



NPTTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 04 : Normal modes of the Earth
Lecture 03: Solving numerical problems

CONCEPTS COVERED

- **Recap**
- **Solving numerical problems**

NPTEL



Recap

- ❖ Normal modes are “Oscillation of the system” and is sum of the standing waves. Example, whole earth vibration after a large earthquake, free oscillation of sun.

For displacement in 3D:

$$\mathbf{u}(r, \theta, \phi) = \sum_n \sum_l \sum_m {}_n A_l^m {}_n y_l(r) \mathbf{x}_l^m(\theta, \phi) e^{i n \omega_l^m t} \quad \text{Displacement}$$

n, l, m -radial, angular and azimuthal order

${}_n y_l(r)$ -scalar radial eigenfunction

$\mathbf{x}_l^m(\theta, \phi)$ - vector surface eigenfunction

${}_n A_l^m$ - excitation amplitudes (weights for eigenfunctions) that depend on the seismic source

Recap

- ❖ Two types of Normal modes: Spheroidal modes; analogous to P-SV motion.
Toroidal Modes; analogous to love of SH motion.
- ❖ Boundary condition on the bounded string (at $x=0$ and $x=L$) produces normal modes, and solution is given by

$$Y(x,t) = \sum_{n=0}^{\infty} A_n Y_n(x, \omega_n) \cos(\omega_n t) \quad Y_n(x, \omega_n) = \sin(\omega_n x / v)$$

- ❖ Similarly, fully normalized and orthogonal spherical harmonics for earth is given by

$$Y_l^m(\theta, \phi) = (-1)^m \left[\left(\frac{2l+1}{4\pi} \right) \frac{(l-m)!}{(l+m)!} \right]^{1/2} P_l^m(\cos \theta) e^{im\phi}$$

Where n, l and m are radial, angular and azimuthal order.

Recap

Surface eigenfunctions for Toroidal modes are given by a vector spherical harmonics:

$$\mathbf{T}_l^m(r, \theta, \phi) = \left(0, \frac{1}{\sin \theta} \frac{\partial Y_l^m(\theta, \phi)}{\partial \phi}, -\frac{\partial Y_l^m(\theta, \phi)}{\partial \theta} \right)$$

The displacements are given by

$$\mathbf{u}^T(r, \theta, \phi) = \sum_n \sum_l \sum_{m=-l}^l \underbrace{A_l^m}_n \underbrace{W_l(r)}_n \underbrace{\mathbf{T}_l^m(\theta, \phi)}_n e^{i_n \omega_l^m t}$$

amplitude radial eigenfunction θ, ϕ motion

Recap

For spheroidal mode, the surface eigenfunctions are given by two other vector spherical harmonics with (r, θ, ϕ) components

$$\mathbf{R}_l^m = (Y_l^m, 0, 0)$$
$$\mathbf{S}_l^m = \left(0, \frac{\partial Y_l^m(\theta, \phi)}{\partial \theta}, \frac{1}{\sin \theta} \frac{\partial Y_l^m(\theta, \phi)}{\partial \phi} \right)$$

Each corresponds to a different radial eigenfunction, ${}_n U_l(r)$ and ${}_n V_l(r)$, so the displacement for spheroidal modes is

$$\mathbf{u}^s(r, \theta, \phi) = \sum_n \sum_l \sum_{m=-l}^l {}_n A_l^m [{}_n U_l(r) \mathbf{R}_l^m(\theta, \phi) + {}_n V_l(r) \mathbf{S}_l^m(\theta, \phi)] e^{n \omega_l^m t}$$

The radial eigenfunction ${}_n U_l(r)$ corresponds to radial motion and ${}_n V_l(r)$ corresponds to horizontal motion.

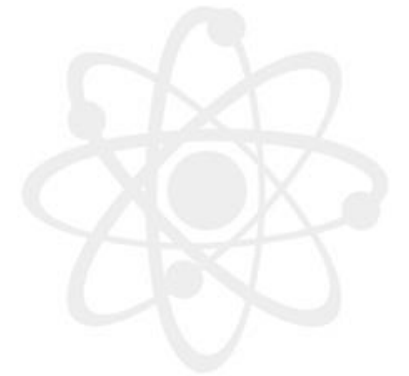
Problem 1. For $m = 1, l = 2$

a) What is $P_1^m(\cos \theta)$?

a) What is $Y_2^1(\theta, \phi)$?

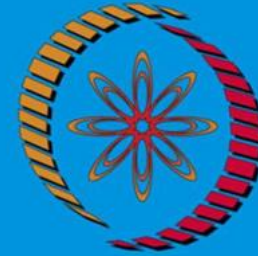
a) What is T_2^1 ?

Problem 2. Calculate surface eigenfunctions R_2^2 , S_2^2 , and T_2^2 ?



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**THANK
YOU!**