NPTEL ONLINE CERTIFICATION COURSES

# EARTHQUAKE SEISMOLOGY

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**Module 09 : Earthquakes, focal mechanisms, moment tensors.**
**Lecture 03: Equivalent Forces, Moment tensors**

# CONCEPTS COVERED

- Moment Tensor
- Equivalent Forces
  - Single forces
  - Force couples
  - Double couples
- Summary

# Recap

- Seismograms recorded at various distances and azimuths are used to study the geometry of faulting during an earthquake, known as the focal mechanism. It uses the fact that the pattern of radiated seismic waves depends on the fault geometry.

- The first motion observed at different azimuths define four quadrants, two compressional and two dilatational. These quadrants are separated by the fault plane and auxiliary plane.

- The elastic radiation can be described as resulting from double couple of forces, these forces are known as equivalent body forces for the fault slip.

# Recap

**Radiation pattern for P-wave and S-wave is given by**

$$u_r = \frac{1}{4\pi\rho\,\alpha^3 r}\dot{M}(t - r/\alpha)\sin 2\theta\,\cos\phi$$

$$u_\theta \hat{e}_\theta + u_\phi \hat{e}_\phi \ , \ where$$

$$u_\theta = \frac{1}{4\pi\rho\beta^3 r}\dot{M}(t - r/\beta)\cos 2\theta\cos\phi$$

$$u_\phi = \frac{1}{4\pi\rho\beta^3 r}\dot{M}(t - r/\beta)(-\cos\theta\sin\phi)$$

# Green's Functions

Lets recall momentum equation consisting of body forces,

$$\rho \frac{\partial^2 u}{\partial t^2} = \partial_j \sigma_{ij} + f_i$$

So far we have ignored the body forces, but these will be considered now because earthquakes are a body forces.

Consider a delta function, a simplest body force, it will have direction f, and location ($x_0$, $t_0$)

The displacement (u(x,t)) of the Earth due to this force at location and time (x,t) will be given by

$$u(x,t) = G(x,t;\, x_0,\, t_0)\, f(x_0,\, t_0)$$

Where $G_{ij}(x,t\,;\,x_0,t_0)$ denoted the Green's function consisting of all path effects. \

In summation notation to sum up the effect of different sources, $u_i = G_{ij}\, f_j$

# Moment tensors

➔ Seismic moment tensor comes into picture, when we generalize to include all type of seismic sources.

➔ Seismic moment tensor gives additional insight into the rupture process and greatly simplifies inverting seismograms to estimate source parameters.

Seismic sources (such as explosions, landslides or impacts on the earth's surface.

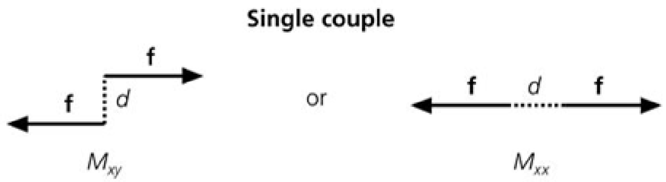
⇨ Generation of observable seismic waves

⇨ Seismic moment tensor assuming equivalent body forces

⇩ Estimation of source parameter

# Force couples

A force couple consists of two forces acting together.  It has two basic force couple types




$M_{xy}$ :  The couple $M_{xy}$ consists  of two forces of magnitude f , separated by a distance d along the

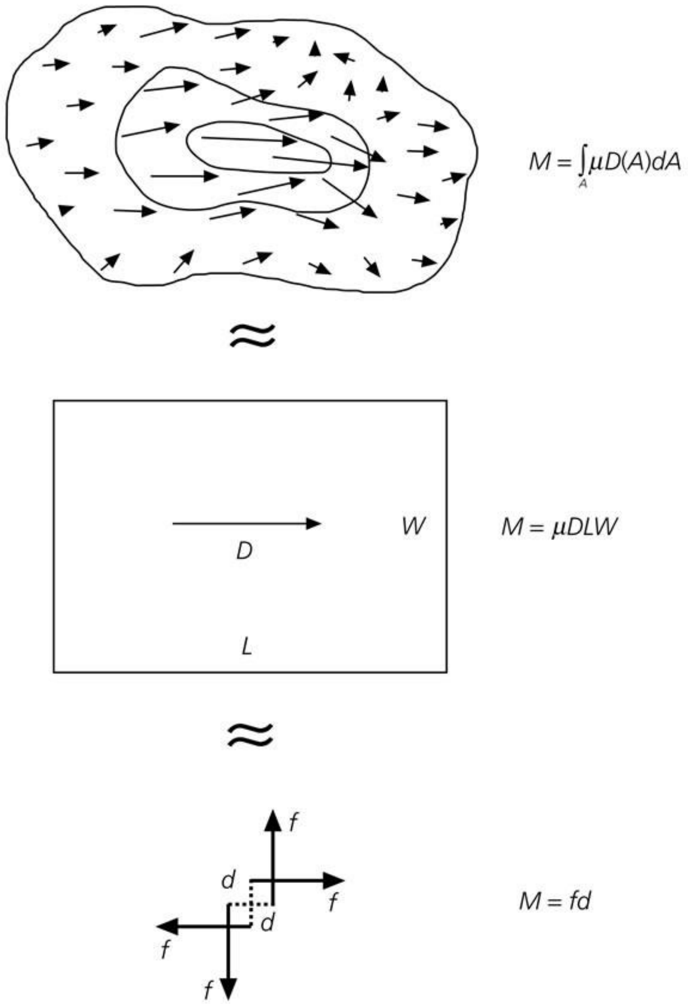
y axis, that act in opposite direction (±x) directions.

The magnitude of $M_{xy}$ is "fd", which in seismology is given in dyn-cm or N-m. To model a couple acting at a point, the limit is taken as d goes to zero such that the product fd stays constant.

$M_{xx}$:  The other type of couple is vector couple, consists of forces offset in the direction of the force. $M_{xx}$ consists of two forces of magnitude f acting in the ±x directions, separated by d along the x axis. The magnitude is "fd"

# Moment tensor

Figure 4.4-5: Approximations made in modeling seismic ruptures.




**Moment tensor is a simple mathematical representation that gives the seismic waves produced by a complex rupture involving displacement varying in space and time on a irregular fault.**

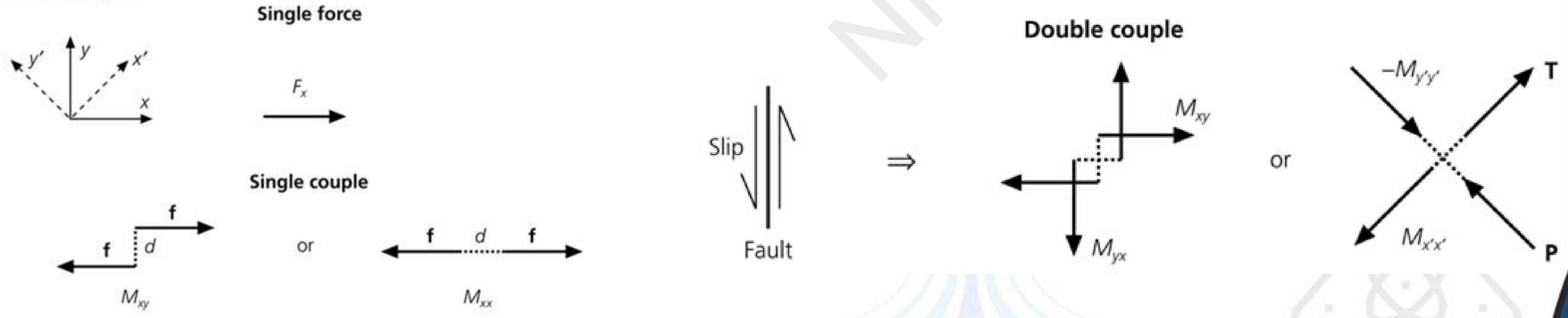
# Moment tensors

## Equivalent forces

**We assume the equivalent body forces which produces same radiation pattern as the slip on the fault for the earthquake and other seismic sources.**

**The seismic sources can be modelled as**
- **Single force**
- **Single couple/Force couple**
- **Double couple**

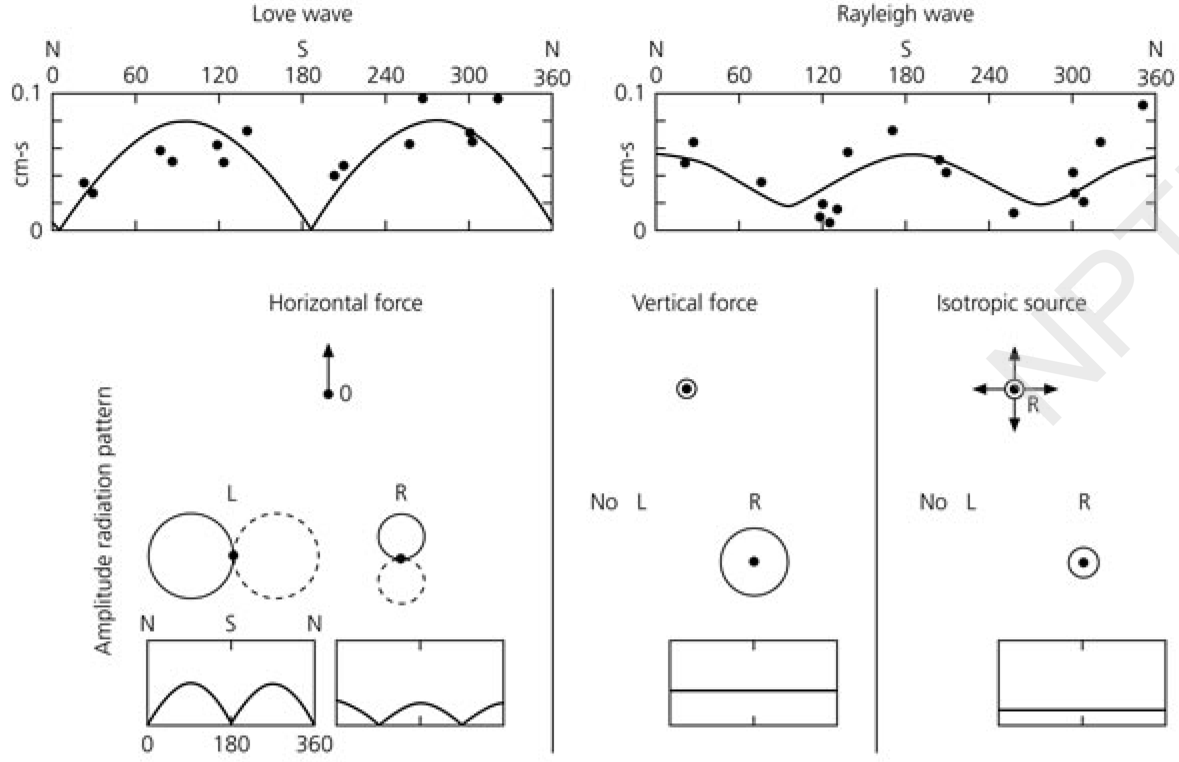
**Figure 4.4-1: Equivalent body forces for a single force, single couple, and double couple.**

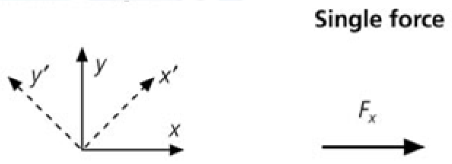


# Single forces

**Geophysical phenomena generate seismic waves that are sometimes modeled as single force sources. It can be volcanic eruptions, landslides, and Meteoritic impacts.**

**Figure 4.4-2: Modeling the 1980 Mt. St. Helens eruption as a single horizontal force.**

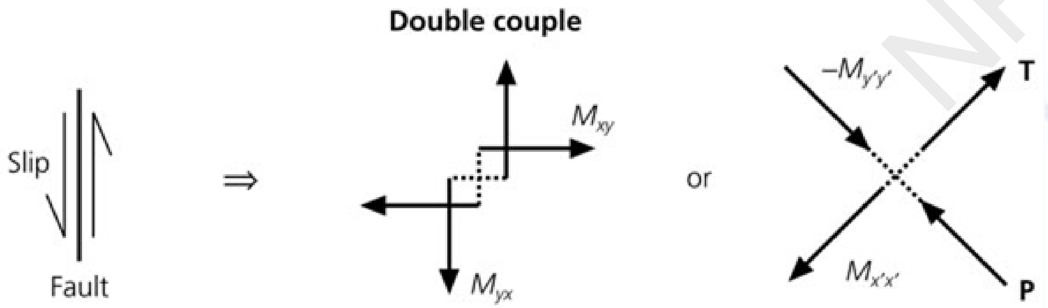


## Double couples

**Left-lateral strike-slip in the ±y directions on a fault in the y–z plane, the equivalent body forces $M_{xy} + M_{yx}$ make up the double-couple source.**

**The $M_{yx}$ couple seems intuitive, because the forces point in the slip directions, but the $M_{xy}$ couple is also needed for reasons including avoiding net torque on the fault.**




**The relation between an earthquake's fault geometry and the double couple of equivalent body forces**

# Moment tensor

**Slip on a fault can be represented as a series of force couples**

$$\overleftrightarrow{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

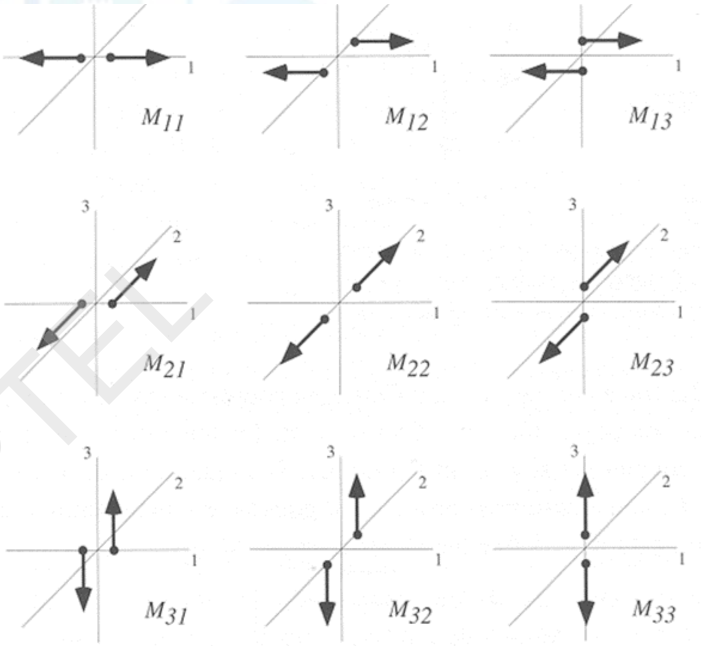
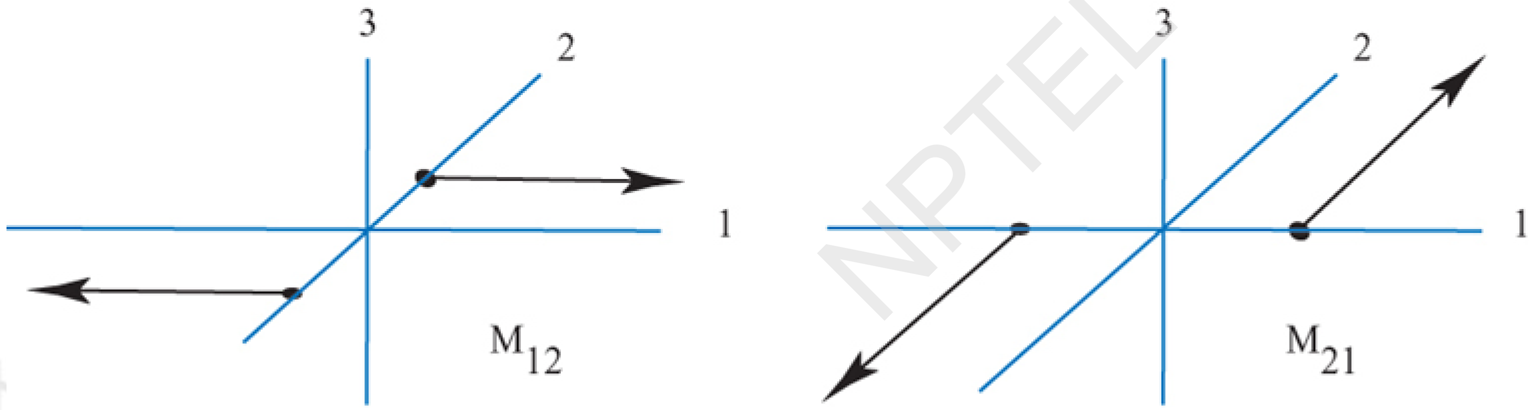


Fig. 9.2. The nine different force couples that make up the components of the moment tensor.

**Because of conservation of angular momentum, each couple must be balanced.**

**Therefore, for example, $M_{12} = M_{21}$**

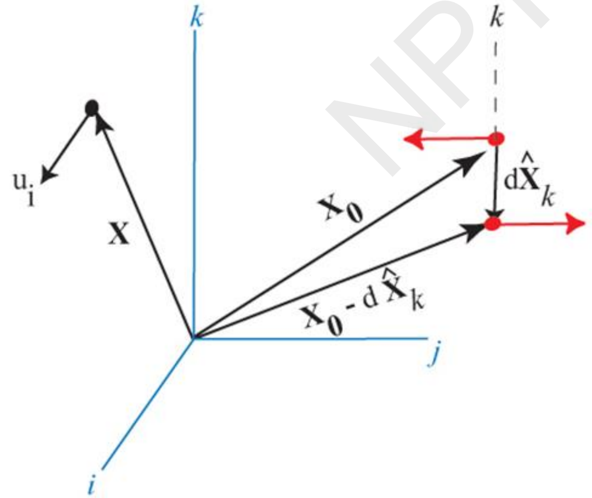


# Using the moment tensor with the Green's function

**Earlier we saw for a point force:** $u_i\,(x,t) = G_{ij}\,(x,t\,;\,x_0,t_0)\; f_j(x_0,t_0)$

**Thus the displacement at x due to a force couple at $x_0$ can be written as:**

$$u_i(\vec{x},t) = G_{ij}(\vec{x},t;\vec{x}_0,t_0)\,f_j(\vec{x}_0,t_0) - G_{ij}(\vec{x},t;\vec{x}_0 - \hat{x}_k d,t_0)\,f_j(\vec{x}_0,t_0)$$




(From Derek Schutt, CSU)

**It can be rewritten as:**

$$u_i(\vec{x},t) = f_j(\vec{x}_0,t_0)\hat{x}_k d\left[\frac{G_{ij}(\vec{x},t;\vec{x}_0,t_0) - G_{ij}(\vec{x},t;\vec{x}_0 - \hat{x}_k d,t_0)}{d}\right]$$

$$u_i(\vec{x},t) = f_j(\vec{x}_0,t_0)\hat{x}_k d\,\frac{\partial G_{ij}}{\partial(x_0)_k}$$

This is derivative

**It shows the displacement in the ‘i’ direction, for a force couple in the ‘j’ direction, separated by a distance ‘d’ in the ‘k’ direction.**

## Double couples

The magnitude of equivalent body forces is $M_0$( the scalar seismic moment of the earthquake), which has units of dyn-cm, similar to those of force couple.

Thus if $M_{xy}$ and $M_{yx}$ are couples of unit magnitude , the moment tensor is

$$\mathbf{M} = M_0(M_{xy} + M_{yx})$$



Moment tensor → Components ☰ Signifies fault geometry

Moment tensor → Scalar moment ☰ Signifies size of the earthquake

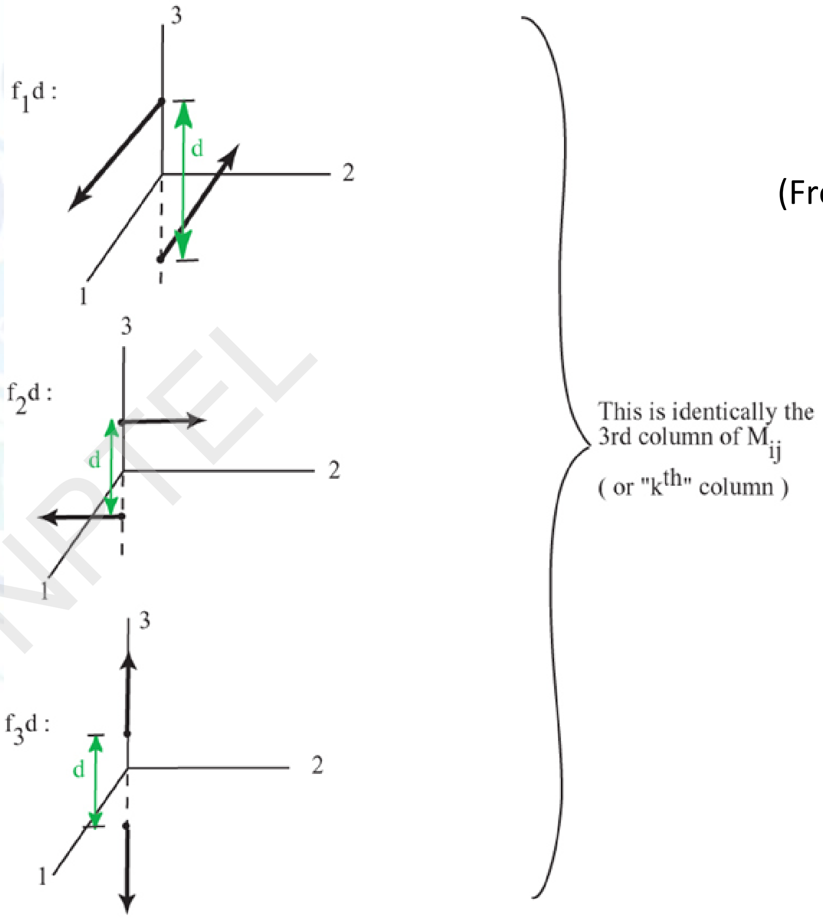
The displacement due to double couple is :

$$u_i(\vec{x},t) = f_j(\vec{x}_0,t_0)\hat{x}_k d \frac{\partial G_{ij}}{\partial (x_0)_k}$$

Here $f_j(\vec{x}_0,t_0)\hat{x}_k d$ = $M_{jk}$

is the force in direction j, offset by d in the k direction.

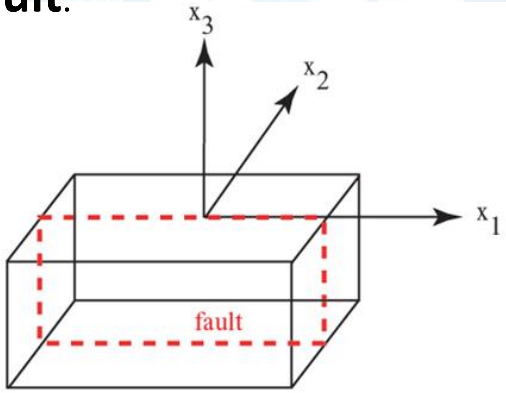
Where 'j' indicates summation over three directions.



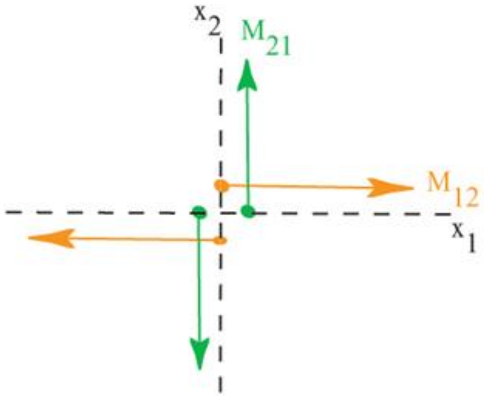

(From Derek Schutt, CSU)

**Now understand how can we relate the moment tensor to the actual geometry of a fault.**

**Let's consider a fault where we have slip along $X_1$ direction.**




(if its a right lateral strike slip fault, this will have this double couple




(From Derek Schutt, CSU)

**Therefore our moment tensor is** $\mathbf{M} = \begin{bmatrix} 0 & M_{12} & 0 \\ M_{21} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **and we know that** $M_{12} = M_{21} = M_0$

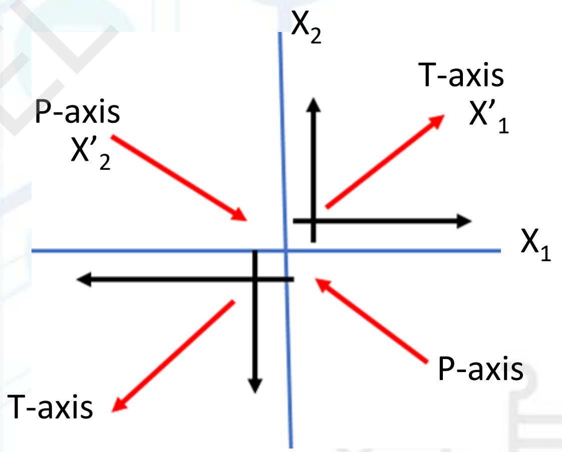
$$\mathrm{M} = \begin{bmatrix} 0 & M_0 & 0 \\ M_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

**Where, $M_0 = \mu D(t)A(t)$. $M_0$ has units of work**

→ **Moment tensor is symmetric in nature and there will be a set of axes which will make off-diagonal elements zero. It is similar to stress and strain tensors.**

→ **The off-diagonal elements will become zero at $45^0$ rotation.**

**In new coordinate system, it gives $M_{11} = M_0$ & $M_{22} = -M_0$**




$$\Rightarrow \mathbf{M}' = \begin{bmatrix} M_0 & 0 & 0 \\ 0 & -M_0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Where, $M_0 = \frac{1}{\sqrt{2}}\left(\sum_{ij} M_{ij}^2\right)^{1/2}$

**Positive moments implies stretching or tension while negative moment implies pressure.**

# Summary

- Seismic moment tensor gives additional insight into the rupture process and greatly simplifies  inverting seismograms to estimate source parameters.

- We assume the equivalent body forces which produces same radiation pattern as the slip on the fault for the earthquake and other seismic sources. The seismic sources can be modeled as single forces, force couples, and double couples.

- Components of moment tensor represents the fault geometry and size of the earthquake represented by scalar moment.

- Moment tensor is symmetric in nature and there will be a set of axes which will make off-diagonal elements zero. It is similar to stress and strain tensors.

# REFERENCES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- Seismology course, Professor Derek Schutt, Colorado State Univ., USA.

# THANK YOU !