NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 05 : Refraction and Reflection seismology
Lecture 04: Earth as a constant velocity Distribution and Intercept-slowness formulation for travel times

CONCEPTS COVERED

- Recap
- Earth as a constant velocity Distribution
- Intercept-slowness formulation for travel times
- Summary

Recap

- Study which use reflected arrivals to determine velocities within the crust, and other geophysical parameters is known as Reflection seismology.
- Travel time curve for reflected arrival is hyperbola , for one layer $T(x)^2 = \frac{x^2}{v_0^2} + t_0^2$
- Travel time curve for n layer geometry $T(x)_{n+1}^2 = \frac{x^2}{\bar{V}_n^2} + t_n^2$,
- Relation between ray path and travel time curve $p = \frac{\sin i}{v} = \frac{dT}{dx}$ (Ray parameter)
- Root mean square and interval velocity $\bar{V}_n^2 = \left(\sum_{j=0}^{n} v_j^2 \, \Delta t_j\right) / \sum_{j=0}^{n} \Delta t_j$

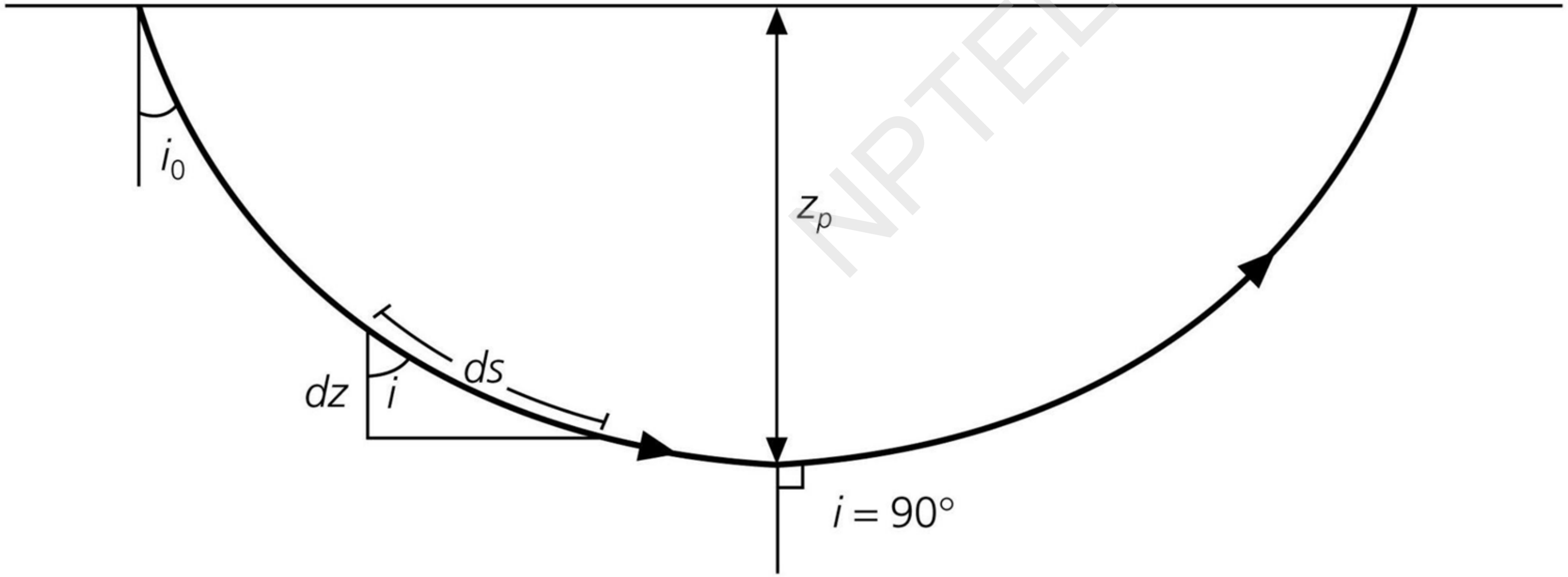
$$v_n^2 = \frac{\bar{V}_n^2 t_n - \bar{V}_{n-1}^2 t_{n-1}}{t_n - t_{n-1}}$$

Earth as continuous velocity distribution

Earth is spherical and the it has continuous distribution of velocity with depth, v(z), rather than a stack of discrete layers.

The ray path shown in Figure, and given by Snell's law, p = sin i/v(z), is constant.

Figure 3.3-6: Ray path in a medium with smoothly increasing velocities.



Earth as continuous velocity distribution

- If velocity increases with depth, sin *i* and thus *i* increase.
- So the ray bends away from the vertical on its way down.
- Once *i* = 90°, the ray turns, becomes horizontal, and then goes upward.
- At the deepest point, the turning, or bottoming, depth z_p the velocity is the reciprocal of the ray parameter, $p = 1/v(z_p)$.

- The expression for the distance traveled by the ray becomes:

$$x(p) = 2\int_0^{z_p} \tan i \, dz = 2p \int_0^{z_p} \left(\frac{1}{v^2(z)} - p^2 \right) dz \qquad \sin i = pv$$

Note: "If on some portion of the ray path the velocity decreases with depth, the ray bends toward the vertical. The ray does not turn upward until it gets below the low-velocity region."

Earth as continuous velocity distribution

In terms of the slowness, as u(z) = 1/v(z)

$$x(p) = 2p \int_0^{z_p} \frac{dz}{\left(u^2(z) - p^2\right)^{1/2}}$$

Similarly, total travel time becomes

$$T(p) = 2\int_0^{z_p} \frac{dz}{v(z)\cos i} = 2\int_0^{z_p} \frac{dz}{v(z)\left(1 - p^2 v^2(z)\right)^{1/2}}$$

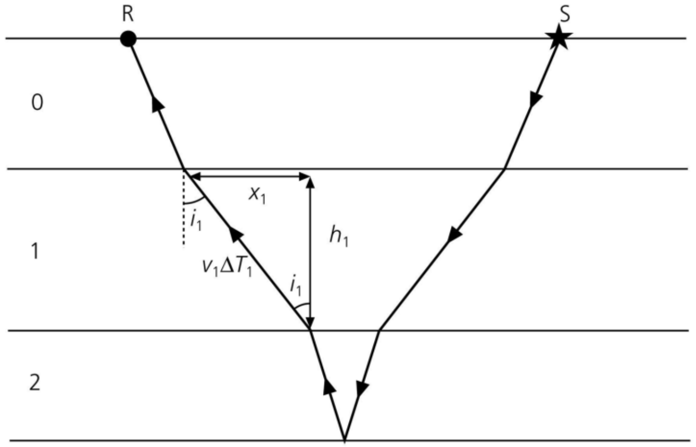
$$= 2\int_0^{z_p} \frac{u^2(z)dz}{\left(u^2(z) - p^2\right)\cos i}$$

This integral is valid everywhere except at the exact bottom of the curve, where u(z) equals p.

Intercept-slowness formulation for travel times

An alternative formulation in terms of intercept-slowness may provide interesting insights for data analysis.

Figure 3.3-3: Ray path through multilayered structure.



From the figure,

$$v_j \Delta T_j = \left(x_j^2 + h_j^2\right)^{1/2}$$

The incident angle i_j for this ray satisfies

$$\sin i_j = \frac{x_j}{\left(x_j^2 + h_j^2\right)^{1/2}} = \frac{x_j}{v_j \Delta T_j} \quad \textbf{and} \quad \cos i_j = \frac{h_j}{\left(x_j^2 + h_j^2\right)^{1/2}} = \frac{h_j}{v_j \Delta T_j}$$

Hence,

$$v_j \Delta T_j = \frac{x_j^2 + h_j^2}{\left(x_j^2 + h_j^2\right)^{1/2}} = x_j \sin i_j + h_j \cos i_j$$

or

$$\Delta T_j = \frac{x_j \sin i_j}{v_j} + \frac{h_j cos i_j}{v_j} = p_j x_j + \eta_j h_j$$

The travel time a ray spends in a layer is the sum of the horizontal slowness times the horizontal distance traveled and the vertical slowness times the vertical thickness.

$$p_j = \frac{\sin i_j}{v_j} = u_j \sin i_j \quad \textbf{and} \quad \eta_j = \frac{\cos i_j}{v_j} = u_j \cos i_j$$ …..equ. 1

These are the components of the slowness vector that has magnitude equal to the slowness, and points in the direction of wave propagation. Hence u_j, the slowness in the layer, is

$$u_j^2 = \frac{1}{v_j^2} = p_j^2 + \eta_j^2$$

Intercept-slowness formulation for travel times

The total travel time is:

$$T(x) = 2\sum_{j=0}^{n} \Delta T_j = 2\sum_{j=0}^{n} p_j x_j + 2\sum_{j=0}^{n} \eta_j h_j$$

By Snell's law, the horizontal ray parameter is constant along the ray path, so $p_j = p$, and

$$T(x) = px + 2\sum_{j=0}^{n} \eta_j h_j$$

We define, $T(x) = px + \tau(p)$ **……….equ 2**

where the function,

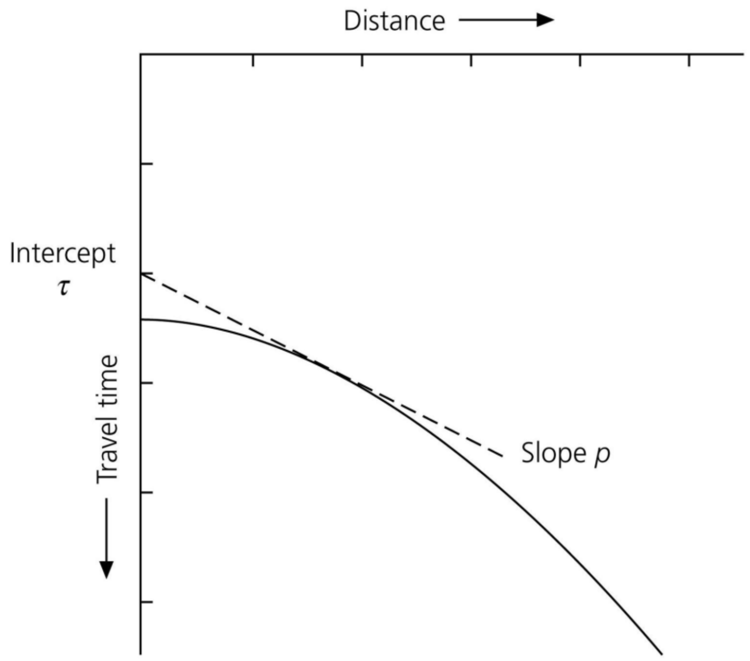
$$\tau(p) = 2\sum_{j=0}^{n} \eta_j h_j = 2\sum_{j=0}^{n} \left(\frac{1}{v_j^2} - p^2\right)^{1/2} h_j = 2\sum_{j=0}^{n} \left(u_j^2 - p^2\right)^{1/2} h_j$$ **……….equ 3**

- Because p is the slope of the travel time curve (dT/dx) and hence of a line tangential to it at the point (T, x), τ is the intercept of the tangent line with the time axis.

- In general τ and *p* differ for different points on the travel time curve, so the travel time curve can be described by the values of either (T, x) or (τ, p).

- “Thus the function τ(p) is called the *intercept-slowness* representation of the travel time curve.”

Figure 3.3-7: Relation between travel time curve, tau, and ray parameter.



On differentiating equ.2 w.r.t. *p*

$$T(x) = px + \tau(p) \qquad \text{……….equ 2}$$

$$\frac{d\tau}{dp} = \frac{dT}{dp} - p\frac{dx}{dp} - x(p) = \frac{dT}{dx}\frac{dx}{dp} - p\frac{dx}{dp} - x(p) = -x(p)$$

"Thus, just as *p* is the slope of the travel time curve, T(x), the distance, x, is minus the slope of the τ(p) curve."

Intercept-slowness formulation for travel times

We may also show that the τ(p) formulation will lead to the travel time curve for the reflected wave in a layer over a halfspace. For this case, x_0= x/2, so, using Eqn 1,

$$p = \frac{x/2}{v_o\left[(x/2)^2 + h_o^2\right]^{1/2}}, \quad \eta_o = \frac{h_o}{v_o\left[(x/2)^2 + h_o^2\right]^{1/2}}$$

Hence,

$$T(x) = px + 2\eta_o h_o = \frac{(x^2/2) + 2h_o^2}{v_o\left[(x/2)^2 + h_o^2\right]^{1/2}}$$

$$= \frac{\left((x/2)^2 + h_o^2\right)^{1/2}}{v_o}$$

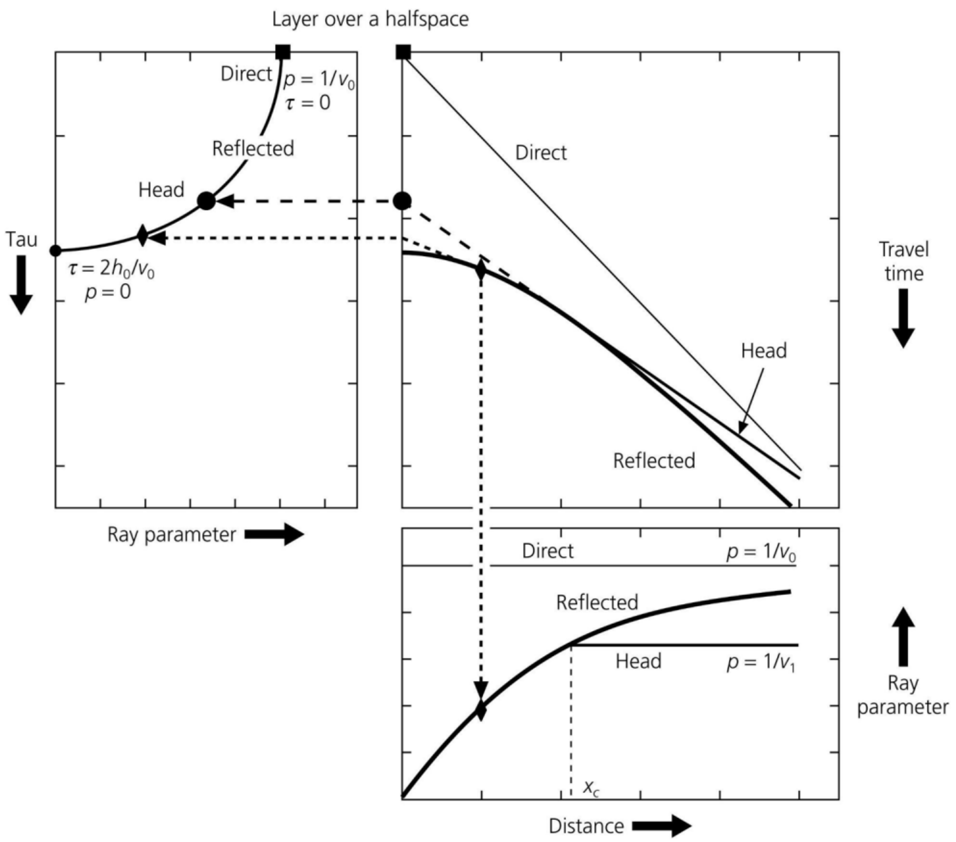
which is the familiar hyperbola. To see how this travel time curve appears when written as τ(p), we write Eqn 3 for a layer over a halfspace:

$$\tau(p) = 2\left(\frac{1}{v_o^2} - p^2\right)^{1/2} h_o \quad \text{……….equ 4}$$

Intercept-slowness formulation for travel times

This can also be written as: $$\frac{v_o^2\tau^2}{4h_o^2} + v_o^2p^2 = 1$$ **ellipse whose axes are the τ and p axes**

Figure 3.3-8: Relation between tau-p, and travel time curves.



- **It intersects the τ axis at ($\tau = t_0 = 2h_0/v_0$ at $p = 0$), which show the travel time curve has zero slope and the time axis intercept is the vertical two-way travel time, corresponds to the zero-offset point x = 0.**

- **Ellipse intersect p axis at ($\tau = 0$, $p = 1/v_0$), where the travel time curve has slope $1/v_0$ and time axis intercept 0, is the τ(p) position of the linear travel time curve for the direct wave.**

Intercept-slowness formulation for travel times

Hence the line for the direct wave maps to a point in the τ(p) plane that is on the ellipse describing the reflected wave.

To understand why this occurs, we use the fact that distance is minus the derivative of the τ(p) curve and differentiate Eqn 4, giving

$$x(p) = -\frac{d\tau}{dp} = 2ph_o\left(\frac{1}{v_o^2} - p^2\right)^{-1/2} \qquad \text{……….equ 5}$$

so at the point $p = 1/v_0$, $x = \infty$. This makes sense, because as $x \rightarrow \infty$, the reflected wave is asymptotic to the direct wave.

The head wave is easily mapped into the τ (p) plane, because its travel time curve is:

$$\begin{aligned} T_H(x) &= x/v_1 + 2h_o\left(1/v_o^2 - 1/v_1^2\right)^{1/2} \\ &= x/v_1 + \tau_1 \end{aligned}$$

a line with slope equal to the reciprocal of the halfspace velocity, $p = 1/v_1$ and intercept τ_1.

Intercept-slowness formulation for travel times

Thus the head wave maps into a point on the ellipse describing the reflected wave, corresponding to the critical distance x_c where the head and reflected waves are the same.
To see this, note that for $p = 1/v_1$, Eqn 5 gives:

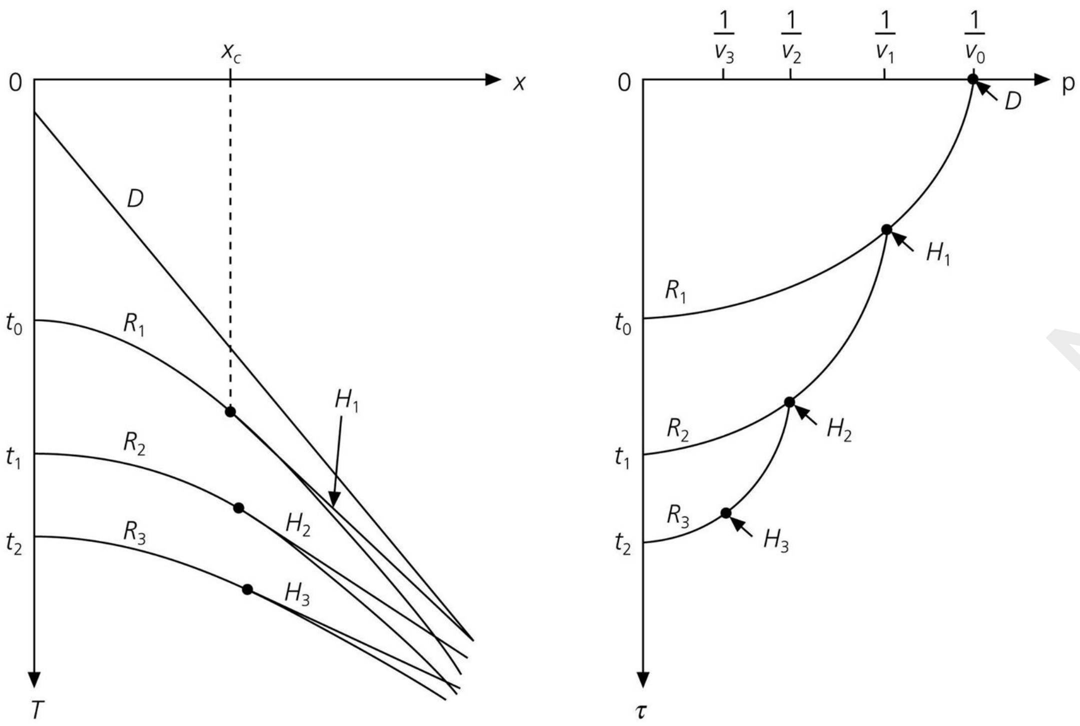
$$x(p) = -\frac{d\tau}{dp} = 2h_o v_o \left(v_1^2 - v_o^2\right)^{-1/2} = x_c$$

This point divides the ellipse describing the reflected wave into a subcritical portion, between the τ axis and the head wave, and a post critical portion, between the head wave and the p axis.

Intercept-slowness formulation for travel times

For multiple layers, the τ(p) curves corresponding to reflections off successive layers are all portions of different ellipses.

Figure 3.3-9: Tau-P and travel time curves for multiple layers.



For a continuous velocity distribution, the summation for τ (Eqn 3) becomes an integral

$$\tau(p) = 2\int_0^{z_p} \eta(z)dz = 2\int_0^{z_p}\left(\frac{1}{v^2(z)} - p^2\right)^{1/2} dz$$

$$= 2\int_0^{z_p}\left(u^2(z) - p^2\right)^{1/2} dz$$

Formulating travel time curves as τ(p) is useful for some techniques that invert for velocity structure.

Summary

- For a continuous increasing velocity model, at the deepest point, the turning, or bottoming, depth z_p the velocity is the reciprocal of the ray parameter, $p = 1/v(z_p)$.

- Distance travelled by the ray: $x(p) = 2\int_0^{z_p} \tan i \, dz = 2p\int_0^{z_p} \left(\frac{1}{v^2(z)} - p^2\right) dz$

- Time taken to cover distance x(p) by the ray: $\Delta T_j = \frac{x_j \sin i_j}{v_j} + \frac{h_j \sin i_j}{v_j} = p_j x_j + \eta_j h_j$

- The function τ(p) is called the *intercept-slowness* representation of the travel time curve.

- *p* is the slope of the travel time curve, T(x), the distance, x, is minus the slope of the τ(p) curve.

- The head wave maps into a point and hyperbola maps as ellipse on τ-p plane

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THANK YOU !