NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 07 : Anisotropic earth structure, Attenuation and Anelasticity.

Lecture 01: Types of Anisotropy; Anisotropy in upper crust and due to minerals and rocks

CONCEPTS COVERED

- **Anisotropic Earth structure**
- **Lattice and shape preferred orientation anisotropy**
- **Transverse and Azimuthal Anisotropy**
- **Anisotropy of upper crust due to cracks**
- **Anisotropy due to minerals and rocks**
- **Summary**

Recap

- The ray parameter for the spherical earth is given as $p = \dfrac{r \sin i}{v}$

- Ray path travel time and angular distance for spherical earth is expressed as

$$T(p) = \int \frac{ds}{v} = 2\int_{r_p}^{r_0} \frac{\zeta^2 dr}{r(\zeta^2 - p^2)^{\left(\frac{1}{2}\right)}} \qquad \Delta(p) = \int d\theta = 2p \int_{r_p}^{r_o} \frac{dr}{r(\zeta^2 - p^2)^{1/2}}$$

- For a triplication, the back branch meets the two forward branches at two points on the travel time and p(Δ) curves, for triplication $\dfrac{dp}{d\Delta} = \infty$

- Herglotz–Wiechert integral is an approach which gives the distance traveled by a ray with ray parameter p as a function of the velocity structure

$$\Delta(p) = 2p \int_{r_p}^{r_o} \frac{dr}{r(\zeta^2 - p^2)^{1/2}}$$

- Travel time data are generated by combining data from numerous earthquake at different epicentral distances.

Recap

- Surface reflections PP and SS are maximum-time phases in contrast to phases reflection phases like ScS, which are minimum phases.
- For the core, the shadow zone occurs for distances between ~98° to ~145° fo the P-wave and seismic energy also enters the shadow zone via P and S waves that diffract around core.
- “K” denotes passage through the outer core, reflections off the CMB are denoted by a lower-case “c” and P waves in the inner core are denoted by “I”.
- Sub-crustal lithosphere is characterised by the high P- and S-wave velocity about 8.1 and 4.5 km/s respectively. It is approximately zero at mid oceanic ridges and ~200 km beneath the stable craton.
- Across entire Earth, there is a ‘Low Velocity Zone (LVZ)’ beneath the lithosphere coincides with the expected mechanically weak asthenosphere underlying the stronger lithosphere.

Recap

- Velocity differences between the crust and the mantle results from their different compositions.
- The velocity discontinuities at depth of 410 km and 660 km is marked as transition zone between the upper and lower mantles
- Upper mantle travel times show two triplications around 15° and 22° caused by the 410 and 660 km discontinuities.
- At the very bottom 10-20 km of the mantle , there is an evidence of ultra-low-velocity zone(ULVZ)
- D'' layer is present at the very base of the mantle, a fascinating and poorly understood region and show strong evidence for significant seismic anisotropy.

Anisotropic earth structure

Some points

- For an isotropic material, elastic properties are same in all directions.

$$\sigma_{ij} = c_{ijkl} e_{kl}$$

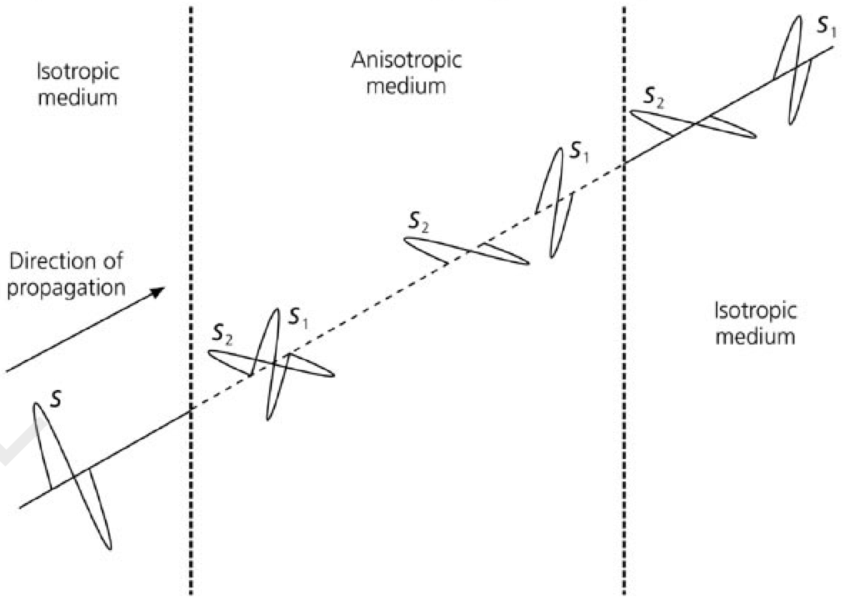
- In such materials the stresses are linearly proportional to the strains via Hooke's law.

- 81-term tensor of elastic moduli, C_{ijkl}, reduces to two independent elastic constants, λ and μ.

On the other hand,

- **Physical properties in an anisotropic material depends on the direction.**
- **It also follows Hooke's law and involves more than two elastic constants.**
- **It can have upto 21 independent elastic constants.**
- **For a material in which more than two elastic constants are needed, is called anisotropic.**

Figure 3.6-1: Cartoon of a shear wave split by an anisotropic medium.



How anisotropy occurs?

Lattice Preferred Orientation Anisotropy (LPO)

- Due to inhomogeneity or material’s being non-uniform also termed as heterogeneity.
- One of the most important anisotropic minerals is olivine which comprises much of the upper mantle.
- For example, mineral olivine is homogenous in that it is composed of the same repeating groups of atoms, but acts anisotropically because it align itself in the direction of mantle flow.

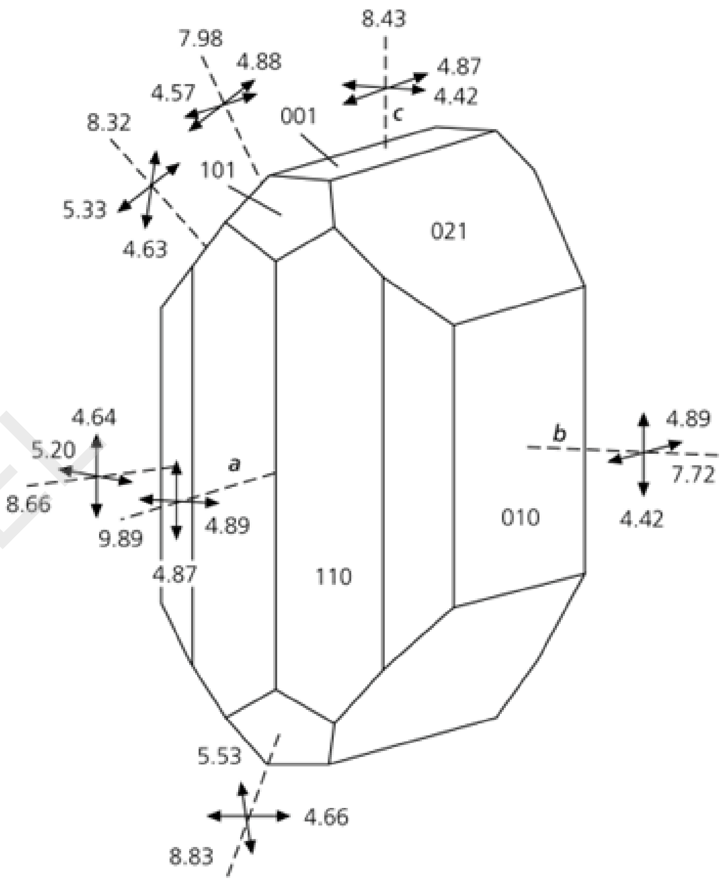


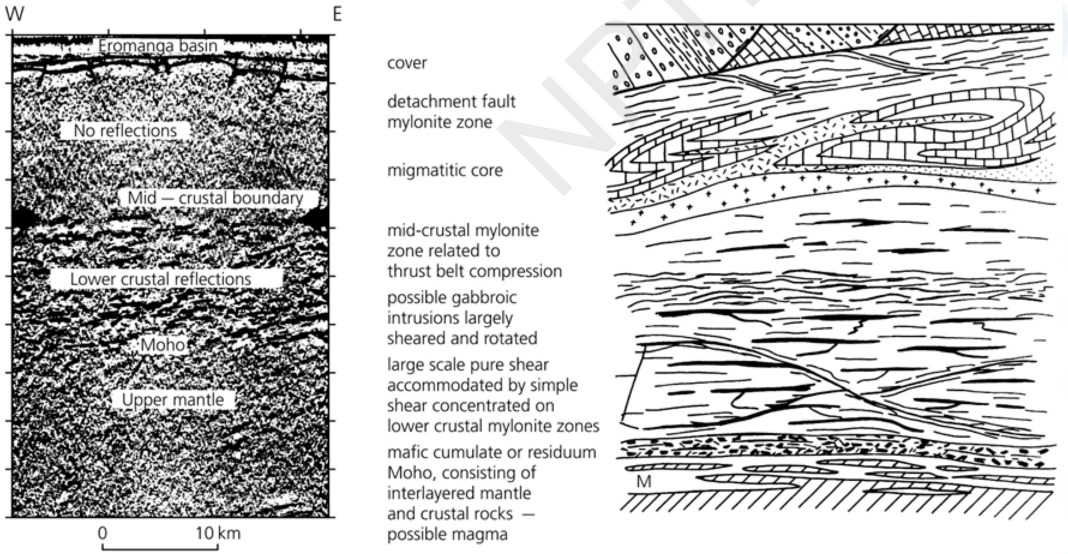
Figure 3.6-3: Anisotropy of an olivine crystal.

How anisotropy occurs?

Shape-Preferred Orientation (SPO) anisotropy

- A common situation is when material has directionality in its structure.
- Seismic waves travel with different speeds parallel and perpendicular to the layers. This situation is called Shape-Preferred Orientation (SPO) anisotropy.

Figure 3.6-5: Seismic reflection profile and cartoon of the crust.



What is Voigt Matrix?

The C_{ijkl} tensor as a matrix C_{mn} , where the indices m and n vary from 1 to 6. For example,

(1,1) → 1 (2,2) → 2

(3,3) → 3 (2,3) → 4

(1,3) → 5 (1,2) → 6

It maps (i,j) to m, and (k,l) to n so $C_{ijkl} = C_{mn}$

$$C_{mn} = \begin{pmatrix} c_{1111} & c_{1122} & c_{1133} & c_{1123} & c_{1113} & c_{1112} \\ c_{2211} & c_{2222} & c_{2233} & c_{2223} & c_{2213} & c_{2212} \\ c_{3311} & c_{3322} & c_{3333} & c_{3323} & c_{3313} & c_{3312} \\ c_{2311} & c_{2322} & c_{2333} & c_{2323} & c_{2313} & c_{2312} \\ c_{1311} & c_{1322} & c_{1333} & c_{1323} & c_{1313} & c_{1312} \\ c_{1211} & c_{1222} & c_{1233} & c_{1223} & c_{1213} & c_{1212} \end{pmatrix}$$

$$= \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{pmatrix}$$

For an isotropic material, the c_{ijkl} tensor can be written in terms of two independent elastic constants OR the Voigt matrix for isotropic matrix may look like as following.

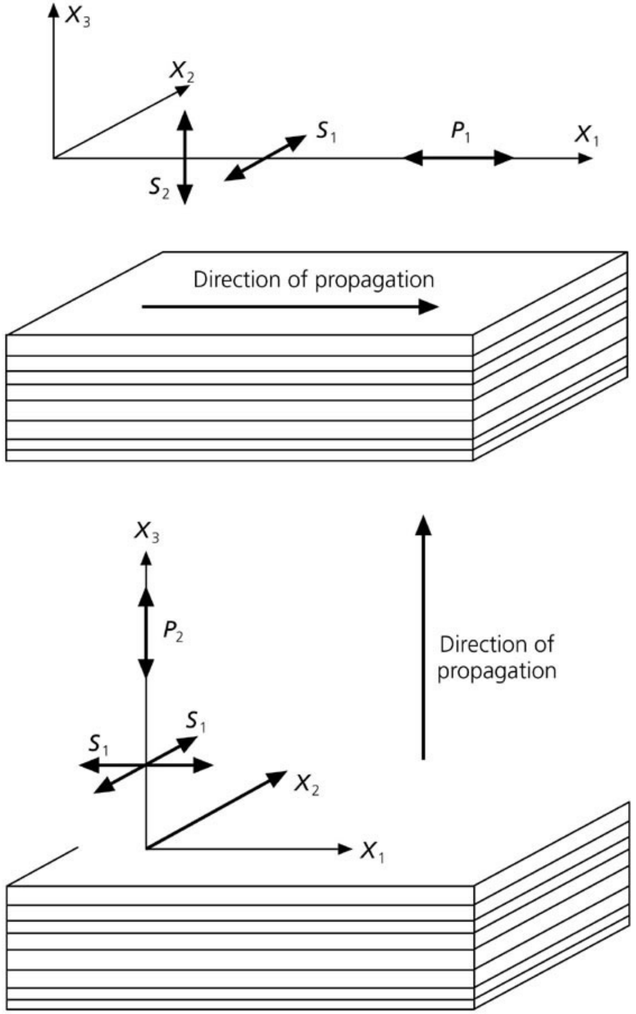
$$c_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

in matrix form

$$C_{mn} = \begin{pmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix}$$

Transverse isotropy and azimuthal anisotropy

Figure 3.6-2: The effects of transverse isotropy due to layering.



- Also termed as radial anisotropy, axis symmetry, and cylindrical symmetry) occurs for a stack of layered materials.

- It is isotropic within the layer, but the properties changes between layers (as in plywood).

- Elastic properties, and hence seismic velocities, of the stack as a whole are identical regardless of the amount of rotation about the axis of symmetry, which is perpendicular to the layers. However, these aggregate properties differ in the perpendicular directions.

Transverse isotropy

- A transverse isotropic materials are represented by five independent elastic coefficients, A, C, F, L, N. These letters are Backus notation.

- If the axis of symmetry is x_3, so properties in that direction differ from those in the x_1 –x_2 plane, the elastic constant matrix becomes,

$$C_{mn} = \begin{pmatrix} A & A-2N & F & 0 & 0 & 0 \\ A-2N & A & F & 0 & 0 & 0 \\ F & F & C & 0 & 0 & 0 \\ 0 & 0 & 0 & L & 0 & 0 \\ 0 & 0 & 0 & 0 & L & 0 \\ 0 & 0 & 0 & 0 & 0 & N \end{pmatrix}$$

$$A \Rightarrow \lambda + \mu \;\rightarrow x_1 \;\; direction$$
$$N \Rightarrow \mu \rightarrow x_2 \;\; direction$$
$$L \Rightarrow \mu \rightarrow \;x_3 \; direction$$

This matrix gives the velocities of waves propagating in different directions.

In the x_1-x_2 direction, $V_{P1} = (A/\rho)^{1/2}$, $V_{S1} = (N/\rho)^{1/2}$, $V_{S2} = (L/\rho)^{1/2}$

In the x_3 direction, $V_{P1} = (A/\ \rho)^{1/2}$, $V_{S1} = V_{S2} = (N/\rho)^{1/2}$

- **The horizontally layered earth shows transverse isotropy about a vertical axis. The SH-wave velocity S_1 is generally faster than the SV velocity S_2, because the SH displacement is preferentially in the fast layers, whereas SV samples both equally.**

- **That is why, $V_{\text{Love waves}} > V_{\text{Rayleigh waves}}$**

- **Transverse isotropy is often characterized by three parameters**

$$\xi = N/L = (S_1/S_2)^2,\ \phi = C/A = (P_2/P_1)^2,\ \eta = F/(A - 2L)$$

  - **If the material were isotropic, $\xi = \phi = \eta = 1$.**
  - **For layered structures, generally $\xi > 1$ and $\phi < 1$**

Azimuthal anisotropy

- A second common type of anisotropy is azimuthal anisotropy, in which velocities vary as a function of horizontal direction.
- One way to obtain this is to have transverse isotropy with the x_3 axis turned to horizontal, which is analogous to standing plywood vertically.
- The P-wave velocity varies with azimuth as:

$$P(\theta) = A_1 + A_2 \cos(2\theta) + A_3 \sin(2\theta) + A_4 \cos(4\theta) + A_5 \sin(4\theta)$$

Azimuthal anisotropy

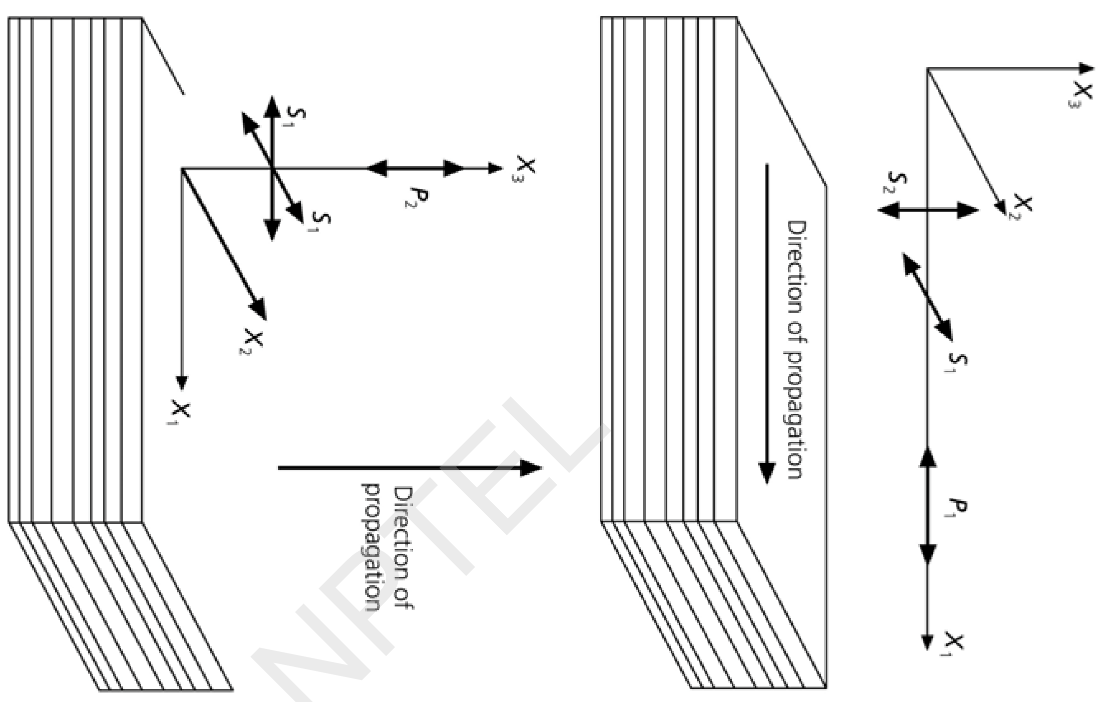


Figure 3.6-2: The effects of transverse isotropy due to layering.

➔ **the P-wave velocity varies with azimuth as:**

$$P(\theta) = A_1 + A_2 \cos(2\theta) + A_3 \sin(2\theta) + A_4 \cos(4\theta) + A_5 \sin(4\theta)$$

A_i depend upon 21 elastic constants. In the lithosphere, the magnitude of the 4θ component may be small, and is often ignored in teleseismic studies.

Anisotropy in the Upper crust due to cracks

- If we are looking at upper crust, where there are lots cracks or joints.

- Then the anisotropy will be a function of ε, which is related to the crack density as $\varepsilon = Na^3/V$, with N being the number of cracks in volume V, and a is the half-width of the cracks.

$$c_{mn} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu(1-\varepsilon) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu(1-\varepsilon) \end{bmatrix}$$

- Cracks may reduce the shear modulus. If the fluid is incompressible, P-wave velocity won't be affected, but S-wave velocity will see some change.

Anisotropy of minerals and rocks

- ➔ The magnitude of anisotropy is characterized by: $k = \frac{v_{\max} - v_{\min}}{v_{mean}}$

For olivine:

- For P-waves in the olivine crystal, α_{max} = 9.89 km/s, α_{min} = 7.72 km/s, and α mean = 8.81 km/s, so k = 25%.
- The maximum and minimum S velocities are 5.53 km/s and 4.42 km/s, so k = 22%.

- ➔ Minerals range from nearly isotropic to extremely anisotropic.

- ➔ One of the most isotropic minerals is garnet, where k for both P and S waves is ≤ 1%.

- ➔ While, sheet silicates like mica can have values of k up to 60% for P waves and 116% for S waves.

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Anisotropy of minerals and rocks

Points to remember!

- A major factor controlling a rock's anisotropy is the anisotropy of the minerals composing it and their relative proportions.
- Another important factor is the presence of deviatoric stresses, which can cause a preferred orientation of anisotropic mineral grains that might otherwise be randomly distributed.
- Crystals are generally oriented with their smallest widths in the direction of maximum compression.
- Shear in a preferred direction can also recrystallize different mineral assemblages, so the resulting anisotropy reflects a combination of the preferred orientation of anisotropic materials and the presence of laminar structures.

Anisotropy of composite structures

- Anisotropy can also result from an asymmetric combination of materials.
- The upper continental crust often contains horizontally layered sedimentary rocks.
- Similarly, oceanic crust is comprised of sediments overlying layers of basalt and gabbro.
- Such layering can yield transverse isotropy, with the symmetry axis oriented vertically.
- On a regional scale, plate collisions often cause significant metamorphism, sometimes yielding transverse isotropy due to the preferred orientation of the foliation of gneisses and schists.

Summary

- **Any material in which more than two elastic constants are need is called anisotropic.**
- **Minerals and rock comprises anisotropy either due to lattice Preferred Orientation (LPO) or Shape-Preferred Orientation (SPO).**
- **Transverse isotropy is often characterized by three parameters**
  $\xi = N/L = (S_1/S_2)^2$, $\phi = C/A = (P_2/P_1)^2$, $\eta = F/(A - 2L)$
  - ➔ **If the material were isotropic, $\xi = \phi = \eta = 1$.**
  - ➔ **For layered structures, generally $\xi > 1$ and $\phi < 1$**
- **In general, the P-wave velocity varies with azimuth as:**

$$P(\theta) = A_1 + A_2 \cos(2\theta) + A_3 \sin(2\theta) + A_4 \cos(4\theta) + A_5 \sin(4\theta)$$

- **The magnitude of anisotropy is characterized by:**

$$k = \frac{v_{\max} - v_{\min}}{v_{mean}}$$

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THANK YOU !