0 are called overtones. (motions reverse direction at different depths)

Figure 2.9-6: Examples of the displacements for several torsional modes.



Spheroidal (Poloidal) modes (involving P-SV motions)

The surface eigenfunctions are given by two other vector spherical harmonics with (r, θ, ϕ) components

$$\mathrm{R}_l^m = (Y_l^m, 0, 0)$$

$$\mathbf{S}_l^m = \left(0, \frac{\partial Y_l^m(\theta, \phi)}{\partial \theta}, \frac{1}{\sin\theta}\frac{\partial Y_l^m(\theta, \phi)}{\partial \phi}\right)$$

Each corresponds to a different radial eigenfunction, $_nU_l(r)$ and $_nV_l(r)$, so the displacement for spheroidal modes is

$$\mathrm{u}^s(r, \theta, \phi) = \sum_n \sum_l \sum_{m=-l}^{l} {}_nA_l^m \left[{}_nU_l(r) R_l^m(\theta, \phi) + {}_nV_l(r) \mathbf{S}_l^m(\theta, \phi)\right] e^{{}_n\omega_l^m t}$$

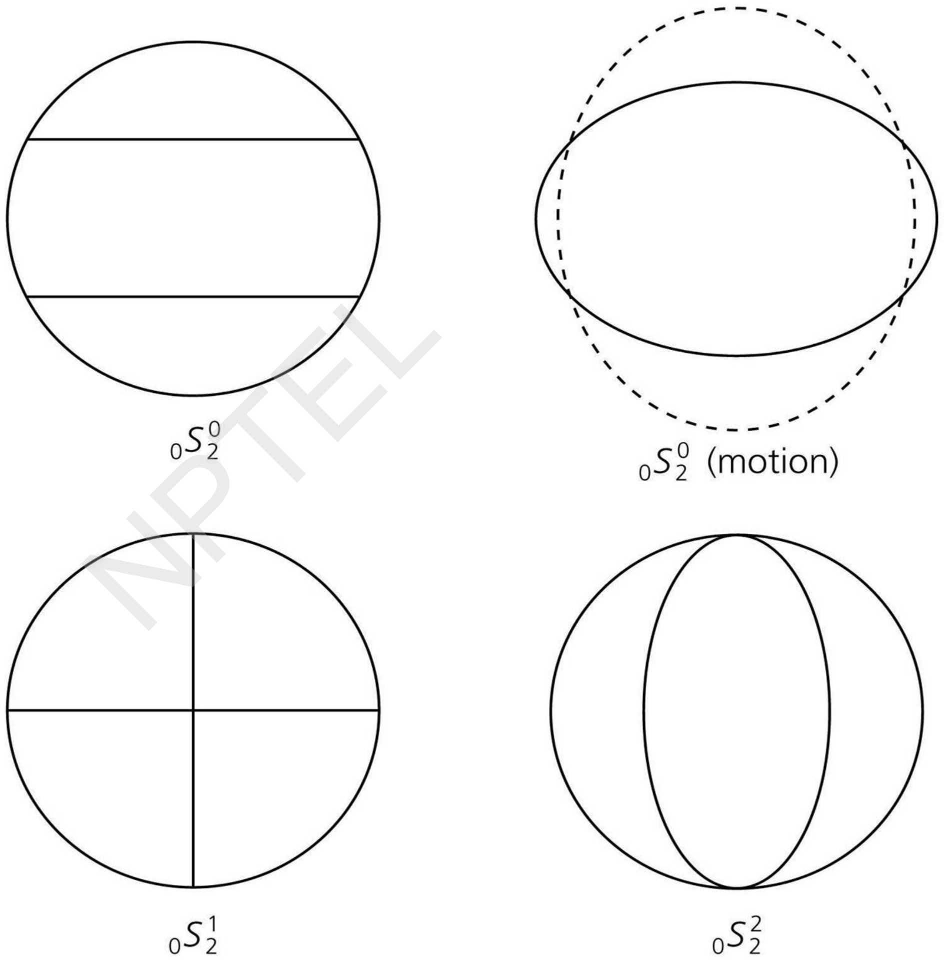
The radial eigenfunction $_nU_l(r)$ corresponds to radial motion and $_nV_l(r)$ corresponds to horizontal motion.

$_0S_2$(football mode) is the gravest (lowest frequency or longest period) of earth's modes, with a period of 3233s, or 54 minutes.

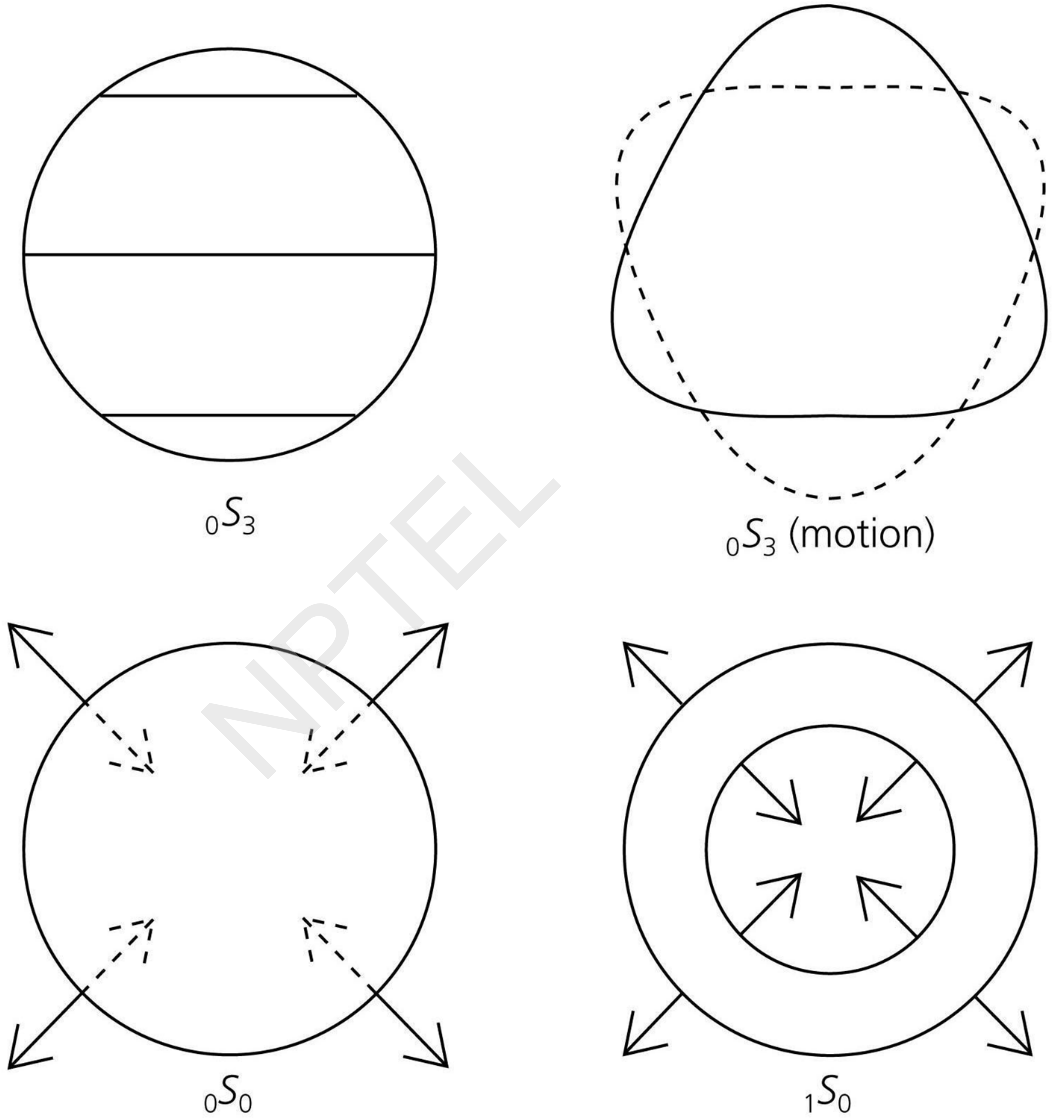
There is no $_0S_1$ mode, which would correspond to a lateral translation of the planet .

The $_1S_1$ slitcher mode due to up and down motion of the inner core through the liquid iron outer core, which has yet to be observed, should in theory have a period of about 5 ½ hours.

Figure 2.9-7: Examples of the displacements for several spheroidal modes.



The "breathing" mode $_{0}S_{0}$ involves radial motions of the entire earth that alternate between expansion and contraction.



How normal modes are useful?

Frequencies of the normal modes tell us

- the shape of the Earth
- bulk modulus
- shear modulus
- density

Normal model can also help with followings:

-Simulating phenomena like diffraction that is not possible with ray theory

-Eigenfrequencies are sensitive to earth structure in a way that compliments other methods

- More efficient way of creating synthetic seismograms that correspond to long time periods, or for longer period surface waves

- inferring distant earthquake focal mechanisms

- Inferring earthquake magnitude

Normal mode synthetic seismograms

There are various techniques to compute a synthetic seismogram, one of these is normal mode summation, analogous to the way the propagating waves on the string were generated.

Spheroidal mode displacements are synthesized by

$$\mathrm{u}^{T}(r_r,\ \theta_r,\ \phi_r)=\sum_{n}\sum_{l}\sum_{m=-l}^{l}{}_{n}A_l^m(r_s, r_r)\,{}_{n}W_l(r_r)\mathbf{S}_l^m(\theta,\ \phi)e^{i\,{}_{n}\omega_l^m t}\,e^{\left(\frac{{}_{n}\omega_l^m t}{2\,{}_{n}Q_l}\right)}$$

$e^{\left(\frac{{}_{n}\omega_l^m t}{2\,{}_{n}Q_l}\right)}$ – **The attenuation of the mode**

$2\,{}_{n}Q_l$ – **quality factor of the mode measures the rate at which the mode's energy is lost by friction.**

After Q cycles of oscillations, the amplitude of a mode has fallen to a level of $e^{-\pi}$ or 4% of the original amplitude

$_nW_l$ - mode's radial eigenfunction

$_n\omega_l$ - mode's eigenfrequencies

determined by the earth's velocity and density structure.

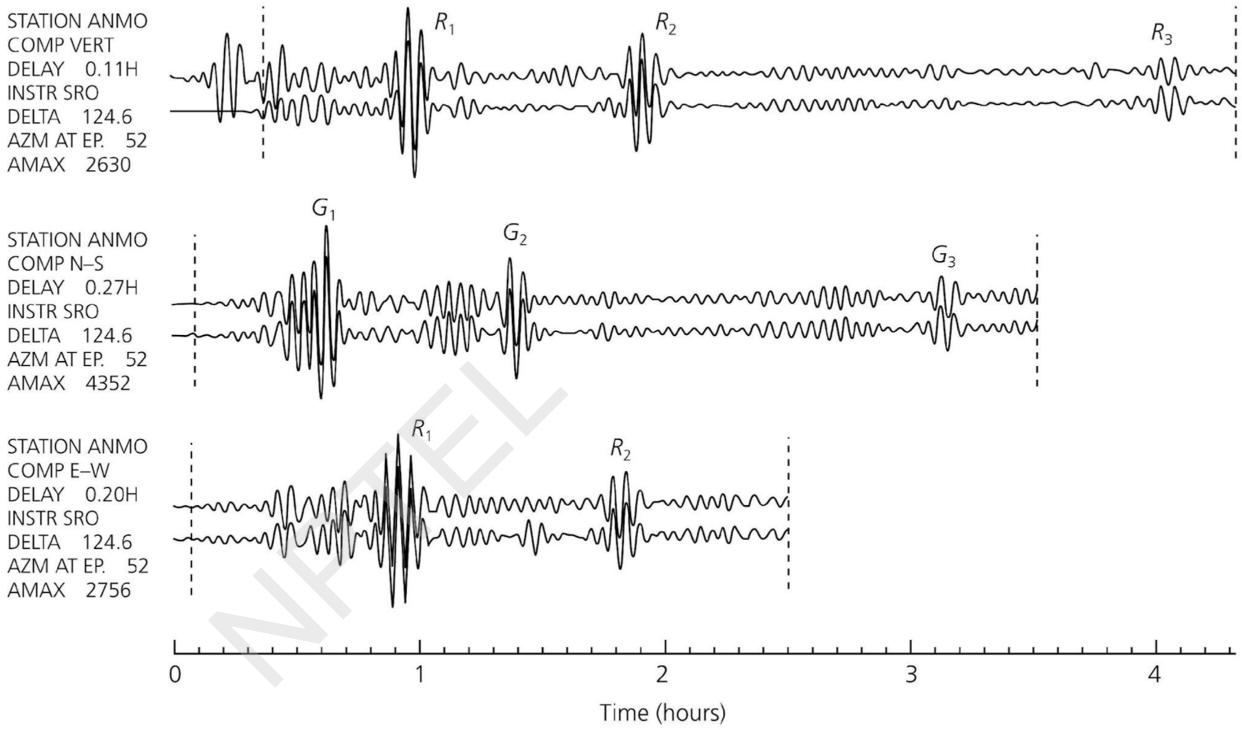
- **These modes are then weighted by excitation amplitudes $_nA^m_l$, determined by the depth, geometry, and time history of the seismic source and the depth of the receiver.**

- **This formulation assumes that all singlets in a multiplet have the same quality factor.**

Figure shows a comparison of observed seismograms created using normal mode summation.

The fits are good enough that many studies use observed normal mode amplitudes to find the fault geometry and focal depth of earthquakes, especially when they are large and remote from seismometers.

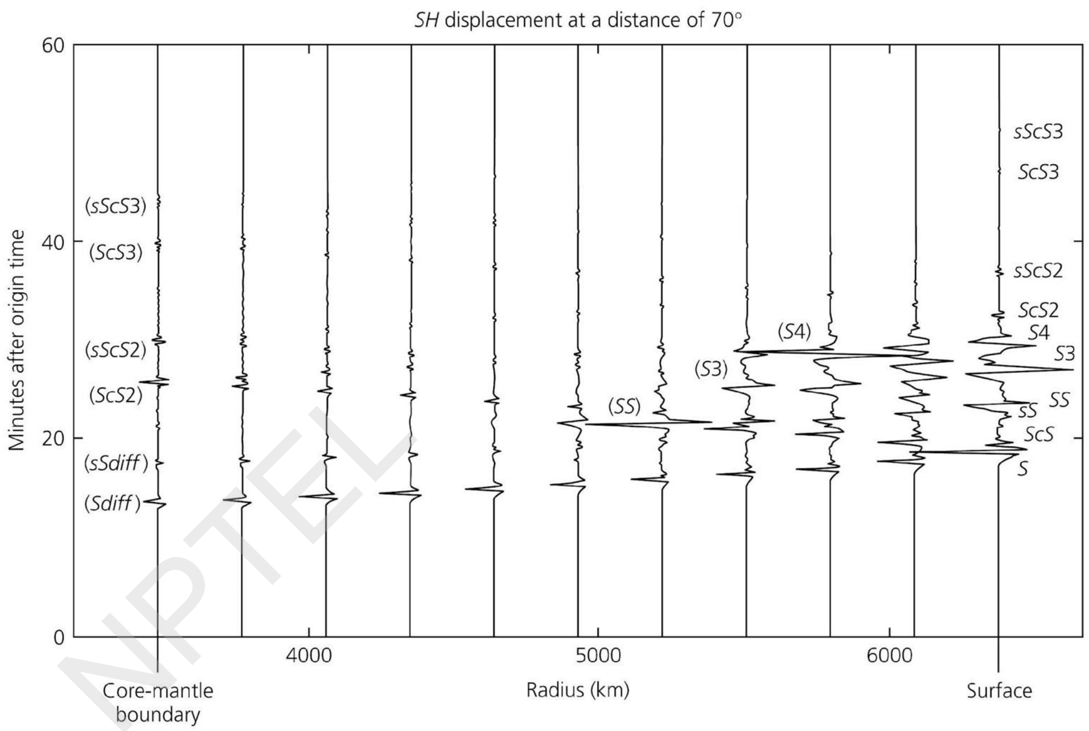
Figure 2.9-13: Example of modeling data with normal mode synthetic seismograms.



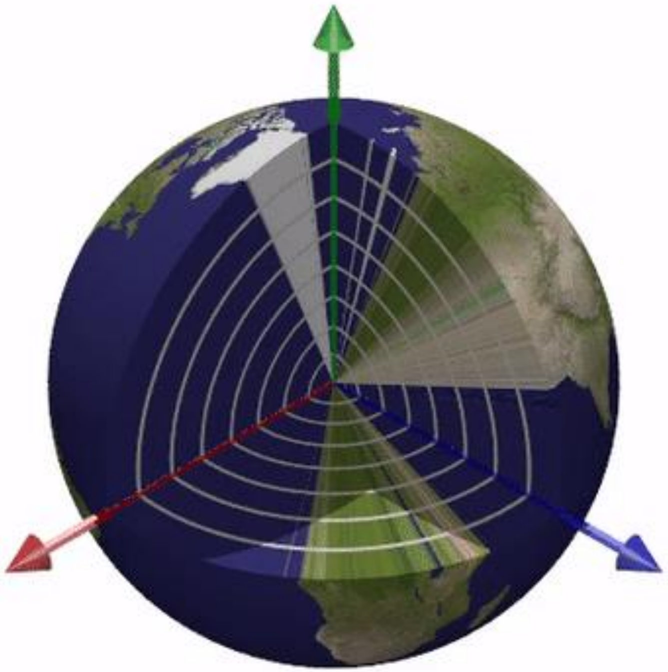
- **It is worth noting that while the synthetic receivers are usually placed at the surface, they can also be computed for any depth within the Earth.**

- **Figure shows a record section that would be recorded at a distance of 70 degrees from an earthquake if seismometers are placed at depths ranging from surface to the core-mantle boundary.**

Figure 2.9-14: Shear wave synthetic seismograms computed at a series of depths.



Animations of different modes



Spheroidal n =0, l = 0, m=0

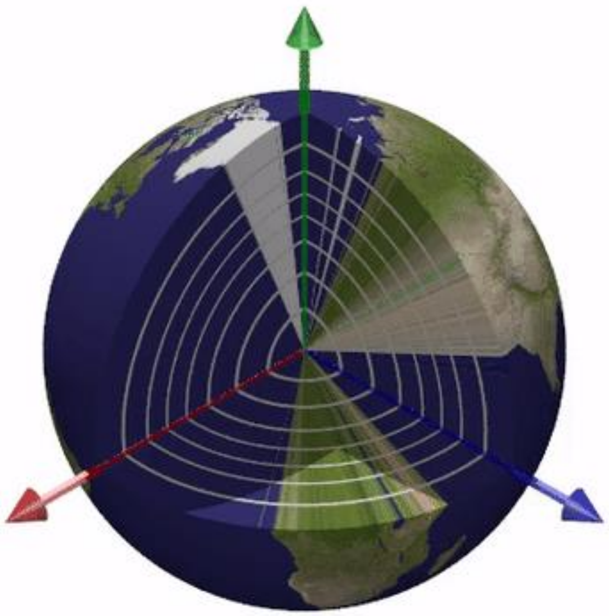
period ~20 min

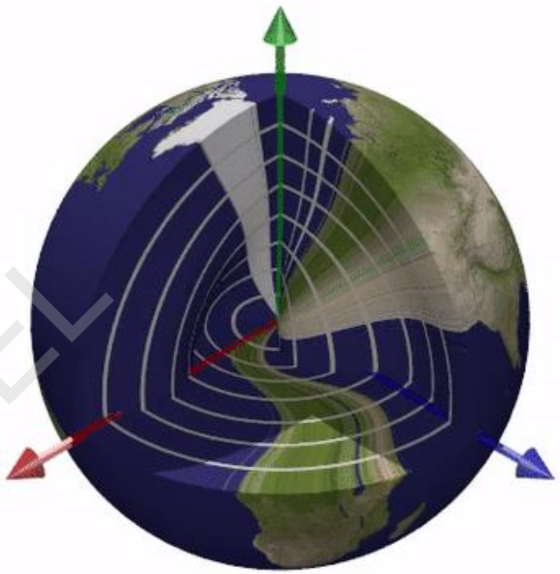
https://saviot.cnrs.fr/terre/index.en.html

Animations of different modes

spheroidal n =1, l = 0, m=0

period ~10 min





Torsional n =1, l = 1, m=0

period ~8 min

https://saviot.cnrs.fr/terre/index.en.html

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- https://www.wikipedia.org/
- Seismology course, Professor Derek Schutt, Colorado State Univ., USA.

THANK YOU !