NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 09 : Earthquakes, focal mechanisms, moment tensors.
Lecture 05: Mechanisms for CLVD moment tensors and its ambiguity; Interpretation of moment tensors

CONCEPTS COVERED

- **Mechanisms for CLVD moment tensors**
- **CLVD moment tensors : Ambiguity**
- **Interpretation of moment tensors**
- **Summary**

Recap

- Seismic moment tensor gives additional insight into the rupture process and greatly simplifies inverting seismograms to estimate source parameters.

- We assume the equivalent body forces which produces same radiation pattern as the slip on the fault for the earthquake and other seismic sources. The seismic sources can be modeled as single forces, force couples, and double couples.

- Components of moment tensor represents the fault geometry and size of the earthquake represented by scalar moment.

- Moment tensor is symmetric in nature and there will be a set of axes which will make off-diagonal elements zero. It is similar to stress and strain tensors.

Recap

- **The Green's function:** tells us about the deformation at point and time (x,t), for a given impulse, $f(x_0,t_0)$. These are linear, for multiple sources it can be summed to find net deformation.
- **Moment Tensors:** Rotation into principal axes, gives P/T/null directions

M_{ij} — i = direction of force couple; j = offset of force couple

$$M_0 = \mu DA$$

$$M_0 = \frac{1}{\sqrt{2}}\left(\sum_{ij} M_{ij}^2\right)^{1/2}$$

$$\overleftrightarrow{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

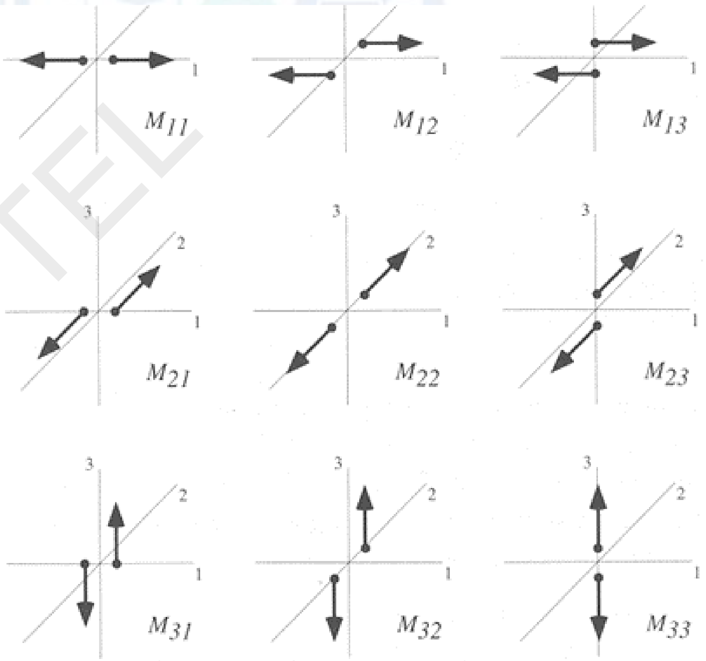


Fig. 9.2. The nine different force couples that make up the components of the moment tensor.

Recap

- **Symmetric property of moment tensor shows that slip on either the fault plane or the auxiliary plane yields the same seismic radiation patterns**

- **The trace (sum of diagonal components) of the tensor is zero**

$$\sum_i M_{ii} = M_{ii} = 2M_0 n_i d_i = 2M_0 \hat{n}.\,\hat{d} = 0$$

- **The scalar moment gives the magnitude of the moment tensor, which is analogous to the magnitude of a vector.**

$$M_0 = \left(\sum_{ij} M_{ij}^2\right)^{1/2} / \sqrt{2}$$

- **Thus, the eigenvectors of the moment tensor are parallel to the T, P, and null axes**

- **The transformation changes the components, but the physical moment tensor stays the same, so these two different-looking force systems give the same radiated seismic waves**

Recap

- **For the isotropic moment tensor has two qualities,**

- **all three diagonal terms of the moment tensor are nonzero and equal.**

- **the polarity of the first motions (focal mechanism) is the same in all directions.**

- **The moment tensor will look like:**

$$\begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix}$$

Recap

- A moment tensor with a nonzero isotropic component represents a volume change.

- Compensated linear vector dipoles (CLVDs) are sets of three force dipoles that are compensated, with one dipole −2 times the magnitude of the others:

$$\begin{pmatrix} -\lambda & 0 & 0 \\ 0 & \lambda/2 & 0 \\ 0 & 0 & \lambda/2 \end{pmatrix}$$

Isotropic and CLVD moment tensors

- Compensated Linear Vector Dipoles (CLVDs) are sets of three force dipoles that are compensated, with one dipole −2 times the magnitude of the others:

$$\begin{pmatrix} -\lambda & 0 & 0 \\ 0 & \lambda/2 & 0 \\ 0 & 0 & \lambda/2 \end{pmatrix}$$

These occur rarely, but are observed, usually in complicated tectonic environments.

The trace of the moment tensor is zero, so there is no isotropic component.

$\frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$		$\frac{1}{\sqrt{6}}\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	
$\frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$		$-\frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$	

Mechanisms for CLVD moment tensors

- **Two primary explanations have been offered for CLVD mechanisms.**

- **In volcanic areas, it is natural to think of an inflating magma dike, which can be modeled as a crack opening under tension.**

- **The moment tensor is for such a crack is**

$$\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda + 2\mu \end{pmatrix}$$

The trace of this tensor is $3\lambda + 2\mu$, which is positive because the crack opened.

Mechanisms for CLVD moment tensors

We can decompose the tensor into two terms:

$$\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda + 2\mu \end{pmatrix} = \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix} + \begin{pmatrix} -2/3\mu & 0 & 0 \\ 0 & -2/3\mu & 0 \\ 0 & 0 & 4/3\mu \end{pmatrix}$$

CLVD Tensor

The first term is an isotropic tensor whose diagonal components E = λ + 2/3μ are one-third of the trace.

Mechanisms for CLVD moment tensors

- An alternative explanation is that CLVDs are due to near simultaneous earthquakes on nearby faults of different geometries.

- For example, consider the sum of two double-couple sources with moments M_0 and $2M_0$, expressed in the principal axis coordinate system

$$\begin{pmatrix} M_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -M_0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -2M_0 & 0 \\ 0 & 0 & 2M_0 \end{pmatrix} = \begin{pmatrix} M_0 & 0 & 0 \\ 0 & -2M_0 & 0 \\ 0 & 0 & M_0 \end{pmatrix}$$

Thus, adding these two double couples yields a CLVD

CLVD moment tensors : Ambiguity

- **Decomposing a CLVD into double couples bears out the concept that the moment tensors can be decomposed in different ways, with different interpretations.**

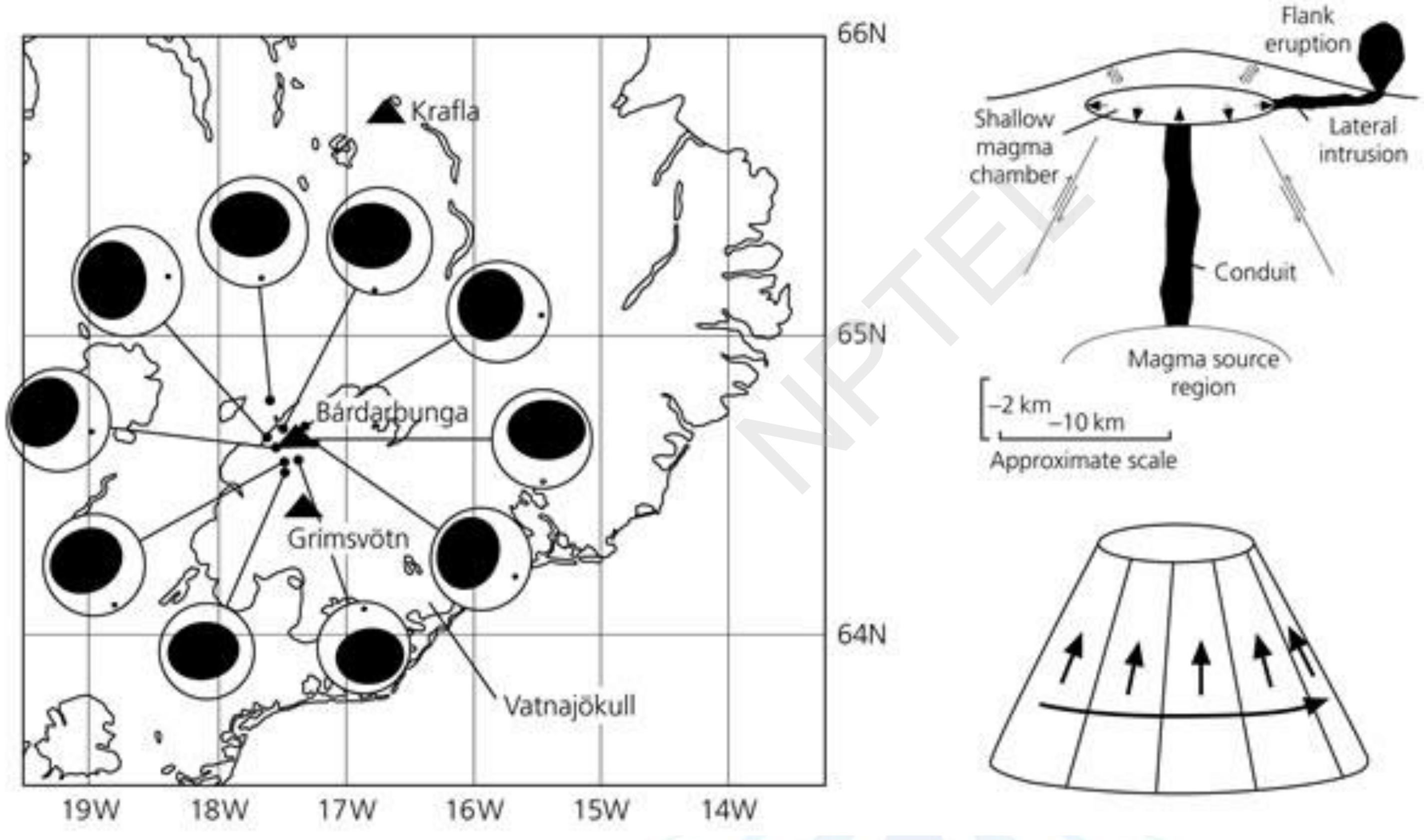
- **This is because the moment tensor represents the equivalent body force system, so different decompositions reflect the same force system and give the same seismic waves.**

- **Hence the seismic waves alone cannot distinguish between alternative decompositions.**

CLVD moment tensors : Ambiguity

- **Multiple faulting events giving rise to apparent CLVDs have been reported.**

Figure 4.4-8: Example of CLVD-type focal mechanisms for caldera earthquakes.



Interpretation of moment tensors

- In general, once a moment tensor has been found by inverting seismograms, it will be more complicated than expected for a double couple.

- Even if the source were a pure double couple, noise in the data and imperfect knowledge of earth structure would likely produce a tensor that, once diagonalized, would look like

$$M = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \qquad |\lambda_1| \geq |\lambda_2| \geq |\lambda_3|,$$

- If M represents a double couple, then $\lambda_1 = -\lambda_2$, and $\lambda_3 = 0$.

$$M = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

Interpretation of moment tensors

- In most cases, $\lambda_1 \approx -\lambda_2$, and $|\lambda_2| >> |\lambda_3|$, so M is approximately, but not exactly, a double couple.

- In this case, we interpret the moment tensor by decomposing it, as we did for the CLVD.

- Since, inversion of moment tensors for shallow earthquakes cannot resolve the isotropic component. We remove it via

$$\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix} + \begin{pmatrix} \lambda_1' & 0 & 0 \\ 0 & \lambda_2' & 0 \\ 0 & 0 & \lambda_3' \end{pmatrix}$$

where $E = (\lambda_1 + \lambda_2 + \lambda_3)/3$. The remaining term is a deviatoric moment tensor, with zero isotropic component and components equal to the deviatoric eigenvalues $\lambda'_1 = \lambda_1 - E$, $\lambda'_2 = \lambda_2 - E$, and $\lambda'_3 = \lambda_3 - E$.

Interpretation of moment tensors

- The deviatoric moment tensor can be decomposed in several ways.

- One is in terms of two double couples, called the major and minor double couples:

$$\begin{pmatrix} \lambda'_1 & 0 & 0 \\ 0 & \lambda'_2 & 0 \\ 0 & 0 & \lambda'_3 \end{pmatrix} = \begin{pmatrix} \lambda'_1 & 0 & 0 \\ 0 & -\lambda'_1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda'_3 & 0 \\ 0 & 0 & \lambda'_3 \end{pmatrix}$$

- The first tensor is the major double couple, with scalar moment $|\lambda'_1|$, and the second is the minor double couple, with scalar moment $|\lambda'_3|$.

- Usually, the magnitude of the major double couple is much larger, and we treat it as the earthquake's source mechanism.

Example: An intermediate depth thrust earthquake in the Kurile subduction zone near Japan (components are in units of 10^{27} dyne-cm):

$$M = \begin{pmatrix} 0.12 & -0.17 & -0.06 \\ -0.17 & -1.54 & -1.44 \\ -0.06 & -1.44 & 1.43 \end{pmatrix}$$

Diagonalizing the matrix yields

$$\begin{pmatrix} -2.14 & 0 & 0 \\ 0 & 2.01 & 0 \\ 0 & 0 & 0.13 \end{pmatrix} = \begin{pmatrix} -2.14 & 0 & 0 \\ 0 & 2.14 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -0.13 & 0 \\ 0 & 0 & 0.13 \end{pmatrix}$$

with eigenvectors $\hat{\mathbf{n}}_1 = (0.80, 0.92, 0.37)$, $\hat{\mathbf{n}}_2 = (0.00, -0.38, 0.93)$ and $\hat{\mathbf{n}}_3 = (-0.99, 0.07, 0.03)$.

The isotropic component was constrained in the inversion to be zero.

Because the minor double couple has a moment only 6% that of the major double couple, we assume that the major double couple represents the earthquake mechanism.

$\hat{\mathbf{n}}_1$ is the P axis of the double couple, $\hat{\mathbf{n}}_2$ is the T axis, and $\hat{\mathbf{n}}_3$ is the null axis.

These convert into thrust on a fault plane striking N189°E and dipping 23° west.

The moment tensor can be decomposed in other ways.

One is into a double couple and a CLVD:

$$\begin{pmatrix} \lambda'_1 & 0 & 0 \\ 0 & \lambda'_2 & 0 \\ 0 & 0 & \lambda'_3 \end{pmatrix} = \begin{pmatrix} \lambda'_1 + \lambda'_3/2 & 0 & 0 \\ 0 & -\lambda'_1 - \lambda'_3/2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} -\lambda'_3/2 & 0 & 0 \\ 0 & -\lambda'_3/2 & 0 \\ 0 & 0 & \lambda'_3 \end{pmatrix}$$

The relative strength of the double couple and CLVD is given by the ratio of the smallest and largest deviatoric eigenvalues, $\varepsilon = \lambda'_3/\lambda'_1$.

$\varepsilon = 0$ indicates a pure double couple, and $\varepsilon = \pm 0.5$ shows a pure CLVD source. About 4% percent of the mechanisms in the Harvard global moment tensor catalog, derived from inversions that are not constrained to yield double couples, have $|\varepsilon| \geq 0.3$.

Some of these may be artifacts of the inversion process similar to spurious minor double couples, and some appear to be real source effects.

Decompositions are non-unique:

$$\begin{pmatrix} \lambda'_1 & 0 & 0 \\ 0 & \lambda'_2 & 0 \\ 0 & 0 & \lambda'_3 \end{pmatrix} = \begin{pmatrix} \lambda'_1 & 0 & 0 \\ 0 & -\lambda'_1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda'_3 & 0 \\ 0 & 0 & \lambda'_3 \end{pmatrix}.$$

But could also do:

$$\begin{pmatrix} \lambda'_1 & 0 & 0 \\ 0 & \lambda'_2 & 0 \\ 0 & 0 & \lambda'_3 \end{pmatrix} = \begin{pmatrix} \lambda'_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\lambda'_1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda'_2 & 0 \\ 0 & 0 & -\lambda'_2 \end{pmatrix}$$

Summary

- **Elastic rebound theory states that strain accumulated in the rock is more than the rocks on the fault can withstand, and the fault slips, resulting in an earthquake.**

- **Interseismic stage consists of long periods between large earthquakes during which elastic strain accumulation occurs in the broad region.**

- **Preseismic stage that can be associated with small earthquakes (foreshocks).**

- **Earthquake itself marks the coseismic phase during which rapid motion on the fault occur and generate seismic waves.**
- **Postseismic phase occurs after the earthquake, and aftershocks and transient afterslip occur for a period of years.**

- **The two different coordinate systems, (ϕ_f, δ, λ) and ($\hat{n}$, $\hat{d}$), are useful to describe the fault geometry.**

Summary

- Unit normal vector to the fault plane is $\hat{n} = \begin{pmatrix} -\sin\delta\sin\phi_f \\ \sin\delta\cos\phi_f \\ \cos\delta \end{pmatrix}$

- Slip vector, a unit vector in the slip direction, is $\hat{d} = \begin{pmatrix} \cos\lambda\cos\phi_f + \sin\lambda\cos\delta\sin\phi_f \\ -\cos\lambda\sin\phi_f + \sin\lambda\cos\delta\cos\phi_f \\ \sin\lambda\sin\delta \end{pmatrix}$

- For pure strike-slip fault λ = 0°, the hanging wall moves to the right, and the motion is called left-lateral and for λ = 180°, right-lateral motion occurs.

- For pure dip-slip fault λ = 270°, the hanging wall slides downward, causing normal faulting and λ = 90°, and the hanging wall goes upward, yielding reverse, or thrust, faulting.

Summary

- **Seismograms recorded at various distances and azimuths are used to study the geometry of faulting during an earthquake, known as the focal mechanism. It uses the fact that the pattern of radiated seismic waves depends on the fault geometry.**

- **The first motion observed at different azimuths define four quadrants, two compressional and two dilatational. These quadrants are separated by the fault plane and auxiliary plane.**

- **The elastic radiation can be described as resulting from double couple of forces, these forces are known as equivalent body forces for the fault slip.**

Summary

Radiation pattern for P-wave and S-wave is given by

$$u_r = \frac{1}{4\pi\rho\,\alpha^3 r}\dot{M}(t - r/\alpha)\sin 2\theta\,\cos\phi$$

$$u_\theta \hat{e}_\theta + u_\phi \hat{e}_\phi \ , \ where$$

$$u_\theta = \frac{1}{4\pi\rho\beta^3 r}\dot{M}(t - r/\beta)\cos 2\theta\cos\phi$$

$$u_\phi = \frac{1}{4\pi\rho\beta^3 r}\dot{M}(t - r/\beta)(-\cos\theta\sin\phi)$$

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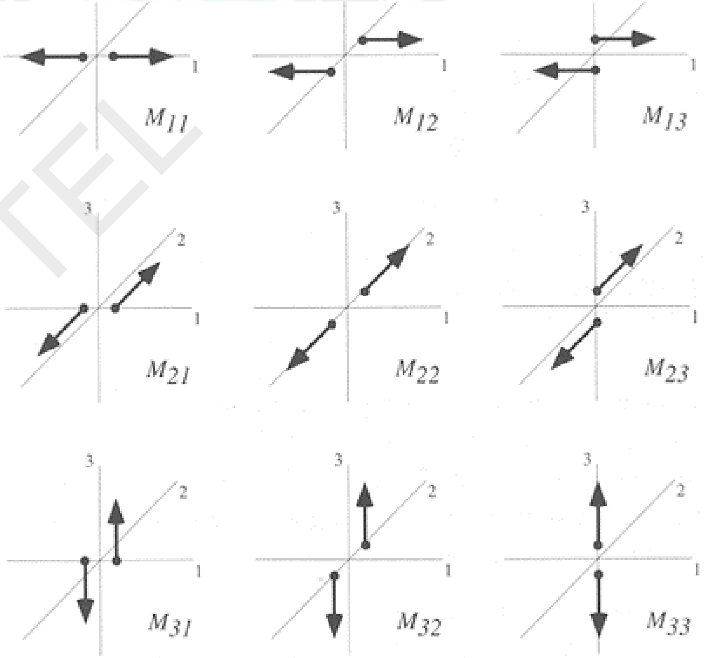

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Summary

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- An alternative explanation is that CLVDs are due to near simultaneous earthquakes on nearby faults of different geometries.

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THANK YOU !