NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 04 : Normal modes of the Earth

Lecture 03: Solving numerical problems

CONCEPTS COVERED

- **Recap**
- **Solving numerical problems**

Recap

- Normal modes are "Oscillation of the system" and is sum of the standing waves. Example, whole earth vibration after a large earthquake, free oscillation of sun.

For displacement in 3D:

$$\mathbf{u}(r,\theta,\phi) = \sum_n \sum_l \sum_m {}_nA_l^m \, {}_ny_l(r)\mathbf{x}_l^m(\theta,\phi)e^{i_n\omega_l^m t}$$

Displacement

n,l, m-radial, angular and azimuthal order

$_ny_l(r)$-scaler radial eigenfunction

$\mathbf{x}_l^m(\theta,\phi)$ - vector surface eigenfunction

$_nA^m_l$- excitation amplitudes (weights for eigenfunctions) that depend on the seismic source

Recap

- Two types of Normal modes: Spheroidal modes; analogous to P-SV motion.
  Toroidal Modes; analogous to love of SH motion.
- Boundary condition on the bounded string (at x=0 and x=L) produces normal modes, and solution is given by

$$Y(x,t)=\sum_{n=0}^{\infty} A_n Y_n(x,\omega_n)\cos(\omega_n t) \qquad Y_n(x,\omega_n) = \sin\left(\omega_n x/v\right)$$

- Similarly, fully normalized and orthogonal spherical harmonics for earth is given by

$$Y_l^m(\theta,\phi) = (-1)^m \left[\left(\frac{2l+1}{4\pi}\right)\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m(\cos\theta)e^{im\phi}$$

Where n,l and m are radial, angular and azimuthal order.

Recap

Surface eigenfunctions for Toroidal modes are given by a vector spherical harmonics:

$$\mathrm{T}_l^m(r,\theta,\,\phi)=\left(0,\;\frac{1}{\sin\theta}\frac{\partial Y_l^m(\theta,\,\phi)}{\partial\phi},\;-\frac{\partial Y_l^m(\theta,\,\phi)}{\partial\theta}\right)$$

The displacements are given by

$$\mathbf{u}^T\left(r,\theta,\phi\right)=\sum_n\sum_l\sum_{m=-l}^{l}\underbrace{{}_nA_l^m}_{amplitude}\;\underbrace{{}_nW_l\left(r\right)}_{radial\;eigenfunction}\;\underbrace{{}_n\mathbf{T}_l^m\left(\theta,\phi\right)}_{\theta,\phi\;motion}e^{i\,{}_n\omega_l^m t}$$

Recap

For spheroidal mode, the surface eigenfunctions are given by two other vector spherical harmonics with (r, θ, ϕ) components

$$\mathrm{R}_l^m = (Y_l^m, 0, 0)$$

$$\mathbf{S}_l^m = \left(0, \frac{\partial Y_l^m(\theta, \phi)}{\partial \theta}, \frac{1}{\sin\theta}\frac{\partial Y_l^m(\theta, \phi)}{\partial \phi}\right)$$

Each corresponds to a different radial eigenfunction, ${}_nU_l(r)$ and ${}_nV_l(r)$, so the displacement for spheroidal modes is

$$\mathbf{u}^s(r, \theta, \phi) = \sum_n \sum_l \sum_{m=-l}^{l} {}_nA_l^m \left[{}_nU_l(r) R_l^m(\theta, \phi) + {}_nV_l(r)\mathbf{S}_l^m(\theta, \phi)\right] e^{{}_n\omega_l^m t}$$

The radial eigenfunction ${}_nU_l(r)$ corresponds to radial motion and ${}_nV_l(r)$ corresponds to horizontal motion.

Problem 1. For m =1, l =2

a) What is $P_1^m(\cos\theta)$?

a) What is $Y_2^1(\theta,\,\phi)$?

a) What is T_2^1 ?

Problem 2. Calculate surface eigenfunctions R_2^2, S_2^2, and T_2^2 ?

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THANK YOU !