NPTEL ONLINE CERTIFICATION COURSES

NPTEL

EARTHQUAKE SEISMOLOGY

Dr. Mohit Agrawal
Department of Applied Geophysics , IIT(ISM) Dhanbad

Module 08 : Composition of the mantle and core
Lecture 01: Density within the Earth, Adams–Williamson equation and Pressure within the earth

CONCEPTS COVERED

- **Density within the earth**
  - **Adams–Williamson equation**
  - **Setbacks**
  - **Solution**
- **Pressure Variation within the earth**
- **Summary**

NPTEL

Density within the Earth

➔ Density of Earth can be combined with velocities to derive elastic constants.

➔ Average Earth's density can be found by the Earth's mass M, which can be found from the law of gravitation,

$$g = GM/a^2$$

where,
$g = 9.8\ m/s^2$, $a = 6371$ km,
$G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$,
$M = 5.97 \times 10^{24}$ kg.

NPTEL

Note: The mass is the volume integral of the density because Earth's density varies only with depth

$$M = 4\pi \int_0^a \rho(r) r^2 dr$$

But the average density, ρ_o, is found by dividing the mass by the volume,

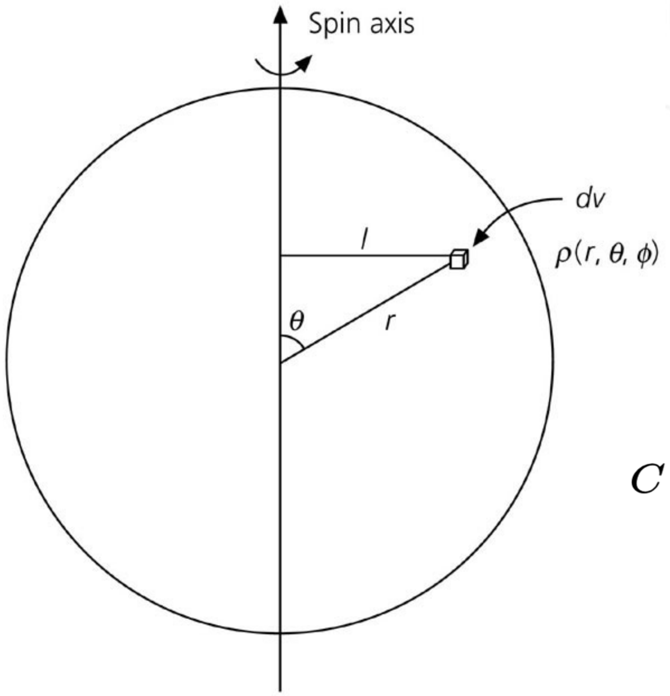
$$\rho_o = M/\left[(4/3)\pi a^3\right]$$

The resulting average density of the earth is about 5.5 g/cm^3

Density within the Earth

- ➔ **The fact that this value is significantly higher than the density of the surface rocks (about 3 g/cm^3) is evidence for a core of much denser, and different material.**

Figure 3.8-1: Moment of inertia of a volume element.



The second indication of the dense core, comes from the moment of inertia about the rotation axis.

This is defined by integrating over volumes dV, each at a distance l = r sin θ from the spin axis,

NPTEL

$$C = \iiint l^2 \rho(r,\theta,\phi) dV = \int_0^{2\pi} \int_0^{\pi} \int_0^{a} \rho(r) \left(r^2 \sin^2 \theta\right) r^2 \sin \theta dr d\theta d\phi$$

$$= \frac{8}{3}\pi \int_0^{a} \rho(r) r^4 dr$$

Density within the Earth

The ratio of the moment of inertia to the mass gives a scalar that depends on the density distribution.

Case I- If the earth were homogeneous, the density everywhere would equal the average density, $\rho(r) = \rho_o$, and

$C = (8/15)\pi a^5 \rho_o$, $M = (4/3)\pi a^3 \rho_o$, $C/Ma^2 = 0.4$

Case II - if all the mass were in a shell at the surface.

$$\rho(r) = \delta(r - a)\rho_s$$

$C = (8/3)\pi a^4 \rho_s$, $M = 4\pi a^2 \rho_s$, $C/Ma^2 = 0.67$

i.e. a distribution with material concentrated towards the outside gives a larger ratio.

NPTEL

Density within the Earth

Case III -A more realistic case is a two-shell planet, with a mantle of density ρ_m and a core of density ρ_c and radius r_c. The integrals are evaluated in pieces as

$$C = \frac{8}{3}\pi\left[\int_0^{r} \rho_c r^4 dr + \int_{r_c}^{a} \rho_m r^4 dr\right] = \frac{8}{15}\pi\left[\rho_m a^5 + (\rho_c - \rho_m) r_c^5\right]$$

$$M = \frac{4}{3}\pi\left[\rho_m a^3 + (\rho_c - \rho_m) r_c^3\right]$$

NPTEL

If we consider the values similar to those for the earth i.e.,

$\rho_c = 12\ g/cm^3$,

$\rho_m = 5\ g/cm^3$

$r_c = 3480$ km,

Density within the earth

➔ It yields a moment of inertia ratio of $C/Ma^2 = 0.35$.

➔ This value is less than the 0.4 which a uniform planet would have, because the material is concentrated toward the center.

➔ It is similar to the value of C/Ma^2 for the earth field, about 0.33, determined from the earth's shape and gravity. Thus indicates the presence of a dense core.

NPTEL

Density within the Earth

- Let's look at the variation of density with depth relating the seismic velocities. Consider a region of uniform material and see how the density increases with depth as the material is self-compressed by its own weight.

- At a radius r, the gradient of the hydrostatic pressure P(r) is:

$$\frac{dP}{dr} = -g\rho$$

where ρ(r) and g(r) are the density and the acceleration of gravity at that depth.

Since, $$g = GM/r^2$$

hence, $$\frac{dP}{dr} = \frac{-\rho Gm}{r^2}$$ ……….Equ.1

We know that, $$\rho = m/V, \quad d\theta = dV/V$$

So, $$d\rho = -(m/V^2)dV = -\rho d\theta$$

NPTEL

Density within the Earth

The bulk modulus K can be expressed as:

$$K = -\frac{dP}{d\theta} = -\frac{dP}{d\rho}\frac{d\rho}{d\theta} = \rho\frac{dP}{d\rho}$$

Combining this with the pressure derivative equation 1

$$\frac{d\rho}{dr} = -\frac{d\rho}{dP}\frac{dP}{dr} = -\frac{\rho^2 Gm}{Kr^2}$$

NPTEL

To include the observations of seismic velocities, we define the seismic parameter, Φ, and bulk sound speed, $\Phi^{1/2}$, such that

$$\Phi = \alpha^2 - (4/3)\beta^2 = K/\rho$$

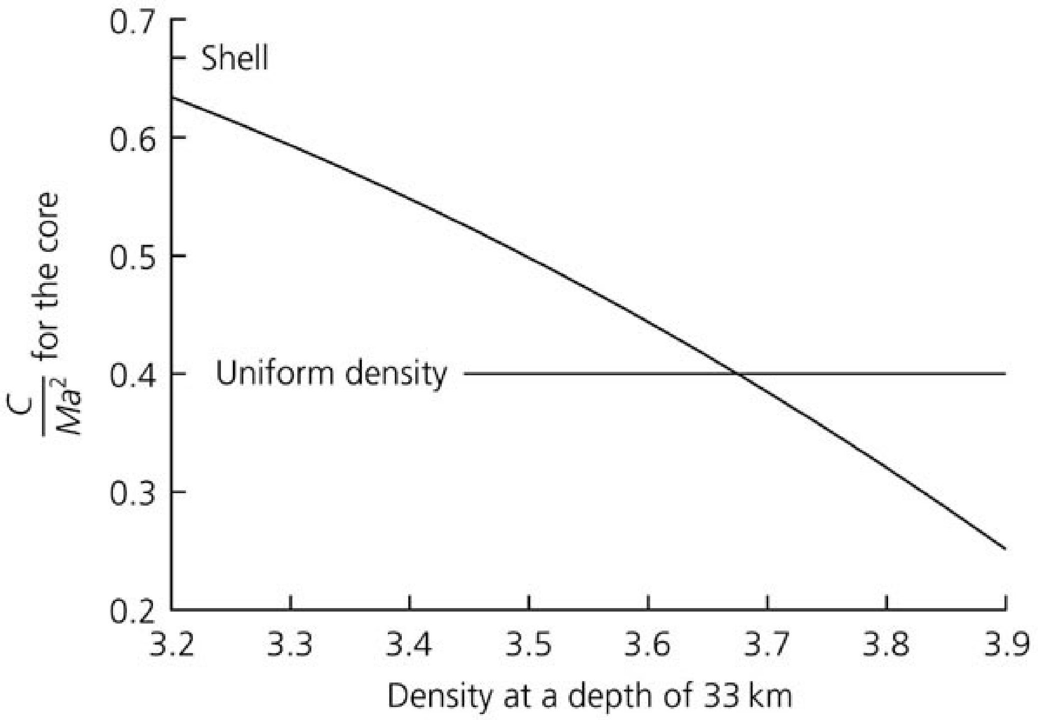
Thus we can write the Adams–Williamson equation relating the velocity structure to the derivative of density with radius,

$$\frac{d\rho(r)}{dr} = \frac{-\rho(r)Gm(r)}{\Phi(r)r^2} = \frac{-\rho(r)g(r)}{\Phi(r)}$$

Density within the earth : Setbacks

- ➔ Adams– Williamson equation is insufficient because density increases with depth as a result of mineral phase changes as well as of self-compression.

Figure 3.8-2: Moment of inertia ratio of the earth's core as a function of density at the top of the mantle.



- ➔ Figure shows the C/Ma^2 value calculated for the core as a function of the assumed density at the top of the mantle, which is the initial density for the Adams–Williamson calculation.
- ➔ If we consider near-surface density ≈ 3.3 g/cm^3,this will give C/Ma^2 for core greater than 0.4, implying that density decreases with depth in the core.
- ➔ It is not true, because the solid inner core should be denser than the liquid outer core.

NPTEL

Density within the earth : Solution!!

- F. Birch (1950s) showed that at least one of two assumptions underlying the method was inappropriate.

- One implicit assumption is that the temperature increases with depth along an adiabatic gradient, or "adiabat," which is not always true.

- Infact, the temperature gradient in the mantle is thought to exceed the adiabatic gradient, because a superadiabatic gradient is required for the thermal convection expected in the mantle.

- The superadiabatic gradient can be incorporate in the Adams–Williamson equation to:

$$\frac{d\rho}{dr} = -\frac{\rho g}{\phi} + g\tau\alpha$$

where, α is the coefficient of thermal expansion and τ is the portion of the temperature gradient exceeding the adiabetic gradient.

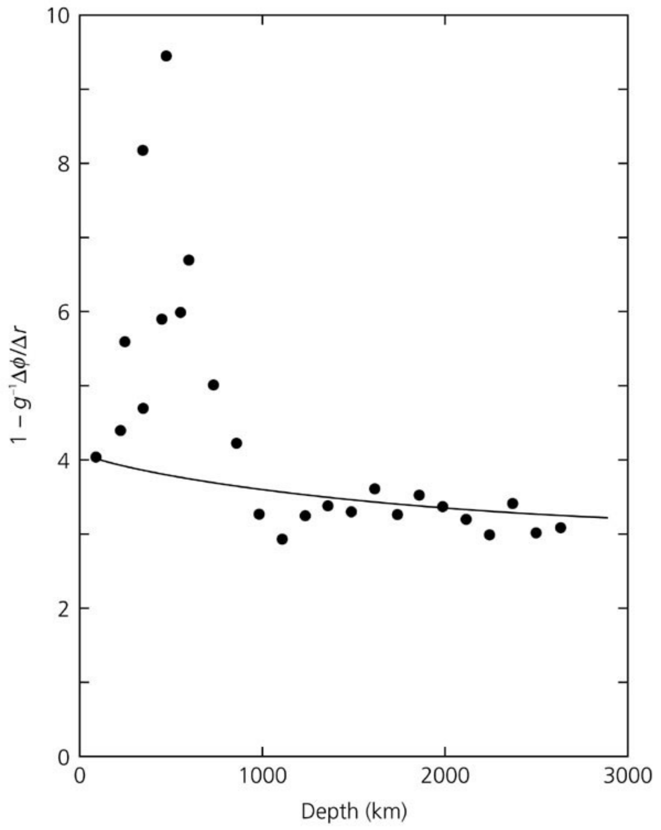
NPTEL

h

Density within the earth : Solution!!

- This correction for higher temperature lowers the calculated mantle densities and increase the C/Ma^2 for the core. So, it makes the issue of core density structure worse.

Figure 3.8-3: Deviation of the mantle from adiabatic conditions.



- Hence the assumption of homogeneous material whose density changes only by self-compression must be incorrect. Birch showed that inhomogeneity can be identified using the function $1 - (1/g)d\phi/dr$.

- Below 1000 km the mantle behaves as a homogeneous material, while at shallower depths it does not.

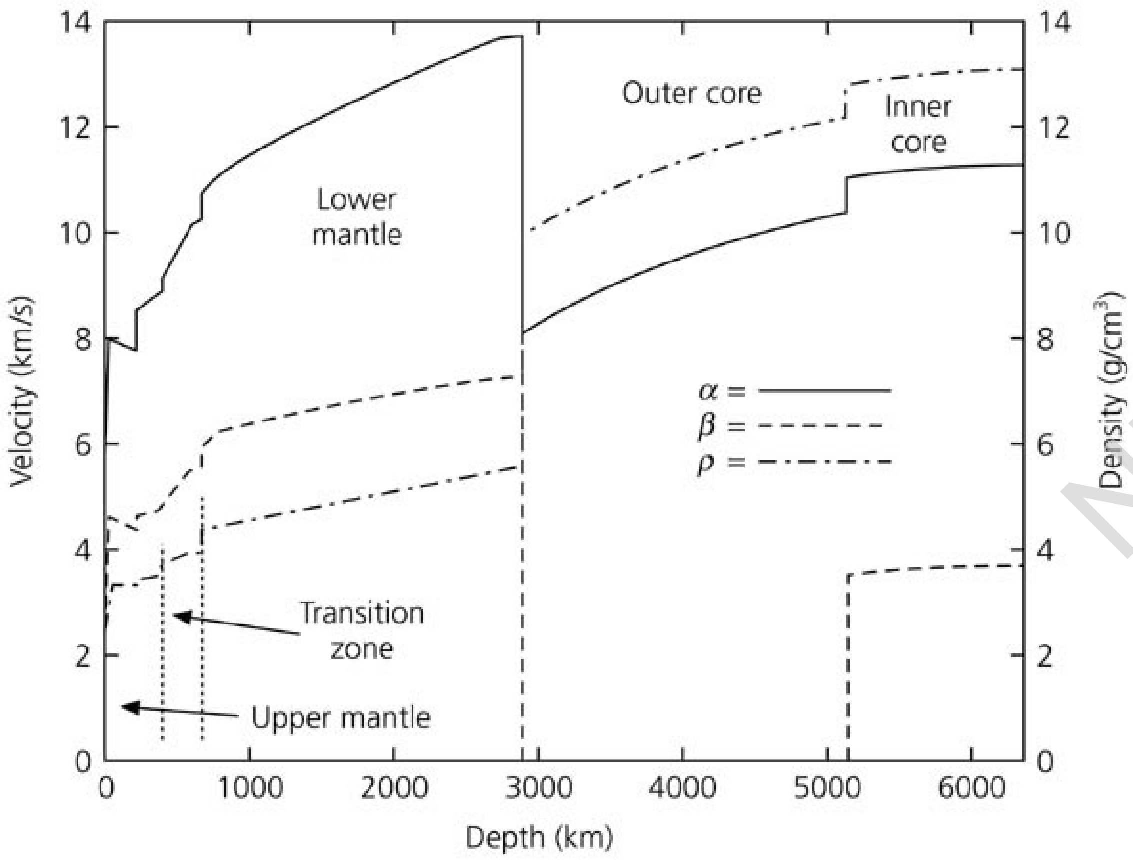
- This is because the mineral phase transitions expected at the 410 and 660 km discontinuities involve denser atomic packings, and therefore transitions to higher densities, than predicted by the Adams–Williamson equation.

NPTEL

h

Density within the earth

Figure 3.8-4: Preliminary Reference Earth Model.



- **As a result, density models of the earth include rapid changes in the transition zone.**
- **Within the lower mantle, outer core, and inner core, density increases smoothly with depth according to the Adams–Williamson equation.**

NPTEL

h

Pressure variation within the Earth

A density profile lets us compute a pressure profile, and thus use the results of experiments showing which mineral phases exist at particular pressures.

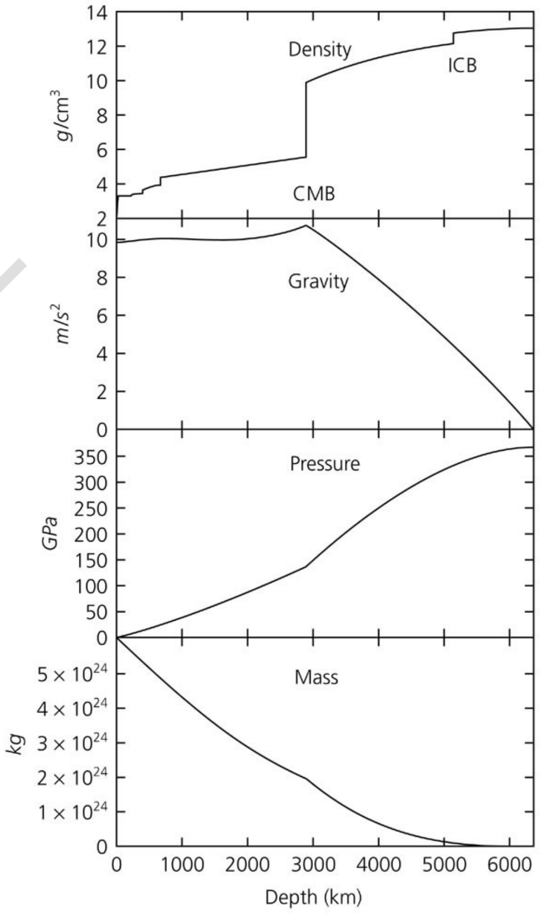
It can be computed as:

$$P(r) = -\int_0^r g(r)\rho(r)dr$$

Points to remember

- **Gravity averages 9.8 m/s^2 at earth's surface, and is zero at earth's center.**
- **Gravity increases slightly across the mantle, reaching a maximum of 10.7 m/s^2 at the CMB because of the high density of the core relative to the mantle.**
- **The core has only 16% of the earth's volume, but has almost one-third of the mass.**

Figure 3.8-5: Density, gravity, pressure, and mass as a function of depth.



NPTEL

Summary

- The ratio of the moment of inertia to the mass gives a scalar that depends on the density distribution.
- Adams–Williamson equation relating the velocity structure to the derivative of density with radius,

$$\frac{d\rho(r)}{dr} = \frac{-\rho(r)Gm(r)}{\Phi(r)r^2} = \frac{-\rho(r)g(r)}{\Phi(r)}$$

- The drawback of Adams– Williamson equation is that, it does not take the account of changes in the mineral phase with depth.
- Modified Adams–Williamson equation which includes superadiabatic gradient is:

$$\frac{d\rho}{dr} = -\frac{\rho g}{\phi} + g\tau\alpha$$

α: coefficient of thermal expansion

τ: portion of the temperature gradient exceeding the adiabatic gradient.

- Inhomogeneity in the earth can be identified using the function $1 - (1/g)d\phi/dr$.
- Pressure inside the earth can be computed as: $P(r) = -\int_0^r g(r)\rho(r)dr$

NPTEL

REFERENCES

- Stein, Seth, and Michael Wysession. An introduction to seismology, earthquakes, and earth structure. John Wiley & Sons, 2009.
- Lowrie, William, and Andreas Fichtner. Fundamentals of geophysics. Cambridge university press, 2020.
- Kearey, Philip, Michael Brooks, and Ian Hill. An introduction to geophysical exploration. Vol. 4. John Wiley & Sons, 2002.
- https://geologyscience.com/geology-branches/structural-geology/stress-and-strain/
- Seismology course, Professor Derek Schutt, Colorado State Univ., USA.

NPTEL

THANK YOU !

NPTEL