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EARTHQUAKE SEISMOLOGY

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Module 07 : Anisotropic earth structure, Attenuation and Anelasticity.
Lecture 05: Spectral Resonance Peaks, Physical dispersion due to anelasticity, and Q from crust to inner core

CONCEPTS COVERED

- ➢ **Recap**
- ➢ **Spectral Resonance Peaks**
- ➢ **Physical dispersion due to anelasticity**
- ➢ **Q from crust to inner core**
- ➢ **Summary**

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Recap

- For the damped harmonic oscillator, displacement can be given: as: $u(t) = A_o e^{i(\omega t + ibt)} = A_o e^{-bt} e^{i\omega t}$ with $\omega = a = \omega_o \left(1 - 1/4Q^2\right)^{1/2}$ and $b = \omega_o / 2Q$
- Relaxation time is given as: $t_{1/e} = 2Q/\omega_o$
- Q is inversely proportional to the damping factor, γ, so the smaller the damping, the greater Q is.
- Energy decay is given as: $E(t) = \frac{1}{2} k A(t)^2 = \frac{1}{2} k A_o^2 e^{-\omega_o t/Q} = E_o e^{-\omega_o t/Q}$
- Quality factor can be formulated as: $Q^{-1} = 2b/a = 2\omega^*/\omega.$ and $Q^{-1} = 2c^*/c$

  $Q_\beta^{-1} = \mu^*/\mu \qquad Q_\alpha^{-1} = (K^* + 4/3\mu^*)/(K + 4/3\mu)$

- $Q = \pi / \ln\left(A_1/A_2\right)$ A_1 and A_2 are the amplitudes of two consecutive cycles

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Spectral Resonance Peaks

We are interested in understanding how anelasticity in the earth causes the attenuation of propagating waves.

Lets us consider a damped harmonic oscillator with a frequency dependent driving force:

$$\frac{d^2u(t)}{dt^2} + \gamma\frac{du(t)}{dt} + \omega_o^2 u(t) = e^{i\omega t} \quad \text{.......equ 1}$$

The solution is found using a trial solution

$$u(t) = A(\omega)e^{i\phi(\omega)}e^{i\omega_o t}$$

Substituting this in Eqn 1 yields the amplitude response, A(ω), and phase response, ϕ(ω),

$$A(\omega) = \frac{1}{\left[(\omega_o^2 - \omega^2)^2 + (\omega\gamma)^2\right]^{1/2}}, \quad \phi(\omega) = \tan^{-1}\left[\frac{-\gamma\omega}{\omega_o^2 - \omega^2}\right]$$

“Amplitude and the phase responses depend on the damping factor γ.”

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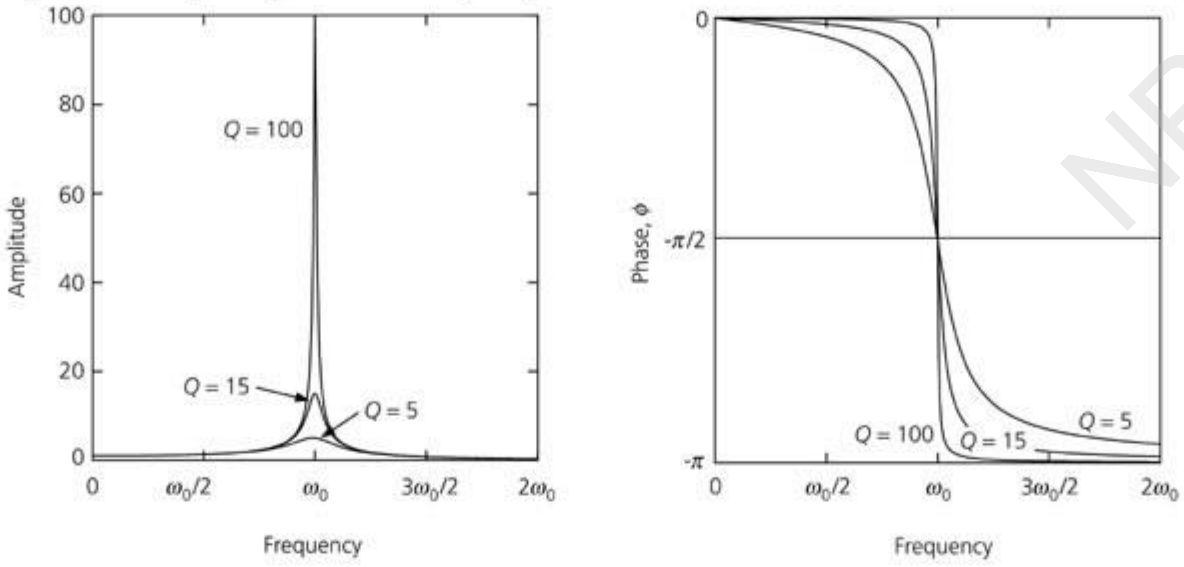
Spectral Resonance Peaks

The resonance curve shows how the damped harmonic oscillator responds to the frequency-dependent driving force.

The closer the driving frequency ω is to the oscillator's natural frequency ω_0, the more the oscillator responds.

The resonance curve can be viewed in terms of the frequency at which the peak occurs:

Figure 3.7-13: Amplitude/phase of a forced, damped harmonic oscillator.



$$\omega_p = \left(\omega_o^2 - \gamma^2/2\right)^{1/2} = \omega_o\left(1 - 1/2Q^2\right)^{1/2}$$

and the amplitude of the peak,

$$A(\omega_p) = Q/\left(\omega_o^2\left(1 - 1/2Q^2\right)^{1/2}\right)$$

If the oscillator is undamped (γ = 0, Q = ∞) the peak occurs at its natural frequency and shows an infinite response.

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Spectral Resonance Peaks

Three commonly considered application of resonance in seismology are-

- **An earthquakes excite the normal modes of earth. These modes form a set of damped harmonic oscillators, so the amplitude spectrum of a long-period seismogram contains peaks that correspond to the net resonance curve for each mode multiplet. The width of a peak depends on the frequencies and amplitudes of the mode's singlets and the mode's damping.**

- **Seismometers can also be viewed as damped harmonic oscillators, whose natural frequency and damping control their response to ground motion.**

- **This concept is important in designing earthquake resistant structures, because buildings are most vulnerable to ground motion with frequencies close to their natural frequencies, so damping is added to reduce the resulting motion.**

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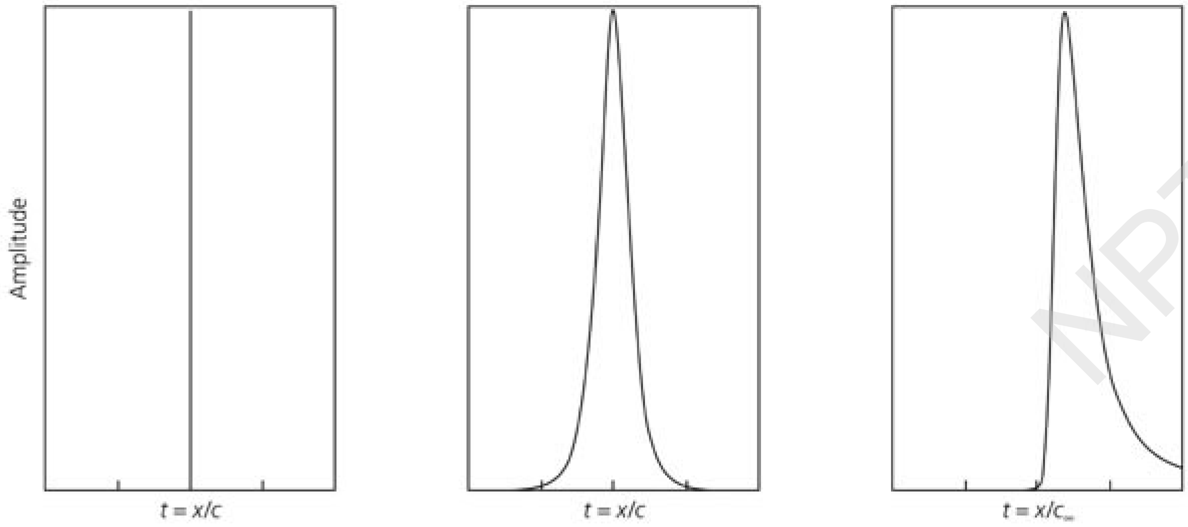
Physical dispersion due to anelasticity

An important consequence of seismic wave attenuation is physical dispersion, in which waves at different frequencies travel at different velocities.

In physical dispersion the intrinsic velocity of waves in the medium varies with frequency.

Figure 3.7-14: Demonstration of physical dispersion for an attenuated pulse.



Left: A propagating wave pulse composed of a delta function. With no dispersion, all frequencies arrive at the same time.

Center: The delta function after broadening by attenuation, showing that energy arrives before the high-frequency arrival time.

Right: The pulse including physical dispersion, which makes the lower frequency waves travel more slowly, so that they do not arrive before the highest frequency component.

Physical dispersion due to anelasticity

Mathematically, it can be seen as:

Assume that a delta function wave, a pulse of infinite height and unit area, propagates through a homogeneous elastic medium with intrinsic velocity c:

$$u(x,t) = \delta(t - x/c)$$

It is a property of the Dirac delta function that

$$\int_{-\infty}^{\infty} \delta(t - t_0) f(t)\,dt = f(t_0)$$

The Fourier transform of the delta function,

$$F(\omega) = \int_{-\infty}^{\infty} u(x,t) e^{-i\omega t} dt = \int_{-\infty}^{\infty} \delta(t - x/c) e^{-i\omega t} dt = e^{(-i\omega x/c)}$$ **…….equ 2**

shows that the delta function is made up of waves of all frequencies. If there is no dispersion, all the frequencies travel at the same speed and arrive at the same time.

The effect of attenuation as a function of distance and frequency,

$$A(\omega) = A_o e^{\frac{-\omega x}{2cQ}}$$ **…….equ 3**

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Physical dispersion due to anelasticity

Previous equation shows that if Q is constant, the rate at which the amplitude decays with distance increases strongly with frequency.

On multiplying Eqn 2 by Eqn 3 and use the inverse Fourier transform to return to the time domain

$$u(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\omega)F(\omega)e^{i\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{\frac{-\omega x}{2cQ}}e^{\frac{-i\omega x}{c}}e^{i\omega t}d\omega$$

Evaluating the integral yields

$$u(x,t) = \left[(x/2cQ)/\left((x/2cQ)^2 + (x/c - t)^2\right)\right]/\pi$$

so the delta function is broadened by attenuation into a wavelet that is symmetric in time about its maximum at t = x/c.
A problem with this solution is that seismic energy arrives before the geometric arrival time of the delta function pulse, t = x/c, which is the arrival time of the infinite-frequency component

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Physical dispersion due to anelasticity

The tails of the wavelet extend to infinity on both sides of $t = x/c$, some energy arrives before the earthquake occurred. This impossible situation, called noncausality

The mathematical condition for causality is that $u(x, t) = 0$ for all $t < x/c_\infty$, where $c_\infty = c(\infty)$ is the phase velocity of the infinite-frequency waves that arrive first.

One such dispersion relation for phase velocity as a function of frequency, called Azimi's attenuation law

$$c(\omega) = c_o\left[1 + \frac{1}{\pi Q}\ln\left(\frac{\omega}{\omega_o}\right)\right] \quad \text{.......equ 4}$$

where c_0 is a reference velocity corresponding to a reference frequency ω_0.

This relation provides the needed causality, because the resulting pulse has high frequencies arriving at or soon after $t = x/c_\infty$, whereas the low frequencies arrive later over a duration depending on the value of Q

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Physical dispersion due to anelasticity

From Eqn 4, the P- and S-wave velocities α and β vary as a function of period T, as

$$\beta(T) = \beta(1)\left(1 - \frac{\ln T}{\pi} Q_\mu^{-1}\right),$$

$$\alpha(T) = \alpha(1)\left[1 - \frac{\ln T}{\pi}\left(L Q_\mu^{-1} + (1 - L) Q_K^{-1}\right)\right]$$

$$\textbf{Here,} \qquad L = \frac{4}{3}\left(\frac{\beta}{\alpha}\right)^2$$

where α(1) and β(1) are the velocities at 1 s.

This phenomenon causes a discrepancy between the seismic velocity structure found by inverting observations of long period normal modes and short-period body waves.

The velocities inferred from normal modes are consistently slower than those from body waves.

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Physical dispersion due to anelasticity

Body wave attenuation is often characterized using the parameter t*. If a ray travels through a region of constant Q

$$t^* = \frac{t}{Q} = \frac{\text{travel time}}{\text{quality factor}}$$

Because Q varies within the earth, we derive t* by integrating along the ray path,

$$t^* = \int \frac{dt}{Q} = \sum_{i=1}^{N} \frac{\Delta t_i}{Q_i}$$

where Δt_i and Q_i are the travel time and Q values on the i^{th} path segment.

- **For P waves, t^*_α is often about 1 s, whereas S waves typically have t^*_β around 4 s.**
- **The values of t* increase with increased distance, but are also affected by the number of passages through the asthenosphere**

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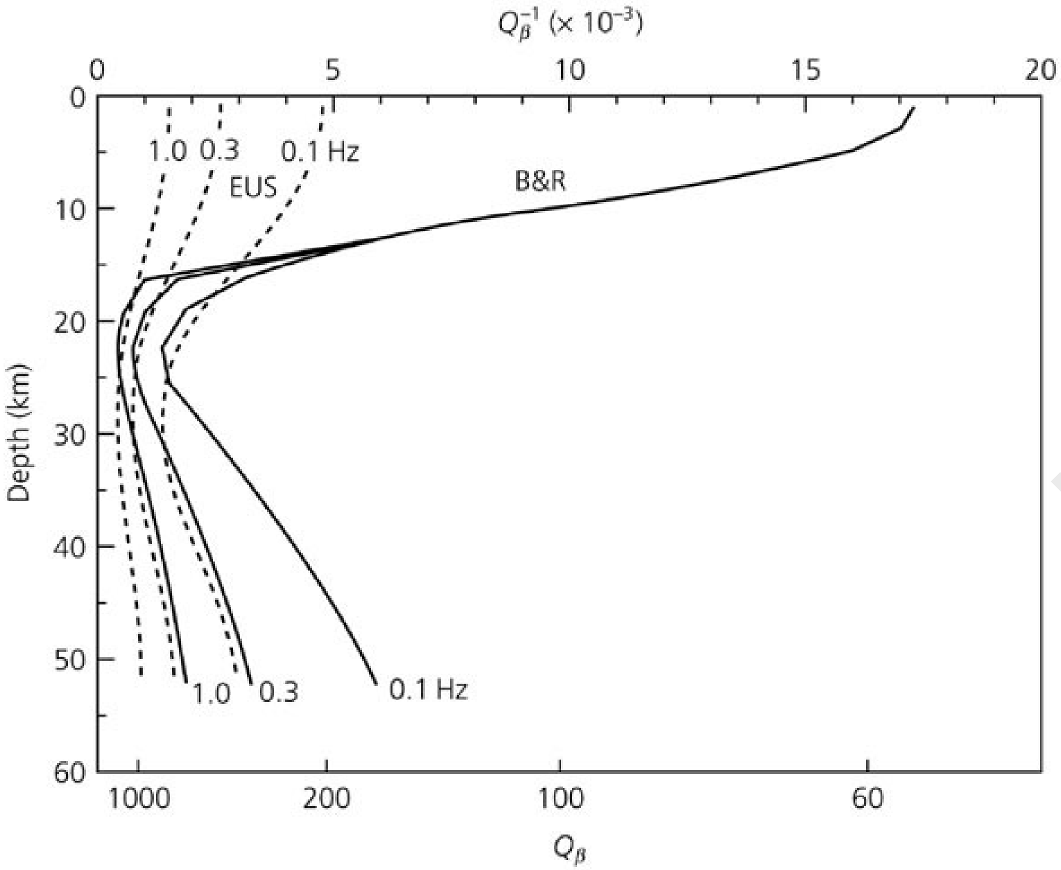
Q from crust to inner core

Crust

Figure 3.7-17: Regional variations in lithospheric attenuation.



- **Attenuation is inferred in all regions of the earth except for the liquid iron outer core, and varies greatly both laterally and vertically.**

- **In the crust, the greatest attenuation (lowest Q or highest Q^{-1}) occurs near the surface, presumably due to the presence of fluids.**

- **Attenuation is lowest at about 20–25 km depth, and then increases again, presumably due to increasing temperature.**

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Q from crust to inner core

- **Q in the upper crust is roughly proportional to the time since the last major tectonic activity in a region, perhaps due to crack generation and fluid flow during tectonism and gradual crack annealing after tectonism ceased.**

- **Regional variations in crustal Q are often studied with L_g waves, a superposition of higher-mode surface waves that give prominent arrivals in continental regions.**

Upper Mantle

- **Attenuation in the upper mantle varies with depth, with the lowest Q in the asthenosphere from about 80 to 220 km depth.**

- **At these depths the temperature approaches, and perhaps exceeds, the melting temperatures of rock, so a small percentage of partial melt may exist.**

- **This pattern of attenuation is similar to that for seismic velocities, which are lowest in the asthenosphere.**

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Q from crust to inner core

- Beneath the asthenosphere, Q increases gradually with depth, presumably because temperature increases at a slower rate than pressure.
- Q_{μ} increases with depth through the lower mantle, reaching values in excess of 500.
- There is some indication that attenuation is enhanced in the D″ region at the base of the mantle.

Outer and Inner Core

- Although no attenuation of P waves is detected for the outer core, there is significant attenuation of PKIKP waves traversing the inner core, yielding Q_{K} estimates in the range of 150–300.

Where temperatures vary over short distances, significant attenuation variations can occur.

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Summary

- **Any material in which more than two elastic constants are need is called anisotropic.**

- **Minerals and rock comprises anisotropy either due to lattice Preferred Orientation (LPO) or Shape-Preferred Orientation (SPO).**

- **Transverse isotropy is often characterized by three parameters**
  $\xi = N/L = (S_1/S_2)^2$, $\phi = C/A = (P_2/P_1)^2$, $\eta = F/(A - 2L)$
  - ➔ **If the material were isotropic, $\xi = \phi = \eta = 1$.**
  - ➔ **For layered structures, generally $\xi > 1$ and $\phi < 1$**

- **In general, the P-wave velocity varies with azimuth as:**

$$P(\theta) = A_1 + A_2 \cos(2\theta) + A_3 \sin(2\theta) + A_4 \cos(4\theta) + A_5 \sin(4\theta)$$

- **The magnitude of anisotropy is characterized by:**

$$k = \frac{v_{\max} - v_{\min}}{v_{mean}}$$

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Summary

- As shear waves travel across the mantle and crust, they can be split when traveling through anisotropic media that serve the basis of the Shear Wave Splitting to study the lithospheric anisotropy.

- We would normally not expect any SKS on the transverse component, but anisotropy yields a combination of both the fast and the slow polarizations on both the radial and the transverse components, given by:

$$R(t) = s(t)\cos^2\phi + s(t-\delta t)\sin^2\phi,$$
$$T(t) = [s(t) - s(t-\delta t)/2]\sin 2\phi$$

$$\delta t = d/v_f - d/v_s$$

v_f and v_s are the velocities of the qS1 (fast) and qS2 (slow),

- Source of anisotropy in the upper crust is the presence of fluid-filled cracks, horizontal sediment layers, intruded dykes, preferred orientation of olivine crystals yielded by spreading process.

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Summary

- Seismic anisotropy within continents is thought to reflect crystal alignment created during a tectonic episode and then "frozen in."

- For plate collisions the fast axis is usually sub-perpendicular to the principal stress axis, or parallel to the resulting orogenic belts.

- Anisotropy in the Pacific ocean is derived by the parameter 'ξ' as a function of age and depth.

When $\xi > 1$, then

→ Love wave velocity is fast than Rayleigh wave and indicates horizontal mantle flow.

When $\xi < 1$, then

→ Love wave velocity is lesser than Rayleigh wave and indicates vertical mantle flow.

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Summary

- Geometric spreading, scattering and multipathing reduces the amplitudes but propagating wavefield is conserved.

- Seismic velocities depend nearly linearly upon temperature, whereas attenuation depends exponentially on temperature. Thus combining velocity and attenuation studies can provide valuable information.

- Because the earth is a sphere, the ring wraps around the globe making the energy per unit wavefront of the surface waves vary as:

  $1/r = 1/(a \sin \Delta)$, where Δ is the angular distance from the source

- Amplitudes decrease as $(a \sin \Delta)^{-1/2}$, with minimum at $\Delta = 90°$, and maxima at $0°$ and $180°$.

- For Body waves, energy per unit wave front decays as $1/r^2$, and the amplitude decreases as $1/r$.

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Summary

- Seismic waves refract towards low-velocity anomalies and away from high velocity anomalies.

- When multipathing occurs, the seismic waves arriving at a receiver can be viewed as having taken some ray paths in addition to the direct path, and so have sampled a larger region of the earth.

- When the heterogeneity is large compared to the wavelength, we regard the wave as following a distinct ray path that is distorted by multipathing.

- Scattering is especially important in the continental crust, which has many small layers and reflectors resulting from billions of years of continental evolution.

- The main arrival has a polarity related to the direction of propagation but the scattered energy arrives from various directions shows little or no preferred particle motion.

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Summary

- For the damped harmonic oscillator, displacement can be given: as: $x(t) = A_o e^{i(\omega t + ibt)} = A_o e^{-bt} e^{i\omega t}$
  with $\omega = a = \omega_o \left(1 - 1/4Q^2\right)^{1/2}$ and $b = \omega_o/2Q$
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- Quality factor can be formulated as: $Q^{-1} = 2b/a = 2\omega^*/\omega.$ and $Q^{-1} = 2c^*/c$

$$Q_\beta^{-1} = \mu^*/\mu \qquad Q_\alpha^{-1} = (K^* + 4/3\mu^*)/(K + 4/3\mu)$$

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Summary

- **Solution of the forced-driven oscillator is:** $u(t) = A(\omega)e^{i\phi(\omega)}e^{i\omega_o t}$

- **Amplitude and the phase responses depend on the damping factor γ and given as:**

$$A(\omega) = \frac{1}{\left[\left(\omega_o^2 - \omega^2\right)^2 + (\omega\gamma)^2\right]^{1/2}}, \quad \phi(\omega) = \tan^{-1}\left[\frac{-\gamma\omega}{\omega_o^2 - \omega^2}\right]$$

- **In physical dispersion the intrinsic velocity of waves in the medium varies with frequency. P- and S-wave velocities α and β vary as a function of period T, as**

$$\beta(T) = \beta(1)\left(1 - \frac{\ln T}{\pi} Q_\mu^{-1}\right), \quad \alpha(T) = \alpha(1)\left[1 - \frac{\ln T}{\pi}\left(L Q_\mu^{-1} + (1 - L) Q_K^{-1}\right)\right]$$

- **Body wave attenuation is often characterized using the parameter t*.**

$$t^* = \int \frac{dt}{Q} = \sum_{i=1}^{N} \frac{\Delta t_i}{Q_i}$$

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