NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

Dr. Mohit Agrawal

Department of Applied Geophysics , IIT(ISM) Dhanbad

Module 06 : Seismic waves in a spherical Earth, Body wave travel time studies

Lecture 02: Velocity distribution, Triplication and Travel time curve inversion

CONCEPTS COVERED

- Recap
- Velocity Distribution
- Triplication and shadow zone
- Travel Time Curve Inversion
- Summary

Recap

- To study the crust and upper mantle the flat layer assumption is more convenient whereas to study structure of deep earth spherical earth is more concise.

- The ray parameter for the spherical earth is given as $p = \dfrac{r \sin i}{v}$

- Rays leaving the source at different angles have different ray parameters and Different rays with different p bottom at different depths

- Ray path and travel time for flat layer and spherical can be visualized as

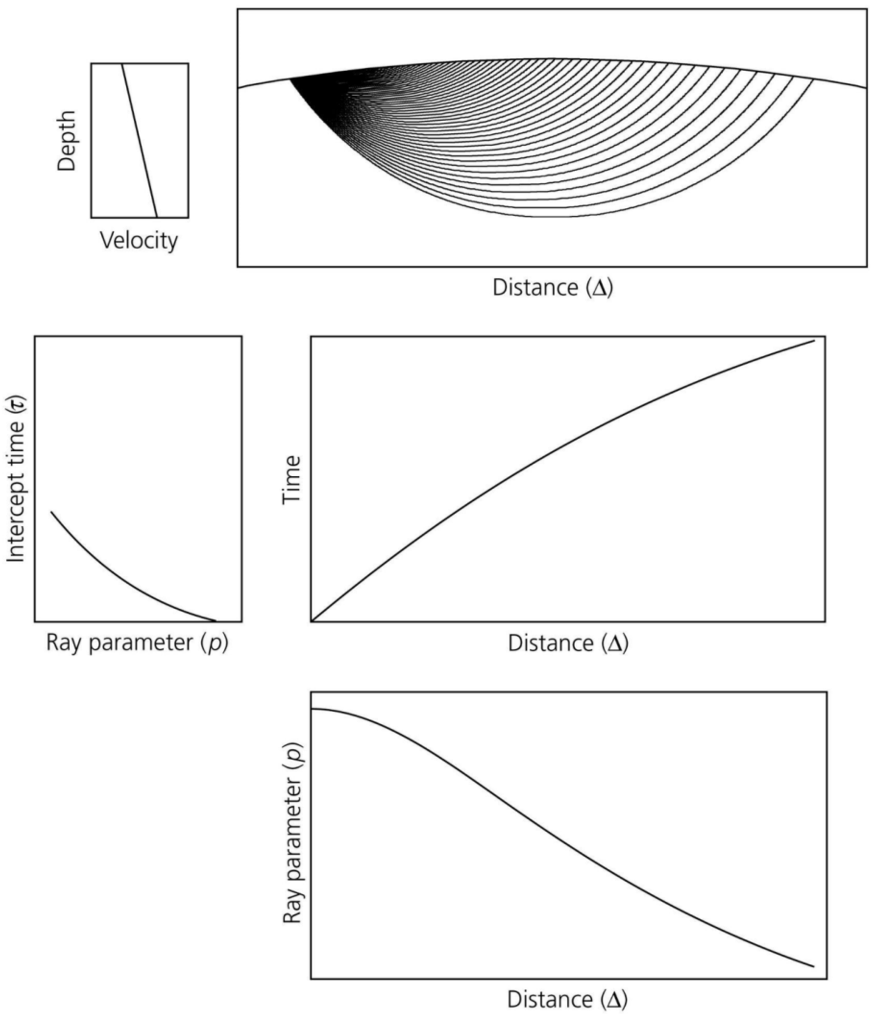
$$p = \frac{1}{c_x} = 1/\left(\frac{d\Delta}{dT}\right) = \frac{dT}{d\Delta} \qquad \frac{dT}{d\Delta} = \frac{r_0 \sin i}{v_0} = p$$

- Ray path travel time and angular distance for spherical earth is expressed as

$$T(p) = \int \frac{ds}{v} = 2\int_{r_p}^{r_0} \frac{\zeta^2 dr}{r(\zeta^2 - p^2)^{\left(\frac{1}{2}\right)}} \qquad \Delta(p) = \int d\theta = 2p\int_{r_p}^{r_o} \frac{dr}{r(\zeta^2 - p^2)^{1/2}}$$

Velocity Distributions : Case I - X increasing with p decreasing

Figure 3.4-5: Ray path effects for increasing velocity.



In a general case, the velocity usually increases slowly with depth.

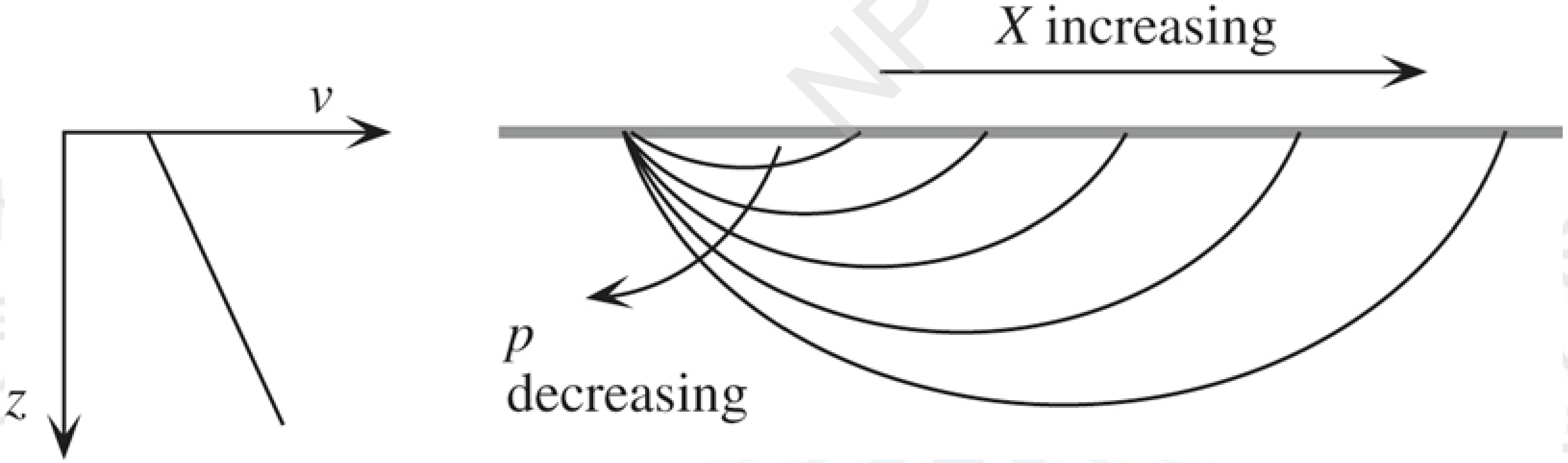
- **Rays that bottoms deeper at a location with smaller r_p and greater v_p, ultimately emerges further from the source because it has a smaller p.**
- **Thus the ray parameter decreases, and travel time increases, monotonically with distance, Δ.**
- **The travel time curve, T(Δ), is concave downward because its slope, p(Δ), decreases with distance ($dp/d\Delta = d^2T/d\Delta^2 < 0$).**
- **The intercept-slowness curve, τ(p), is smooth.**

Velocity Distributions : Case I - X increasing with p decreasing

Generally in Earth, X(p) will increase as p decreases; that is, as the takeoff angle decreases, the range increases. In this case, where dx/dp or dΔ/dp <0 we say that this branch of travel time curve is prograde.

$$\frac{d^2T}{d\Delta^2} = \frac{dp}{d\Delta} < 0$$

(Prograde motion)

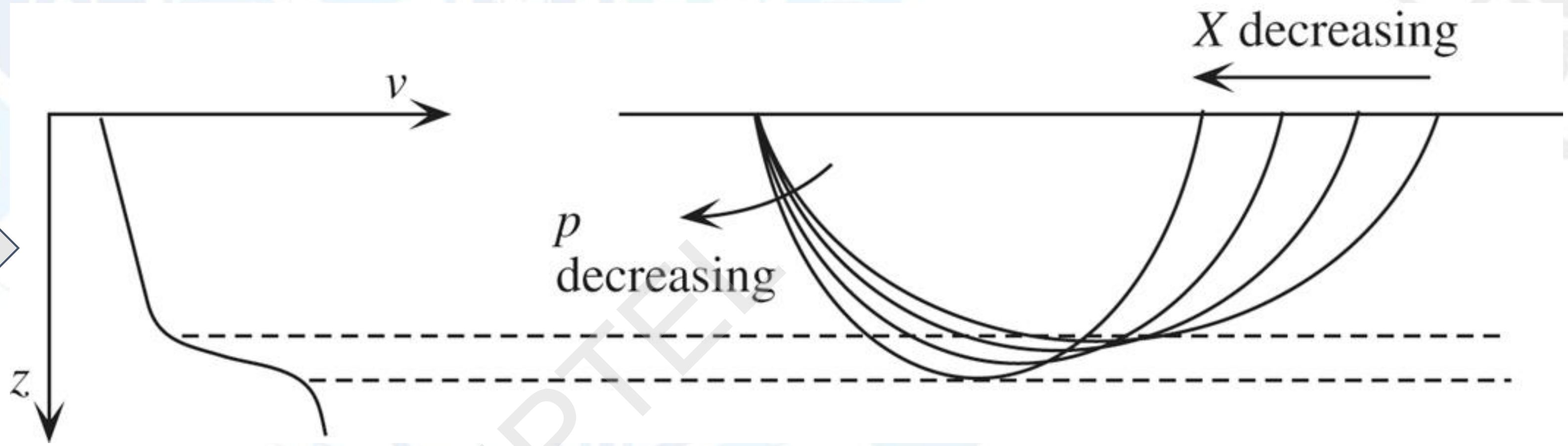


Velocity Distributions : Case II - Triplication

For a high velocity zone, where velocity increase abruptly with depth will create a situation where X decreases when p decreases.

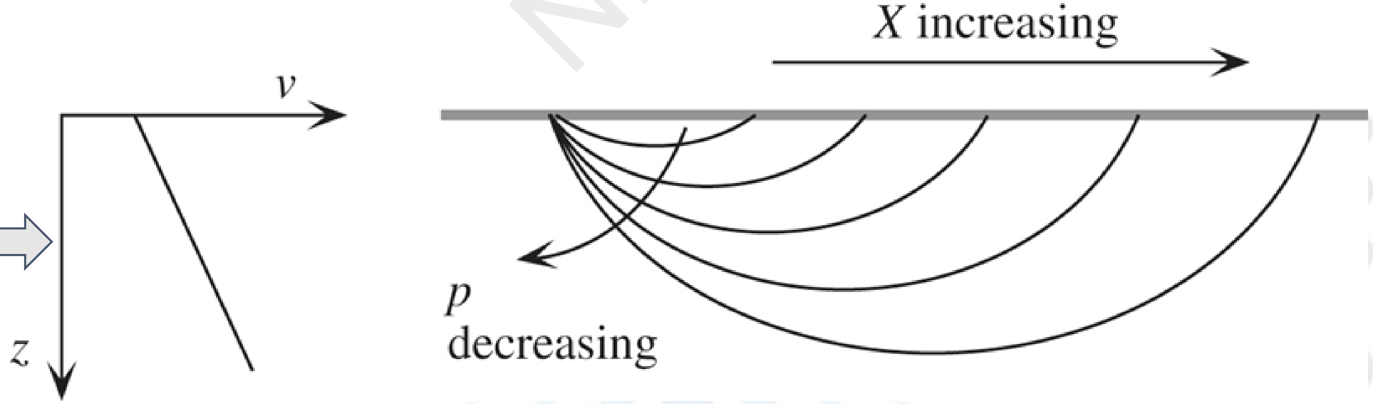
When dX/dp > 0

Retrograde motion

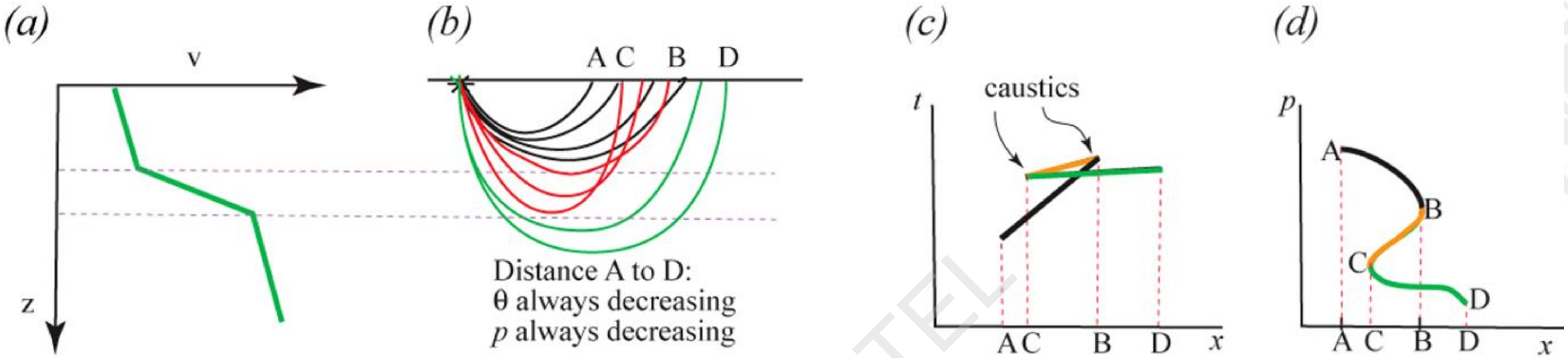


When dX/dp < 0

Prograde motion



Velocity Distributions : Case II - Triplication

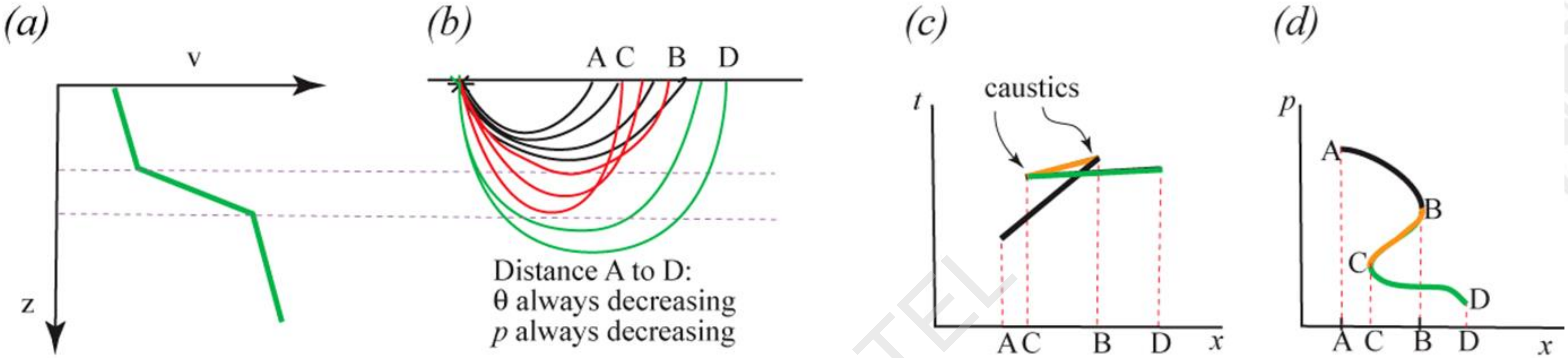


Prograde A to B (black) and
Prograde C to D (green)
Retrograde from B to C (red/orange)

Between C and B, there are multiple rays (three), hence this is called a triplication.

(From Derek Schutt, CSU and Peter M. Shearer book)

Velocity Distributions : Case II - Triplication



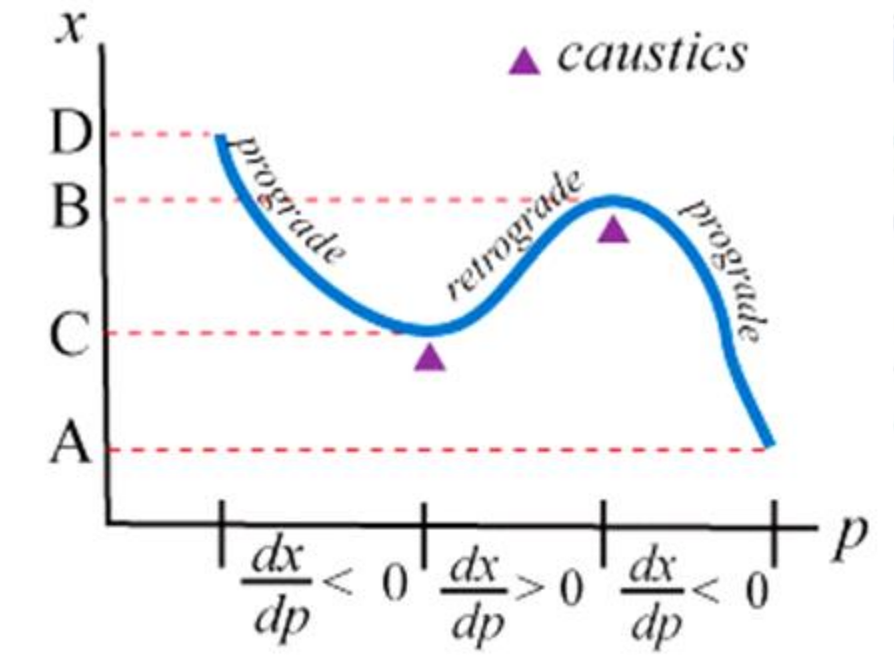
At 'B' and 'C', dp/dx = ∞ or dx/dp = 0

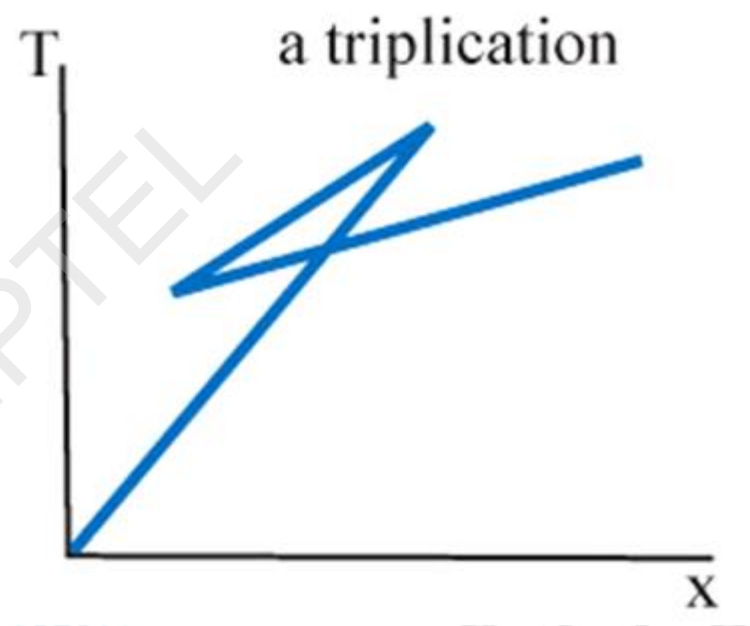
The point 'B' and 'C' are the endpoints of triplication are termed 'caustics'. Energy will be focused at these points since rays at different takeoff angles arrive at same range and would theoretically have infinite amplitude (in reality they have large amplitude).

(From Derek Schutt, CSU and Peter M. Shearer book)

Velocity Distributions : Case II - Triplication

In other ways on X-p and T-X plots

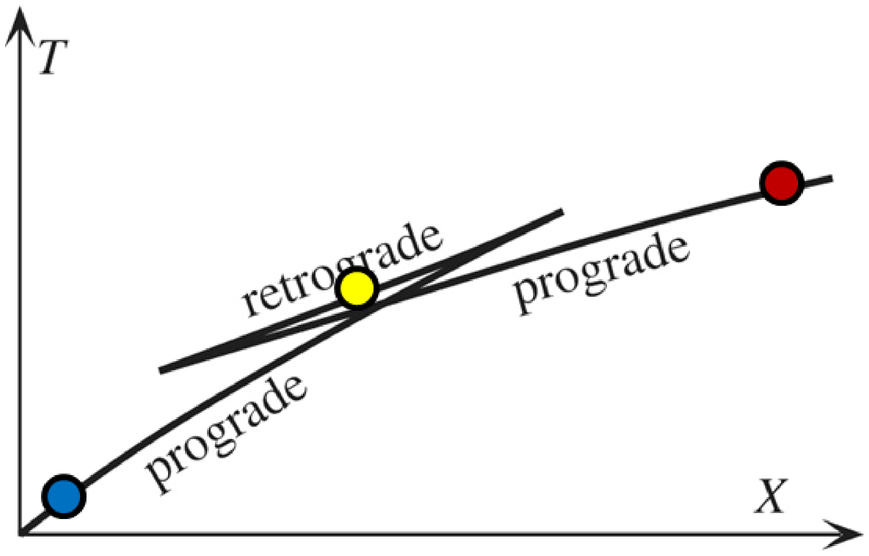


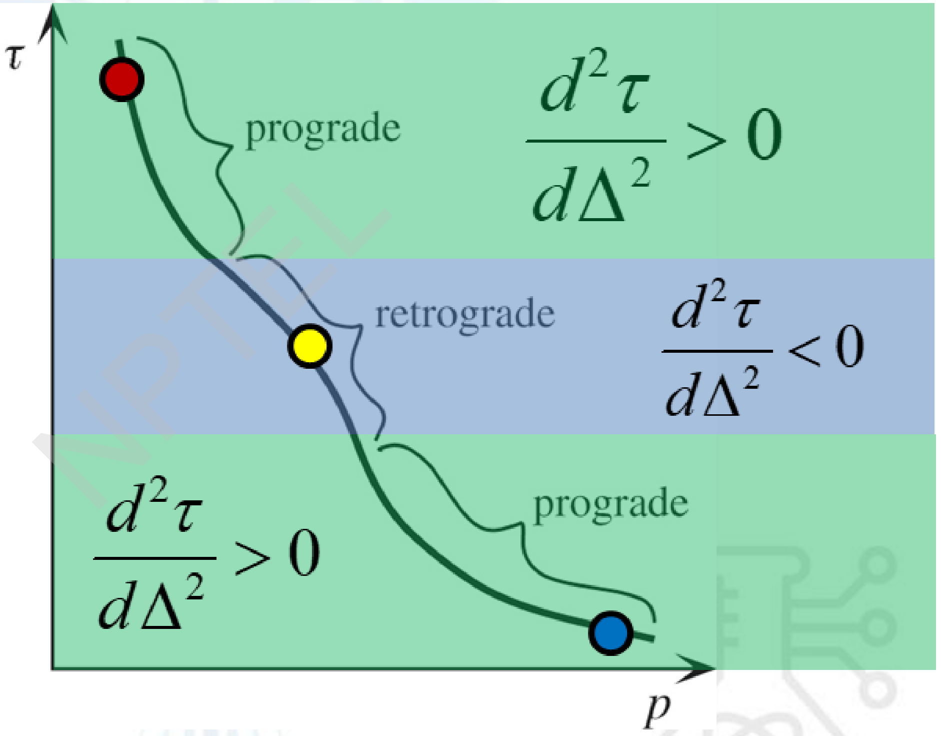


(From Derek Schutt, CSU and Peter M. Shearer book)

Velocity Distributions : Case II - Triplication

On tau-p plane, the triplication looks like as follows





(From Derek Schutt, CSU and Peter M. Shearer book)

Mathematically, the concentration of rays is proportional to di/dΔ, the range of incidence angles for the rays that arrive in a given distance.

To find this, we differentiate the definition of the ray parameter

$$\frac{d^2T}{d\Delta^2} = \frac{dp}{d\Delta} = \frac{d(r\sin i/v)}{d\Delta} = \frac{r}{v}\cos i \frac{di}{d\Delta}$$

For a triplication, the back branch meets the two forward branches at two points on the travel time and p(Δ) curves.

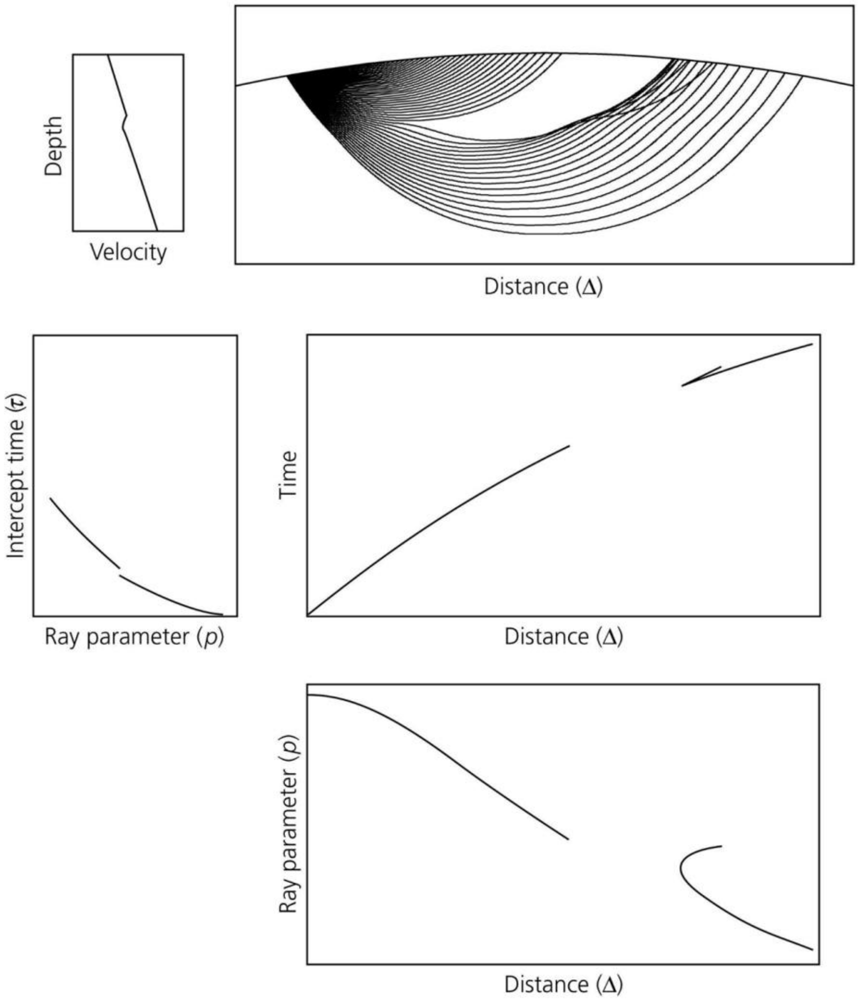
For triplication,

$$\frac{dp}{d\Delta} = \infty$$

so large amplitudes are expected. This situation is called a caustic.

Velocity Distributions: Case III - Shadow Zone

Figure 3.4-7: Ray path shadow-zone effects for a velocity decrease.



- **A low-velocity zone gives rise to a shadow zone because the rays turns more vertically creating a shadow zone.**

- **For a ray to bottom, it must turn upward (to a larger angle of incidence) as it goes deeper (to smaller values of r), so that di/dr < 0.**

- **In low velocity layer, di/dr > 0, the ray turns downward and cannot bottom.**

Velocity Distributions: Case III - Shadow Zone

These conditions can be written in terms of the velocity–depth function by differentiating both sides of

$$\sin i = \frac{pv}{r},$$ gives,

$$\cos i \frac{di}{dr} = p\left(\frac{1}{r}\frac{dv}{dr} - \frac{v}{r^2}\right) = \sin i\left(\frac{1}{v}\frac{dv}{dr} - \frac{1}{r}\right)$$

and thus

$$\frac{di}{dr} = \tan i\left(\frac{1}{v}\frac{dv}{dr} - \frac{1}{r}\right)$$

As mentioned in previous slide, di/dr > 0 for low velocity zone. Hence,

$$\frac{dv}{dr} > \frac{v}{r}$$

This situation causes a shadow zone, a region of the earth's surface where no rays arrive.

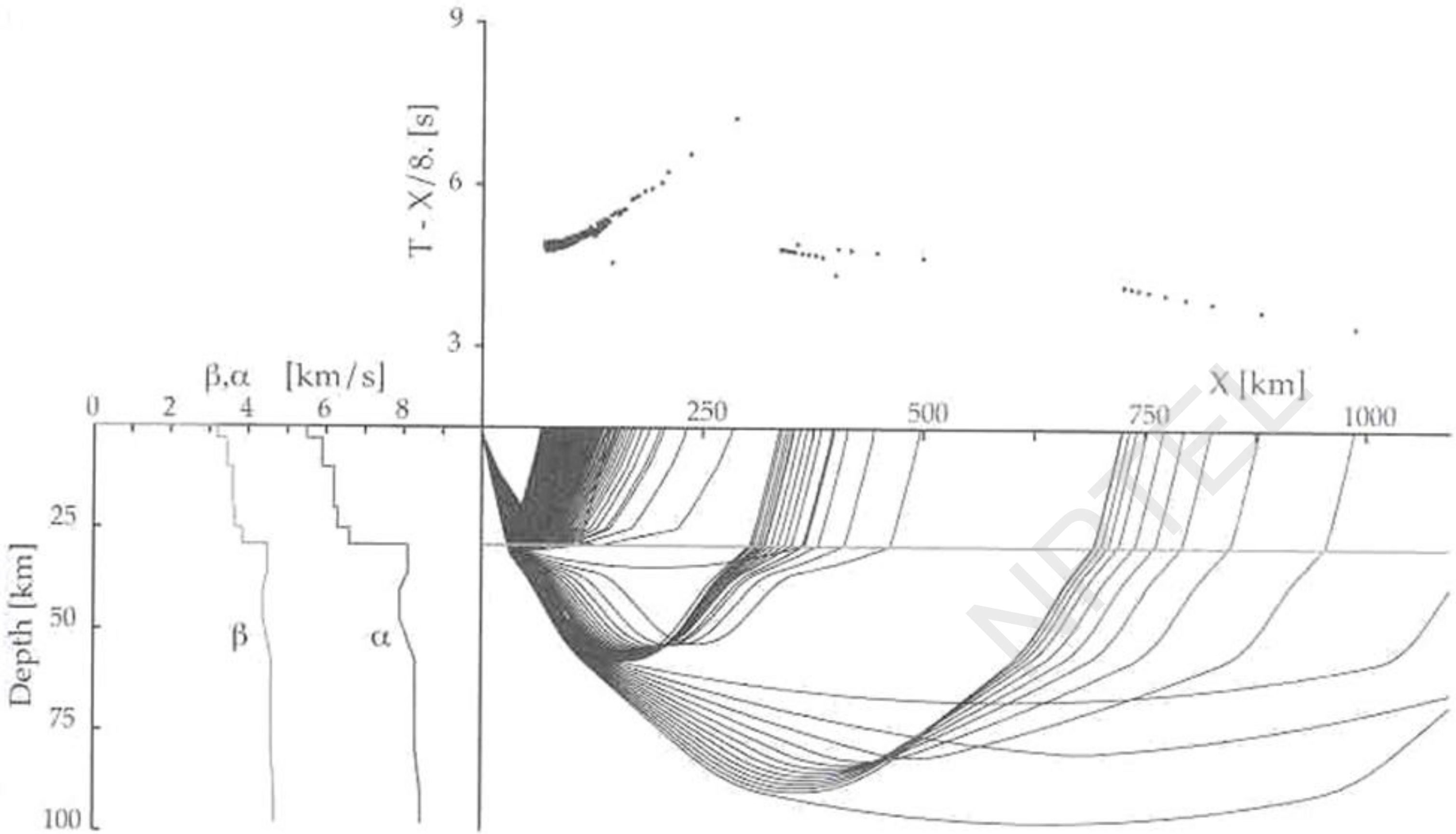


Figure 5.5. Ray and travel-time behaviour for a crustal and upper mantle model derived from long-range refraction profiles across France.

(From Kennet book)

Travel time curve inversion

- Used to infer the velocity distribution with depth.

- Requires sophisticated computing based on Snell's law in order to trace the rays within different velocity layers and compute the corresponding travel time curves. It is then used to derive velocity structure from T (Δ) curves.

- It is an iterative inverse problem where forward problem is solved repeatedly until a satisfactory solution is found.

- Herglotz–Wiechert integral is is used

$$\Delta(p) = 2p \int_{r_p}^{r_o} \frac{dr}{r(\zeta^2 - p^2)^{1/2}}$$

Travel time curve inversion

where $\zeta = r/v$, and p is the ray parameter for the ray arriving at Δ. This can be converted to

$$\int_0^{\Delta_1} \cosh^{-1}\left(\frac{p(\Delta)}{\zeta_1}\right) d\Delta = \pi \ln\left(\frac{r_o}{r_1}\right),$$

where $\zeta_1 = r_1/v_1$ at radius r_1, the bottoming point of the ray that emerges at Δ_1.

- This formula is used by starting with an observed travel time curve, $T(\Delta)$, and forming its derivative $dT/d\Delta = p(\Delta)$ numerically.
- The integral is done numerically from $\Delta = 0$ to $\Delta = \Delta_1$, using the fact that $\zeta_1 = dT/d\Delta$ at a distance Δ_1. The equation then gives the radius, r_1, at which the velocity is r_1/ζ_1.

Note: This method sometimes fails when velocity decreases with depth, giving a low-velocity zone.

Summary

- The travel time curve, T(Δ), is concave downward because its slope, p(Δ), decreases with distance ($dp/d\Delta = d^2T/d\Delta^2 < 0$).
- Rays that bottom in the region of high velocity gradient are bent upward more and emerge at smaller values of Δ than would otherwise be the case.
- The p(Δ) and T(Δ) curves have three distinct branches, two normal forward branches $dp/d\Delta < 0$ and the the back branch Δ decreases with decreasing p, so $dp/d\Delta > 0$.

- For a triplication, the back branch meets the two forward branches at two points on the travel time and p(Δ) curves, for triplication $\frac{dp}{d\Delta} = \infty$
- Herglotz–Wiechert integral is an approach which gives the distance traveled by a ray with ray parameter p as a function of the velocity structure

$$\Delta(p) = 2p \int_{r_p}^{r_o} \frac{dr}{r(\zeta^2 - p^2)^{1/2}}$$

REFERENCES

- Stein, Seth, and Michael Wysession. An introduction to seismology, earthquakes, and earth structure. John Wiley & Sons, 2009.
- Lowrie, William, and Andreas Fichtner. Fundamentals of geophysics. Cambridge university press, 2020.
- Kearey, Philip, Michael Brooks, and Ian Hill. An introduction to geophysical exploration. Vol. 4. John Wiley & Sons, 2002.
- https://geologyscience.com/geology-branches/structural-geology/stress-and-strain/
- Seismology course, Professor Derek Schutt, Colorado State Univ., USA.

THANK YOU !