NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 09 : Earthquakes, focal mechanisms, moment tensors.
Lecture 04: Earthquake moment tensors, Isotropic and CLVD moment tensors

CONCEPTS COVERED

- ➢ **Earthquake moment tensor**
  - **Moment tensor in arbitrary coordinate system**
  - **Relation between the eigenvectors of the moment tensor and T, P and null axes**
  - **Transformation characteristics**
- ➢ **Isotropic and CLVD moment tensors**
- ➢ **Summary**

Recap

- Elastic rebound theory states that strain accumulated in the rock is more than the rocks on the fault can withstand, and the fault slips, resulting in an earthquake.

- **Interseismic stage** consists of long periods between large earthquakes during which elastic strain accumulation occurs in the broad region.

- **Preseismic stage** that can be associated with small earthquakes (foreshocks).

- Earthquake itself marks the **coseismic phase** during which rapid motion on the fault occur and generate seismic waves.
- **Postseismic phase** occurs after the earthquake, and aftershocks and transient afterslip occur for a period of years.

- The two different coordinate systems, $(\phi_f, \delta, \lambda)$ and $(\hat{n}, \hat{d})$, are useful to describe the fault geometry.

Recap

- **Unit normal vector to the fault plane is** $\hat{n} = \begin{pmatrix} -\sin\delta\sin\phi_f \\ \sin\delta\cos\phi_f \\ \cos\delta \end{pmatrix}$

- **Slip vector, a unit vector in the slip direction, is** $\hat{d} = \begin{pmatrix} \cos\lambda\cos\phi_f + \sin\lambda\cos\delta\sin\phi_f \\ -\cos\lambda\sin\phi_f + \sin\lambda\cos\delta\cos\phi_f \\ \sin\lambda\sin\delta \end{pmatrix}$

- **For pure strike-slip fault λ = 0°, the hanging wall moves to the right, and the motion is called left-lateral and for λ = 180°, right-lateral motion occurs.**

- **For pure dip-slip fault λ = 270°, the hanging wall slides downward, causing normal faulting and λ = 90°, and the hanging wall goes upward, yielding reverse, or thrust, faulting.**

Recap

- Seismograms recorded at various distances and azimuths are used to study the geometry of faulting during an earthquake, known as the focal mechanism. It uses the fact that the pattern of radiated seismic waves depends on the fault geometry.

- The first motion observed at different azimuths define four quadrants, two compressional and two dilatational. These quadrants are separated by the fault plane and auxiliary plane.

- The elastic radiation can be described as resulting from double couple of forces, these forces are known as equivalent body forces for the fault slip.

Recap

Radiation pattern for P-wave and S-wave is given by

$$u_\theta \hat{e}_\theta + u_\phi \hat{e}_\phi \ , \ where$$

$$u_r = \frac{1}{4\pi\rho\,\alpha^3 r}\dot{M}(t - r/\alpha)\sin 2\theta\,\cos\phi$$

$$u_\theta = \frac{1}{4\pi\rho\beta^3 r}\dot{M}(t - r/\beta)\cos 2\theta\cos\phi$$

$$u_\phi = \frac{1}{4\pi\rho\beta^3 r}\dot{M}(t - r/\beta)(-\cos\theta\sin\phi)$$

Recap

- Seismic moment tensor gives additional insight into the rupture process and greatly simplifies inverting seismograms to estimate source parameters.

- We assume the equivalent body forces which produces same radiation pattern as the slip on the fault for the earthquake and other seismic sources. The seismic sources can be modeled as single forces, force couples, and double couples.

- Components of moment tensor represents the fault geometry and size of the earthquake represented by scalar moment.

- Moment tensor is symmetric in nature and there will be a set of axes which will make off-diagonal elements zero. It is similar to stress and strain tensors.

Recap

•**The Green's function:** tells us about the deformation at point and time (x,t), for a given impulse, $f(x_0,t_0)$. These are linear, for multiple sources it can be summed to find net deformation.

•**Moment Tensors:** Rotation into principle axes, gives P/T/null directions

M_{ij} i = direction of force couple
j = offset of force couple

$$M_0 = \mu DA$$

$$M_0 = \frac{1}{\sqrt{2}}\left(\sum_{ij} M_{ij}^2\right)^{1/2}$$

$$\overleftrightarrow{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

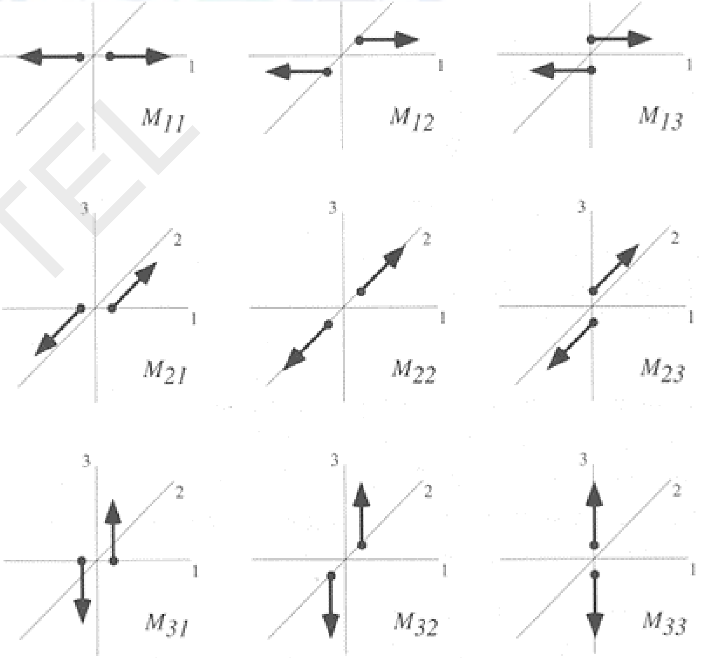


Fig. 9.2. The nine different force couples that make up the components of the moment tensor.

Earthquake moment tensors

As we studied, seismic moment tensor, M, with nine force couples can represent equivalent body forces for seismic sources of different geometries

$$M = \begin{pmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{pmatrix}$$

we can write it in an orthogonal coordinate system and the earthquake is represented as

$$M = \begin{pmatrix} 0 & M_0 & 0 \\ M_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = M_0 \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Moment tensor for a double couple earthquake in an arbitrary coordinate system

The components are given by the scalar moment and components of $\hat{n}$, the unit normal vector to the fault plane, and $\hat{d}$, the unit slip vector.

$$M_{ij} = M_0(n_i d_j + n_j d_i)$$

$$M = M_0 \begin{pmatrix} 2n_x d_x & n_x d_y + n_y d_x & n_x d_z + n_z d_x \\ n_y d_x + n_x d_y & 2n_y d_y & n_y d_z + n_z d_y \\ n_z d_x + n_x d_z & n_z d_y + n_y d_z & 2n_z d_z \end{pmatrix}$$

The above formulation depicts two things:

1. **First, the interchangeability of $\hat{n}$ and $\hat{d}$ makes the tensor symmetric ($M_{ij} = M_{ji}$). Physically, this shows that slip on either the fault plane or the auxiliary plane yields the same seismic radiation patterns.**

2. The trace (sum of diagonal components) of the tensor is zero

$$\sum_i M_{ii} = M_{ii} = 2M_0 n_i d_i = 2M_0 \hat{n}.\,\hat{d} = 0$$

Since slip vector lies in the fault plane and is thus perpendicular to the normal vector. Hence,

- moment tensors corresponding to slip on a fault plane have zero trace.
- A non zero trace implies a volume change (explosion or implosion). Such an isotropic components does not exists for a pure double-couple source.

3. The scalar moment gives the magnitude of the moment tensor, which is analogous to the magnitude of a vector.

$$M_0 = \left(\sum_{ij} M_{ij}^2 \right)^{1/2} / \sqrt{2}$$

How the eigenvectors of the moment tensor are related to T, P and null axes?

As we know,that vectors in these three orthogonal directions t, p, and b can be written in terms of the fault normal, $\hat{n}$, and slip vector, $\hat{d}$, as

$$\mathbf{t} = \hat{n} + \hat{d} \qquad t_i + d_i$$

$$\mathbf{p} = \hat{n} - \hat{d} \qquad t_i - d_i$$

$$\hat{b} = \hat{n} \times \hat{d} \qquad b_i = \epsilon_{ijk} n_j d_k$$

The eigenvectors of the moment tensor are parallel to the T, P, and null axes.

- As with all tensors, the moment tensor can be rotated using a transformation matrix.
- A useful rotation is one that orients the axes into the P/T axes coordinate system.

$$U = \begin{pmatrix} t_1 & b_1 & p_1 \\ t_2 & b_2 & p_2 \\ t_3 & b_3 & p_3 \end{pmatrix}$$

$$\mathbf{t} = \hat{n} + \hat{d} \qquad t_i + d_i$$

$$\mathbf{p} = \hat{n} - \hat{d} \qquad t_i - d_i$$

$$\hat{b} = \hat{n} \times \hat{d} \qquad b_i = \epsilon_{ijk} n_j d_k$$

- Here, t and p are the unit vectors indicating the T and P axes.

- Diagonal moment tensor for a double couple in the principal axis coordinate system.

$$U^{-1} M U = \begin{pmatrix} M_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -M_0 \end{pmatrix}$$

Transformation characteristics

- The trace (M_{xx} + M_{yy} + M_{zz}), which is not changed by an orthogonal transformation, started as zero

$$\sum_i M_{ii} = M_{ii} = 2M_0 n_i d_i = 2M_0 \hat{n}.\,\hat{d} = 0$$

and so remains zero.

So, the isotropic component is an invariant of the moment tensor and does not depend on the coordinate system.

- The transformation changes the components, but the physical moment tensor stays the same, so these two different-looking force systems give the same radiated seismic waves.

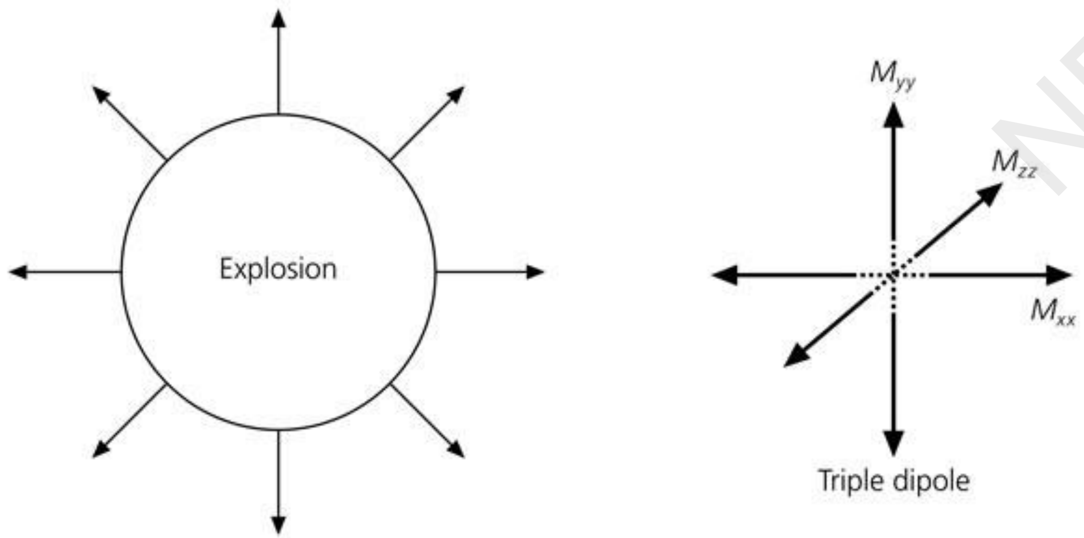
Isotropic and CLVD moment tensors

- For the isotropic moment tensor, all three diagonal terms of the moment tensor are nonzero and equal, the polarity of the first motions (focal mechanism) is the same in all directions.

- It has a triple vector dipole of three equal and orthogonal force couples.

- These can be produced by explosion or an implosion.

Figure 4.4-7: Modeling an explosive source as a triple force dipole.



The moment tensor looks like

$$\begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix}$$

Isotropic and CLVD moment tensors

- A moment tensor with a nonzero isotropic component represents a volume change.
- Meteorites impact could also be regarded as explosive sources.
- An explosion involves a sudden increase in pressure, which causes nonlinear deformation that can melt and even vaporize rock.
- As this shock wave of pressure expands, its amplitude decreases until the deformations are small enough to occur elastically, yielding a spherical P wave.

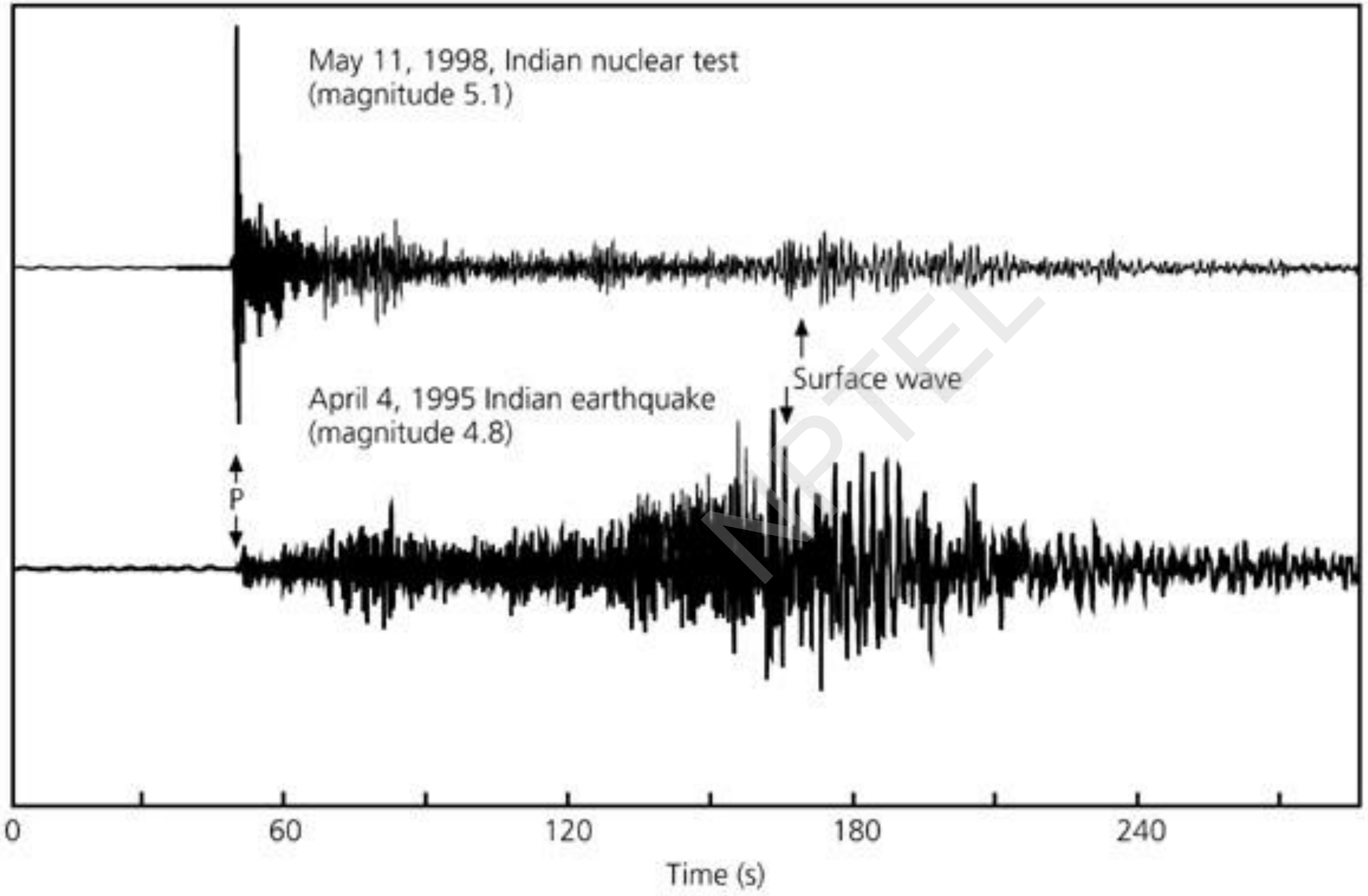
Figure 4.4-6: Selected moment tensors and their associated focal mechanisms.

Moment tensor	Beachball	Moment tensor	Beachball
$\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$		$-\frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	
$-\frac{1}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&-1\\0&0&0\\-1&0&0\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0\\0&0&-1\\0&-1&0\end{pmatrix}$	
$\frac{1}{\sqrt{2}}\begin{pmatrix}-1&0&0\\0&0&0\\0&0&1\end{pmatrix}$		$\frac{1}{\sqrt{2}}\begin{pmatrix}0&0&0\\0&-1&0\\0&0&1\end{pmatrix}$	

The top row shows an explosion (left) and an implosion (right). The next three rows are for double-couple sources.

Isotropic and CLVD moment tensors

Figure 1.2-19: Differences is seismic waves from an earthquake and explosion.



Isotropic and CLVD moment tensors

- Compensated Linear Vector Dipoles (CLVDs) are sets of three force dipoles that are compensated, with one dipole −2 times the magnitude of the others:

$$\begin{pmatrix} -\lambda & 0 & 0 \\ 0 & \lambda/2 & 0 \\ 0 & 0 & \lambda/2 \end{pmatrix}$$

These occur rarely, but are observed, usually in complicated tectonic environments.

The trace of the moment tensor is zero, so there is no isotropic component.

$\frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$		$\frac{1}{\sqrt{6}}\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$		$-\frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$	

Summary

- Symmetric property of moment tensor shows that slip on either the fault plane or the auxiliary plane yields the same seismic radiation patterns

- The trace (sum of diagonal components) of the tensor is zero

$$\sum_i M_{ii} = M_{ii} = 2M_0 n_i d_i = 2M_0 \hat{n}.\,\hat{d} = 0$$

- The scalar moment gives the magnitude of the moment tensor, which is analogous to the magnitude of a vector.

$$M_0 = \left(\sum_{ij} M_{ij}^2\right)^{1/2} / \sqrt{2}$$

- Thus, the eigenvectors of the moment tensor are parallel to the T, P, and null axes

- The transformation changes the components, but the physical moment tensor stays the same, so these two different-looking force systems give the same radiated seismic waves

Summary

- For the isotropic moment tensor, all three diagonal terms of the moment tensor are nonzero and equal, the polarity of the first motions (focal mechanism) is the same in all directions. The moment tensor will look like:

$$\begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix}$$

- A moment tensor with a nonzero isotropic component represents a volume change.

- Compensated linear vector dipoles (CLVDs) are sets of three force dipoles that are compensated, with one dipole −2 times the magnitude of the others:

$$\begin{pmatrix} -\lambda & 0 & 0 \\ 0 & \lambda/2 & 0 \\ 0 & 0 & \lambda/2 \end{pmatrix}$$

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THANK YOU !