NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 05 : Refraction and Reflection seismology
Lecture 03: Reflection seismology, Travel time curve, RMS velocity and Dix equation

CONCEPTS COVERED

- Introduction
- Reflection Seismology
- Travel time curve for reflection event
- Relation between travel time and ray path
- Travel time curve for multiple layers
- Dix equation

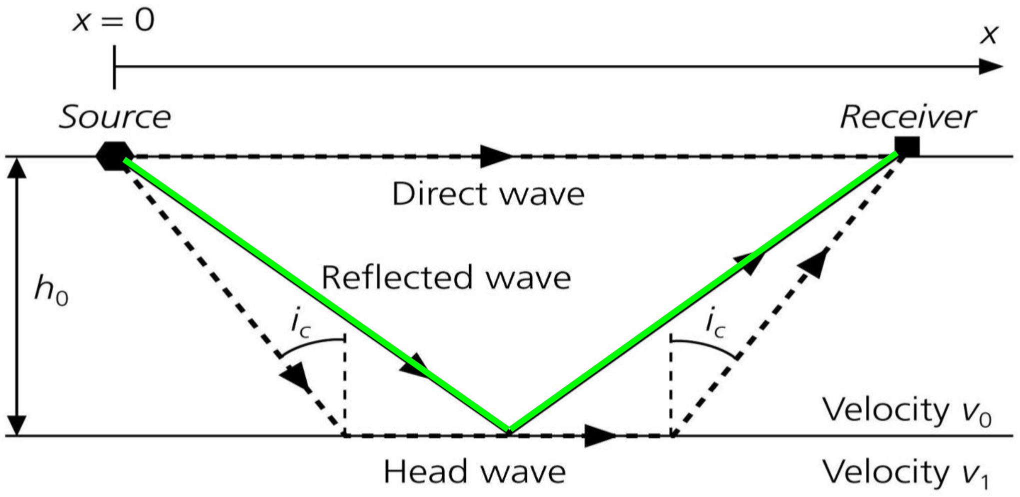
Reflection Seismology

- **Studies which use reflected arrivals to determine velocities within the crust, and other geophysical parameters is known as "Reflection seismology". It is essential in oil and gas exploration.**

- **Due to its abundant use in oil and gas exploration, the data acquisition and processing methods are first developed by reflection seismologists.**

Travel time curve for reflection event

Figure 3.2-1: Ray paths for a layer over a halfspace.



Travel time as a function of source-to-receiver distance known as offset in reflection seismology is

$$T(x)^2 = \frac{x^2}{v_0^2} + 4\frac{h_0^2}{v_0^2} = \frac{x^2}{v_0^2} + t_0^2$$

$$T(x)^2 = \frac{x^2}{v_0^2} + t_0^2$$

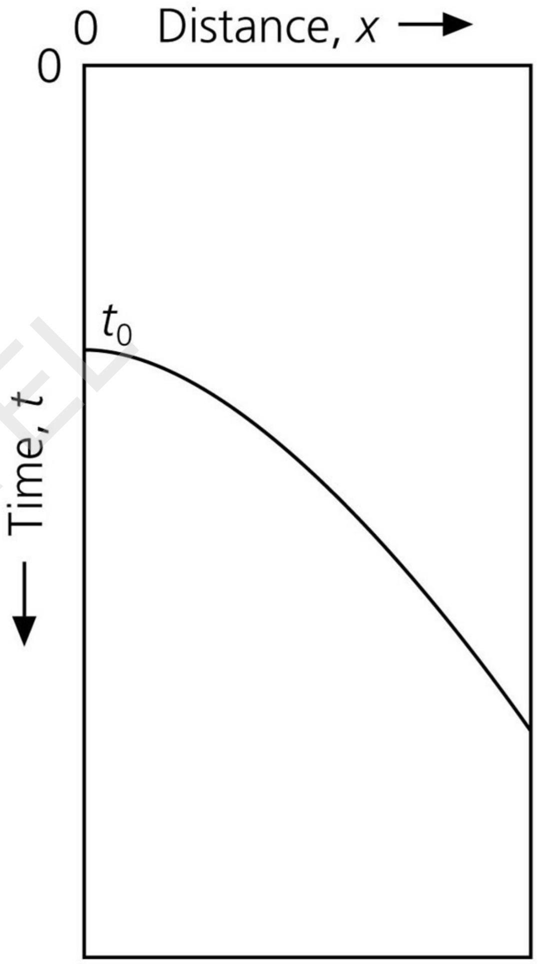
$$T(x)^2 - \frac{x^2}{v_0^2} = t_0^2$$

Travel time curve for reflection

$$T(x)^2 = \frac{x^2}{v_0^2} + t_0^2$$

- The travel time curve T(x) is **hyperbola** that intercepts the T axis at $t_0 = 2\frac{h_0}{v_0}$, the travel time at zero offset.
- This time is also called *two-way vertical time* corresponding ray travel vertically down to the reflector and back.

Figure 3.3-1: Hyperbolic travel time curve for an interface reflection.



Why time increasing downward ?

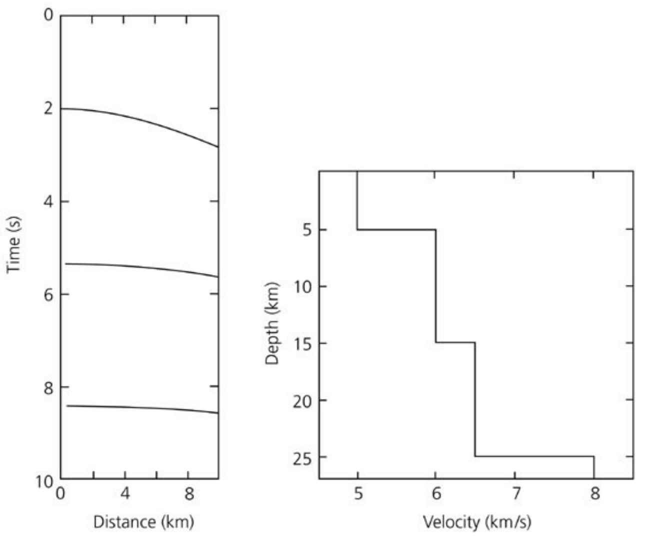
What about hyperbola ?

Layer velocity is found from slope of hyperbola. Because the slope decreases with increasing velocity, "flatter" travel time curves indicate higher velocities.

$$T(x)^2 = \frac{x^2}{v_0^2} + t_0^2$$

$$y = mX + c\,; \quad c = t_0^2\,,\ y \to T(x)^2 \ \ m = \frac{1}{v_0^2} \quad X \to x^2$$

Figure 3.3-4: Travel time curves for multiple layer reflections.



Normal moveout(Normal Move Out)

Variation of travel time with offset is stated in terms of NMO. Difference in travel time at a offset and zero offset is given by

$$T(x) - t_0 = \left(\frac{x^2}{v_0^2} + t_0^2\right)^{\frac{1}{2}} - t_0$$

The travel time curve and ray paths

The angle of incidence of the ray path is

$$\sin i = \frac{vdT}{dx}$$

or

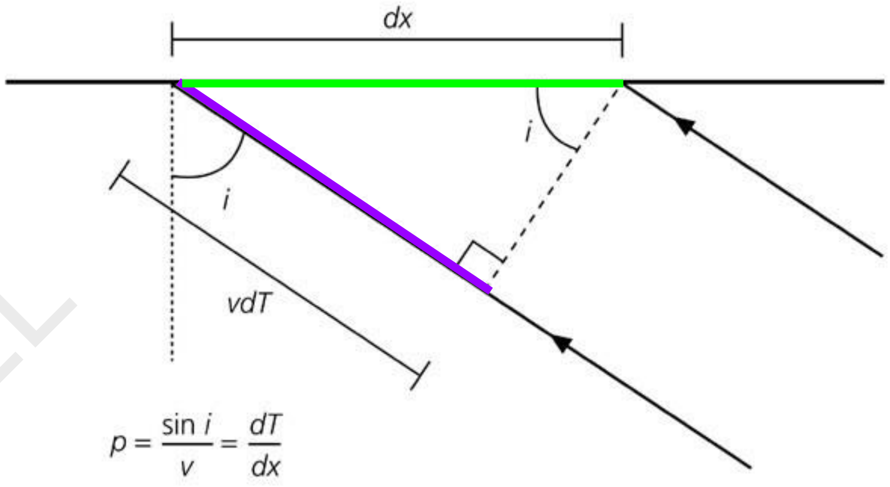
$$\frac{\sin i}{v} = \frac{dT}{dx}$$

or

$$p = \frac{\sin i}{v} = \frac{dT}{dx}$$

In terms of ray parameter ***p***

Figure 3.3-2: Cartoon demonstration of ray parameter.



Thus the ray parameter and the angle of incidence of the ray emerging at a distance x can be found from dT/dx, the slope of the travel time curve evaluated at x.

Travel time curve (Multiple horizontal layer)

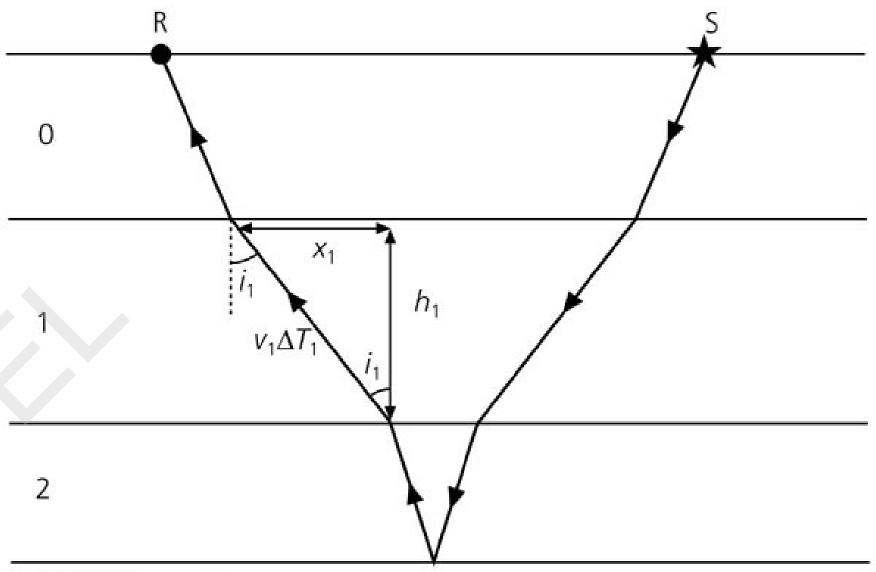
For multiple layer reflection,

Reflection R_{n+1} from the top of the $(n + 1)^{th}$ layer (or the bottom of the n^{th} layer) has traveled through n layers, each of thickness h_j and velocity v_j

As the ray parameter is constant along a ray, so

$$p = \frac{\sin i_j}{v_j} = \frac{\sin i_0}{v_0}$$

Figure 3.3-3: Ray path through multilayered structure.



A downgoing ray, which travels a horizontal distance x_j in the j^{th} layer, spends a time ΔT_j in the layer. Thus, in going down and up again, the ray travels a total horizontal distance

$$x(p) = 2\sum_{j=0}^{n} x_j = 2\sum_{j=0}^{n} h_j \tan i_j$$ **(Total horizontal distance)**

$$T(p) = 2\sum_{j=0}^{n} \Delta T_j = 2\sum_{j=0}^{n} \frac{h_j}{\cos i_j}$$ **(Total time)**

For single layer total time yields

$$T(x) = \frac{2\left[\left(\frac{x}{2}\right)^2 + h_0^2\right]^{\frac{1}{2}}}{v_0}$$

$$\because \cos i_0 = h_0\left(x_0^2 + h_0^2\right)^{\left(\frac{-1}{2}\right)}$$

For multiple layers , the travel time curve for reflection R_{n+1} off the top of the $(n+1)^{th}$ layer as a hyperbola.

$$T(x)^2_{n+1} = \frac{x^2}{\bar{V}_n^2} + t_n^2 \ ,$$

and find the two parameters, $\bar{V}_n$ and t_n .

t_n is the total two-way (up and down) vertical travel time at zero offset, which is twice the sum of the one-way vertical travel times Δt_j for each layer

$$t_n = 2\sum_{j=0}^{n} \Delta t_j = 2\sum_{j=0}^{n}\left(\frac{h_j}{v_j}\right)$$

$$x_j = v_j \Delta T_j \sin i_j = \left(\frac{v_j^2}{v_0}\right) \Delta T_j \; \sin\left(i_0\right),$$

$$x = 2\sum_{j=0}^{n} x_j = 2\frac{\sin\; i_0}{v_0} \sum_{j=0}^{n} v_j^2 \, \Delta T_j$$

$$\frac{dT}{dx} = \frac{\sin i_0}{v_0} = x/\left(2\sum_{j=0}^{n} v_j^2 \, \Delta T_j\right)$$

$$\frac{dT}{dx} = \frac{x}{\bar{V}_n^2 T} \quad \text{so we define } \bar{V}_n^2 = \left(2\sum_{j=0}^{n} v_j^2 \, \Delta T_j\right)/T$$

For vertical incidence

$$\Delta T_j = \Delta t_j$$

$$T = 2 \sum_{j=0}^{n} \Delta t_j$$

$$\bar{V}_n^2 = \left(\sum_{j=0}^{n} v_j^2 \, \Delta t_j \right) \Big/ \sum_{j=0}^{n} \Delta t_j$$

$\bar{V}_n$ the appropriate average velocity for the travel time curve, is the time-weighted root mean square, or rms, velocity for the first n layers.

But how to find the layer velocity or interval velocity?

Dix equation

Given a reflection from the top of the n^{th} layer, with vertical two-way travel time t_{n-1} and rms velocity $\bar{V}_{n-1}$,and a reflection from the top of the $(n + 1)^{th}$ layer, with vertical two-way travel time t_n and rms velocity $\bar{V}_n$, the velocity in the n^{th} layer is

$$v_n^2 = \frac{\bar{V}_n^2 t_n - \bar{V}_{n-1}^2 t_{n-1}}{t_n - t_{n-1}}$$

The relationship is called the “Dix equation. The resulting velocity is called an interval velocity, is better determined for larger offsets, where the slope of the travel time curve is greater.

Summary

- Study which use reflected arrivals to determine velocities within the crust, and other geophysical parameters is known as Reflection seismology
- Travel time curve for reflected arrival is hyperbola , for one layer $T(x)^2 = \frac{x^2}{v_0^2} + t_0^2$
- Travel time curve for n layer geometry $T(x)_{n+1}^2 = \frac{x^2}{\bar{V}_n^2} + t_n^2$,
- Relation between ray path and travel time curve $p = \frac{\sin i}{v} = \frac{dT}{dx}$ (Ray parameter)
- Root mean square and interval velocity $\bar{V}_n^2 = \left(\sum_{j=0}^{n} v_j^2 \Delta t_j \right) / \sum_{j=0}^{n} \Delta t_j$

$$v_n^2 = \frac{\bar{V}_n^2 t_n - \bar{V}_{n-1}^2 t_{n-1}}{t_n - t_{n-1}}$$

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- Seismology course, Professor Derek Schutt, Colorado State Univ., USA.

THANK YOU !