NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

Dr. Mohit Agrawal
Department of Applied Geophysics , IIT(ISM) Dhanbad

Module 06 : Seismic waves in a spherical earth, Body wave travel time studies
Lecture 01: Seismic waves in spherical earth, Ray path and travel time

CONCEPTS COVERED

- Seismic Waves in Spherical earth
- Ray path and travel time
- Ray path and travel time in polar coordinate
- Summary

Seismic waves in spherical earth

Why not flat layers?

The ray paths between the source and receiver are short enough (less than few hundred kilometers) that the earth's curvature can be neglected. → *Flat layer* → **To study structure in the crust and uppermost mantle.**

Greater distance and greater depths → *Spherical earth* → **Structure of deep earth**

Ray paths and travel times

Consider the earth as a series of concentric spherical shells of uniform-velocity material.

By Snell's law we have

$$\frac{r_1 \sin i_1}{v_1} = \frac{r_1 \sin i_1'}{v_2}$$

$$As \qquad r_1 \sin i_1' = r_2 \sin i_2$$

Snell's law for spherical Earth.

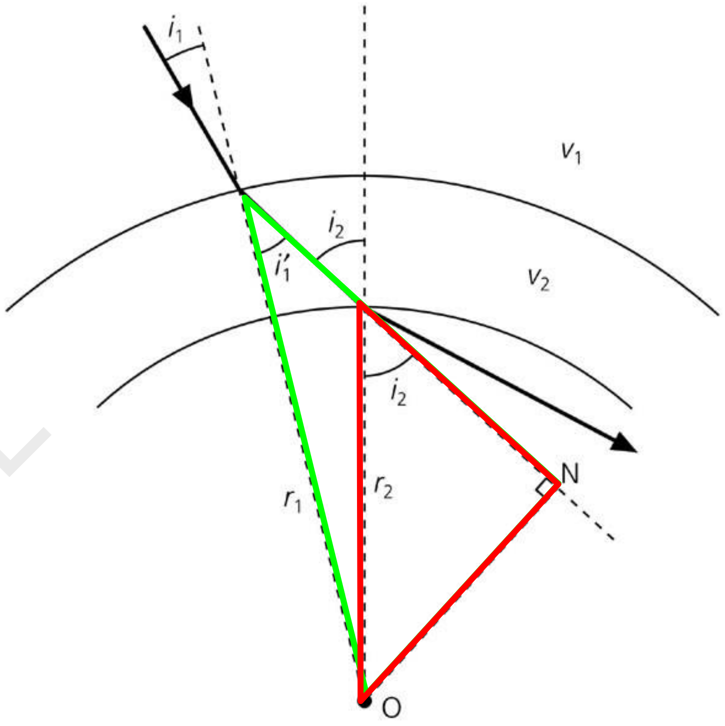
$$\frac{r_1 \sin i_1}{v_1} = \frac{r_2 \sin i_2}{v_2}$$

Thus the ray parameter p for a spherical earth is given by

$$p = \frac{r \sin i}{v},$$

Where r is radial distance from the centre of the Earth.

Figure 3.4-1: Geometry of Snell's law for a spherical earth.

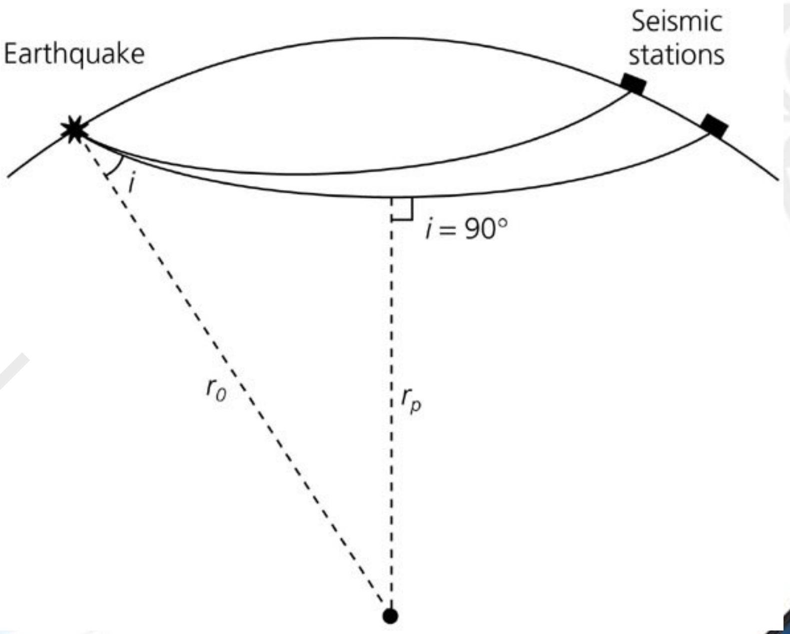


Ray paths and travel times

- For a source at a radius r_0 (the Earth's radius for a surface source) where the velocity is v_0

$$p = \frac{r_0 \sin i}{v_0}$$

Figure 3.4-2: Geometry of a ray path in a spherical earth.



- Rays leaving the source at different angles have different ray parameters.

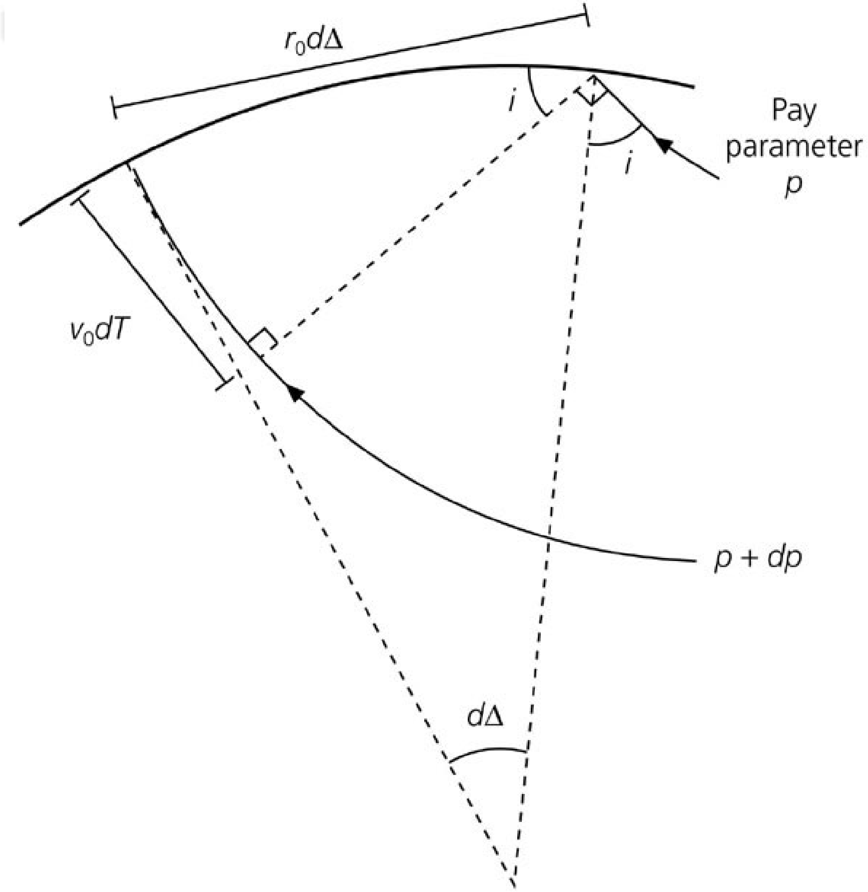
- As the velocity increases with depth, ray eventually 'bottoms' and turns upward when $i=90^0$

- At bottoming depth, $r=r_p$ and $p = \frac{r_p \sin 90}{v_p} = \frac{r_p}{v_p}$

Ray paths and travel times

Different rays with different p bottom at different depths.

Figure 3.4-3: Derivation of the ray parameter in a spherical earth.



$$rays\ with\ ray\ parametes\ p\ and\ p + dp$$

$$p \rightarrow T \rightarrow \Delta$$
$$p + dp \rightarrow T + dT \rightarrow \Delta + d\Delta$$

$$\frac{v_0 dT}{r_0 d\Delta} = \sin i\,,$$

$so,$

$$\frac{dT}{d\Delta} = \frac{r_0 \sin i}{v_0} = p$$

Ray paths and travel times

Thus, similar to the flat layer case the ray parameter is the reciprocal of the apparent velocity along the surface, c_x

$$p = \frac{1}{c_x} = 1/\left(\frac{d\Delta}{dT}\right) = \frac{dT}{d\Delta}$$

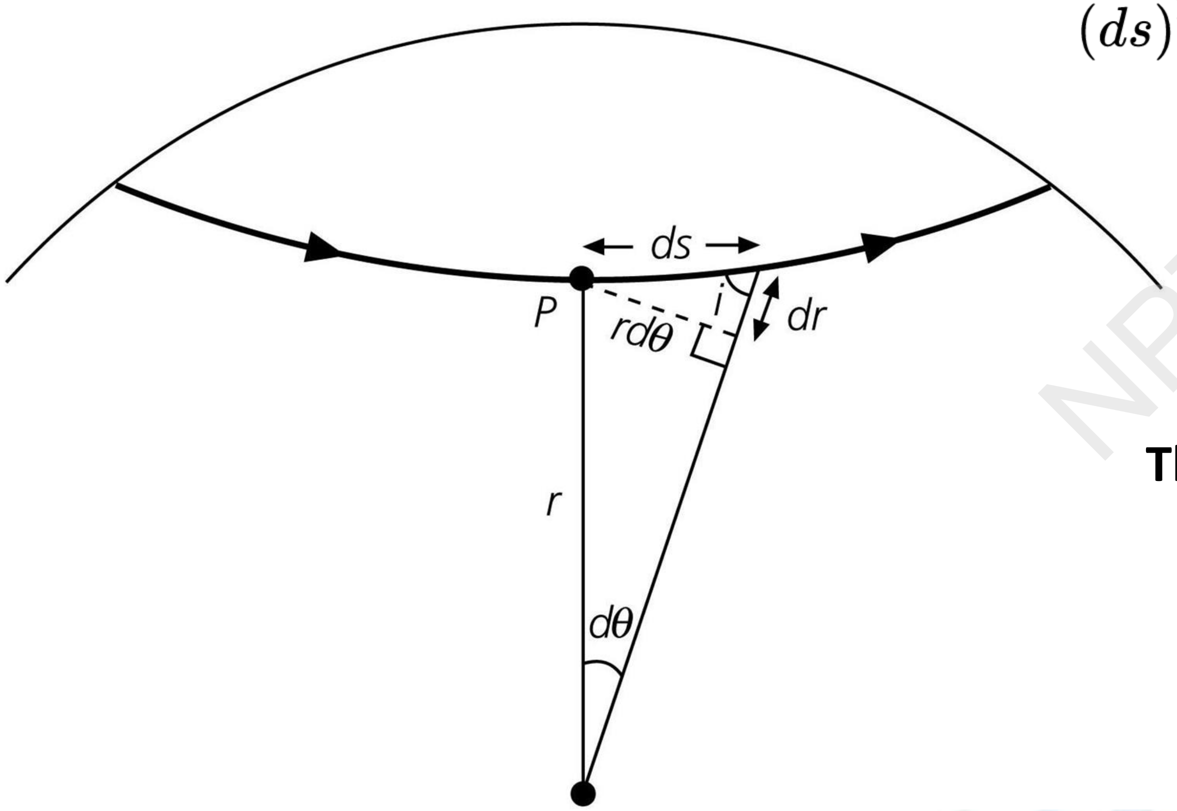
- **Hence, the ray parameter can be measured from the difference in arrival times at nearby stations.**

- **Conversely, the slope of a travel time curve T(Δ) is the ray parameter of the ray emerging at a distance Δ.**

Ray Path In Polar Coordinate

Consider, the point P on the ray path with polar coordinates (r, θ). A small portion of the ray path, ds, subtends an angle at the center of the earth dθ, so

Figure 3.4-4: Definition of the ray portion *ds*.



$$(ds)^2 = (dr)^2 + r^2(d\theta)^2 \quad \textbf{and} \quad \sin i = r\frac{d\theta}{ds}$$

Substitution in Snell's law gives

$$p = \frac{r\sin i}{v} = \frac{r^2}{v}\frac{d\theta}{ds}$$

Thus, above equation can be rewritten as

$$\frac{r^4}{p^2v^2} = \left(\frac{dr}{d\theta}\right)^2 + r^2$$

Ray Path In Polar Coordinate

and manipulate them to obtain

$$d\theta = \frac{\pm p dr}{r(\zeta^2 - p^2)^{1/2}} \qquad \textbf{Here,} \quad \zeta = r/v$$

Integrating this expression from the the surface r_0, to the deepest point on the ray r_p, and doubling to account for the upward path, gives

$$\Delta(p) = \int d\theta = 2p \int_{r_p}^{r_o} \frac{dr}{r(\zeta^2 - p^2)^{1/2}}$$

This integral gives the angular distance Δ traveled by the ray with ray parameter p in an Earth with a velocity distribution v(r).

Ray Path In Polar Coordinate

Integral expression for travel time of this ray can be given as

$$\frac{p^2v^2}{r^2} = r^2\left(\frac{d\theta}{ds}\right)^2 = 1 - \left(\frac{dr}{ds}\right)^2$$

so that a portion of the ray path is

$$ds = \pm\frac{r}{v}\frac{dr}{(\zeta^2 - p^2)^{\left(\frac{1}{2}\right)}}$$

Thus , travel time , which is defined as integral of slowness along the ray path, expressed as

$$T(p) = \int \frac{ds}{v} = 2\int_{r_p}^{r_0} \frac{\zeta^2 dr}{r(\zeta^2 - p^2)^{\left(\frac{1}{2}\right)}}$$

Comparison

Flat layer assumption

$$x(p) = 2p \int_0^{z_p} \frac{dz}{\left(u^2(z) - p^2\right)^{1/2}}$$

$$T(p) = 2 \int_0^{z_p} \frac{u^2(z)dz}{\left(u^2(z) - p^2\right)\cos i}$$

Spherical earth

$$\Delta(p) = \int d\theta = 2p \int_{r_p}^{r_o} \frac{dr}{r\left(\zeta^2 - p^2\right)^{1/2}}$$

$$T(p) = \int \frac{ds}{v} = 2 \int_{r_p}^{r_0} \frac{\zeta^2 dr}{r\left(\zeta^2 - p^2\right)^{\left(\frac{1}{2}\right)}}$$

These integral expressions for the Δ(p) and travel time T(p) of a ray in spherical geometry are analogous to those for x(p) and T(p) in layered material.

For the flat geometry, we observe the usefulness of describing the travel time curve in terms of its slope, the ray parameter, p , and the time axis intercept of its tangent, τ.

To do the same for spherical geometry , we have $T(p) = p\Delta p + \tau(p)$

It can be rewritten as, $\tau(p) = T(p) - p\Delta(p)$

From the previous slide, we can use the integral expressions for travel time and angular distance expression as a function of p.

$$\tau(p) = 2\int_{r_p}^{r_0} \left(\frac{(\zeta^2 - p^2)^{\frac{1}{2}}}{r} \right) dr$$

This formulation can be used to invert travel time curves for the velocity structure.

Summary

- To study the crust and upper mantle the flat layer assumption is more convenient whereas to study structure of deep earth spherical earth is more concise.
- The ray parameter for the spherical earth is given as $p = \dfrac{r \sin i}{v}$
- Rays leaving the source at different angles have different ray parameters.
- Different rays with different p bottom at different depths
- Ray path and travel time for flat layer and spherical can be visualized as

$$p = \frac{1}{c_x} = 1/\left(\frac{d\Delta}{dT}\right) = \frac{dT}{d\Delta} \qquad \frac{dT}{d\Delta} = \frac{r_0 \sin i}{v_0} = p$$

- Ray path travel time and angular distance for spherical earth is expressed as

$$T(p) = \int \frac{ds}{v} = 2\int_{r_p}^{r_0} \frac{\zeta^2 dr}{r(\zeta^2 - p^2)^{\left(\frac{1}{2}\right)}} \qquad \Delta(p) = \int d\theta = 2p \int_{r_p}^{r_o} \frac{dr}{r(\zeta^2 - p^2)^{1/2}}$$

REFERENCES

- Stein, Seth, and Michael Wysession. An introduction to seismology, earthquakes, and earth structure. John Wiley & Sons, 2009.
- Lowrie, William, and Andreas Fichtner. Fundamentals of geophysics. Cambridge university press, 2020.
- Kearey, Philip, Michael Brooks, and Ian Hill. An introduction to geophysical exploration. Vol. 4. John Wiley & Sons, 2002.
- https://geologyscience.com/geology-branches/structural-geology/stress-and-strain/
- Seismology course, Professor Derek Schutt, Colorado State Univ., USA.

THANK YOU !