NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

Dr. Mohit Agrawal
Department of Applied Geophysics , IIT(ISM) Dhanbad

Module 11: Source parameters, Earthquake statistics.
Lecture 02: Source spectra and magnitude saturation

CONCEPTS COVERED

- **Source Spectra**
  - **Convolution of two boxcar functions**
  - **Corner frequency and moment magnitude**
  - **Saturation Of M_s and m_b**
- **Summary**

Recap

- The general form of earthquake magnitude is $M = \log_{10}(A/T) + F(h, \Delta) + C$

  A is the amplitude of the signal,
  T is its dominant period,
  F is a correction for the variation of amplitude with the earthquake's depth h
  Δ is epicentral distance,
  C is a regional scale factor.

- Richter Scale magnitude is: $M_L = \log A + 2.76 \log \Delta - 2.48$

- Body wave magnitude m_b is $m_b = \log_{10}(A/T) + Q(h, \Delta)$

- Surface wave magnitude M_s is: $M_s = \log_{10}(A/T) + 1.66 \log_{10} \Delta + 3.3$

- Body and surface wave magnitudes do not correctly reflect the size of large earthquakes and saturate about 6.2 and 8.3 respectively.

Recap

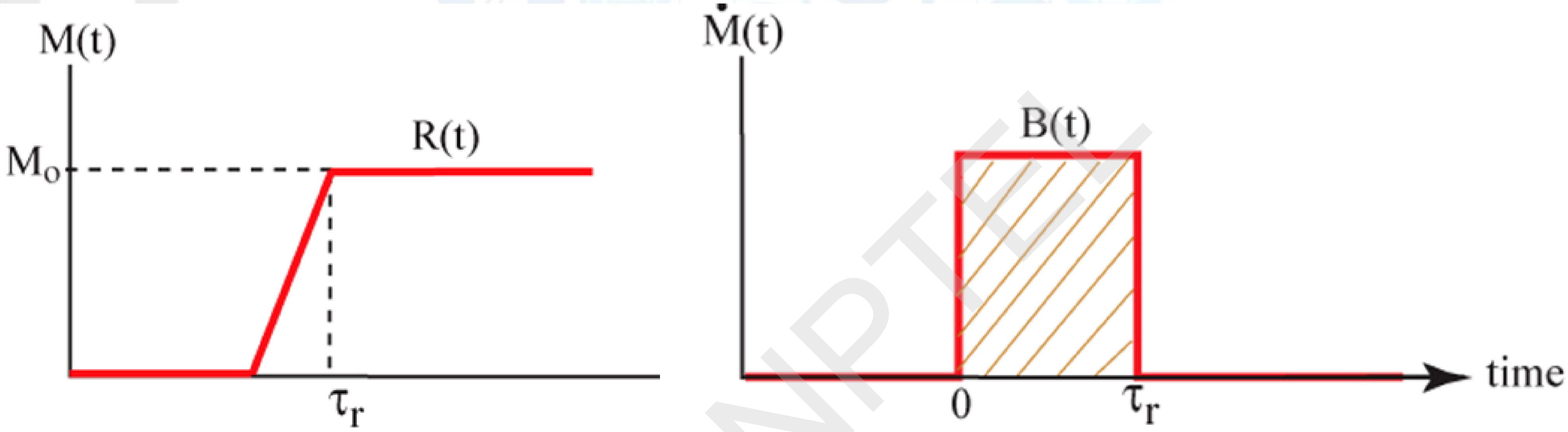
- **Different techniques (body waves, surface waves, geodesy, geology) can yield different estimates.**

$$M_w = \frac{\log M_o}{1.5} - 10.73$$

- **Moment magnitude is given as:**

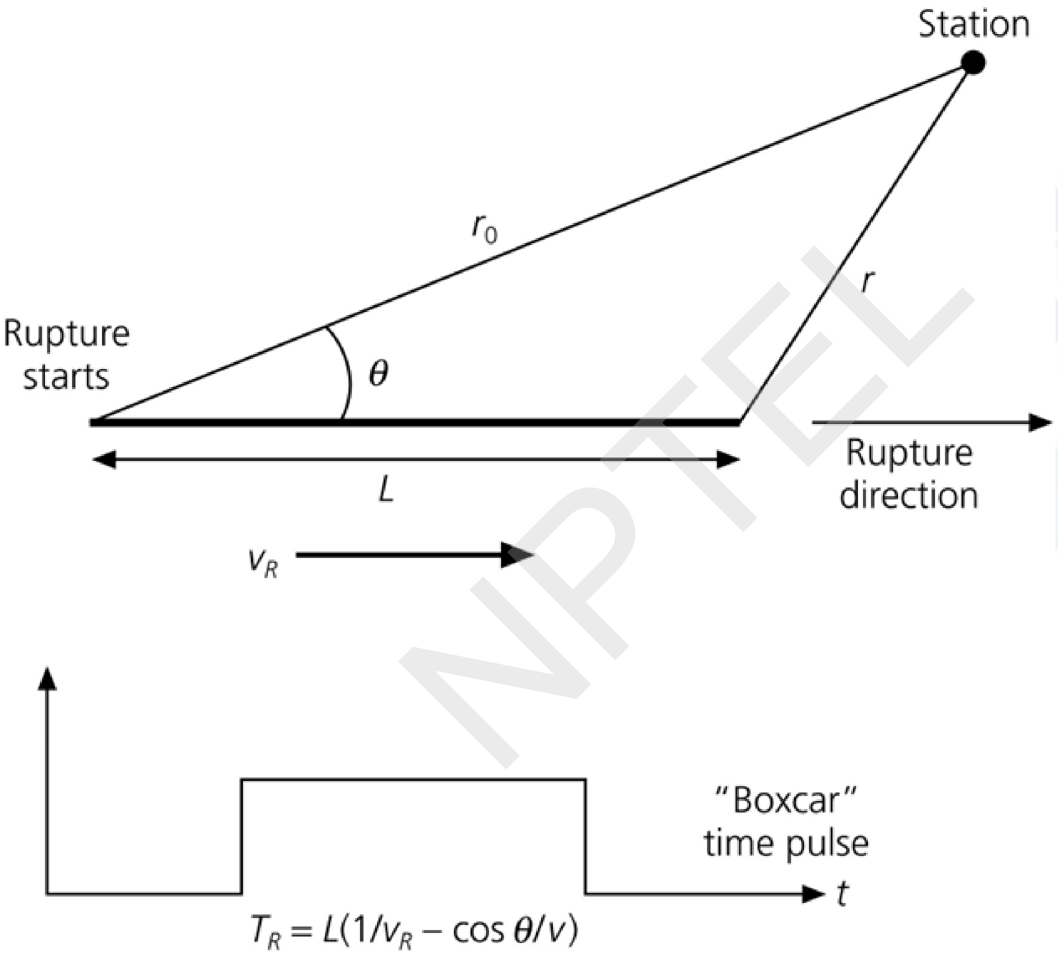
- **It gives a magnitude directly tied to earthquake source processes that does not saturate.**

We may begin with a simple model where rupture start from rest with rupture velocity v_r. They reach to its full velocity in time τ_r.



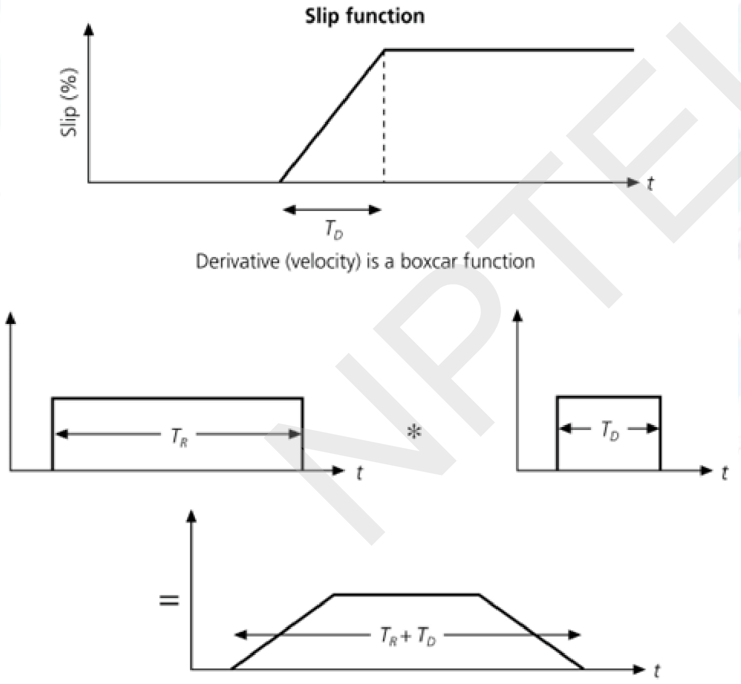
This is also known as Haskell source.

Figure 4.3-2: Derivation of a boxcar rupture time pulse.



Displacement pulses (s(t)) can be represented by a convolution of boxcar functions with width τ_r (rise time) and τ_d (rupture duration time). This will yield a trapezoid:

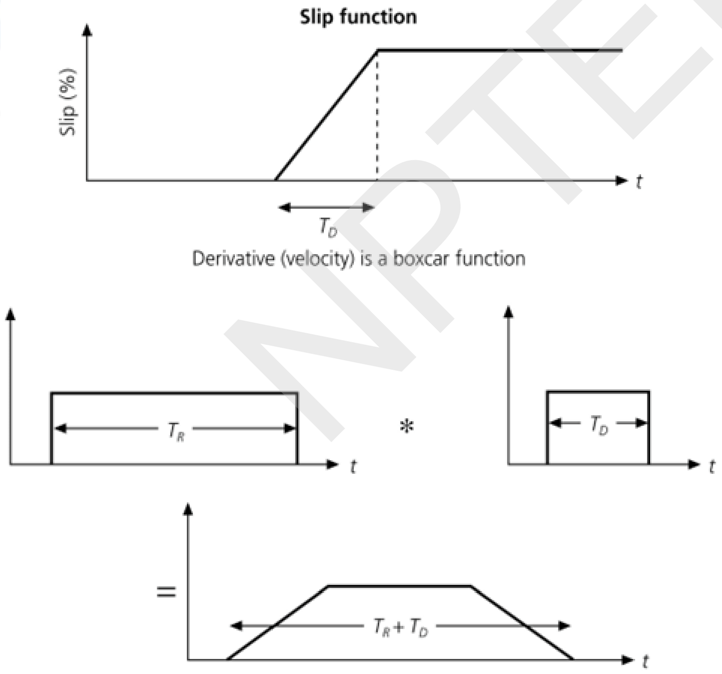
Figure 4.3-3: Derivation of a trapezoidal source time function.



Source spectra

- **Our aim:** In order to understand magnitude saturation of M_s and M_b, we first have to briefly discuss about source spectra.
- The source-time spectrum of an earthquake can be approximated by slip function that is shown below.
- This in turn can be represented by T_D*T_R

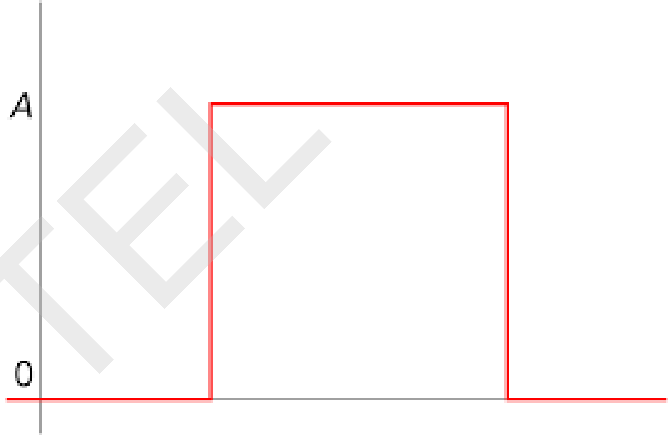
Figure 4.3-3: Derivation of a trapezoidal source time function.



A simple mode for time function: Convolution of two boxcar time functions

- A boxcar function is chosen due to finite length of the fault and the infinite rise time of the faulting at any point

- The Fourier transform of the resulting time function is the product of the transforms of boxcars



Convolution of two boxcar time functions

The transform of a boxcar of height 1/T and length T is

$$F(\omega) = \int_{-T/2}^{T/2} \frac{1}{T} e^{i\omega t} dt = \frac{1}{Ti\omega}\left(e^{i\omega T/2} - e^{-i\omega T/2}\right) = \frac{\sin(\omega T/2)}{\omega T/2}$$

This function sometimes written as $x = (\sin x)/x$ **, it appear in applications in which only part of the signal is selected.**

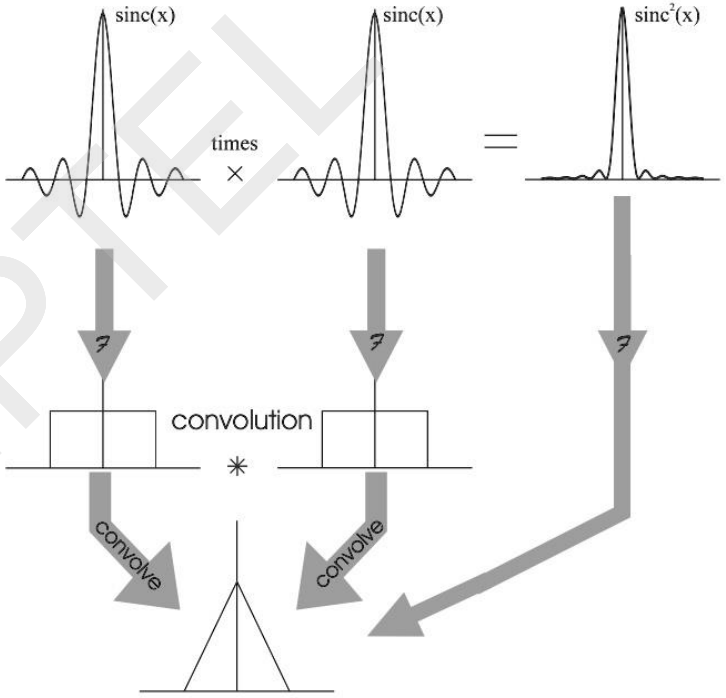


Fig. 11.5: A shortcut to the Fourier transform of the product of two sinc functions through the convolution of two boxcars.

https://www.google.com/search?client=ubuntu&hs=W9p&hl=en&sxsrf=AB5stBhNmeA9HQ-6XtUVFXFBSAwFt-hl8g:1688102525153&q=fourier+transform+of+convolution+of++two+box+car+function&tbm=isch&sa=X&ved=2ahUKEwiNq-z6n-r_AhWHcGwGHSzRDysQ0pQJegQICxAB&biw=1920&bih=995&dpr=1#imgrc=cNEAM2juXeZE6M

Convolution of two boxcar time functions

Thus the spectral amplitude of the source signal is the product of the seismic moment and two sinc term

$$|A(\omega)| = M_0 \left| \frac{\sin(\omega T_R/2)}{\omega T_R/2} \right| \left| \frac{\sin(\omega T_D/2)}{\omega T_D/2} \right|$$

where T_R and T_D are the rupture and rise times. The above equation is used in the form of logarithm

An approximation can be made for sinc(x)

$$\text{sinc}(x) \approx \begin{cases} 1, & x < 1 \\ \dfrac{1}{x}, & x > 1 \end{cases}$$

If $T_R > T_D$, we can divide the equation up into three segments

$\omega < \frac{2}{T_R}$ then

$$\operatorname{sinc}\left(\frac{\omega T_R}{2}\right) \approx 1 \quad \operatorname{sinc}\left(\frac{\omega T_D}{2}\right) \approx 1$$

$$\log\left[A(\omega)\right] \approx \log M_0 + \underbrace{\log(1)}_{=0} + \underbrace{\log(1)}_{=0}$$

$\frac{2}{T_R} < \omega < \frac{2}{T_D}$ then

$$\operatorname{sinc}\left(\frac{\omega T_R}{2}\right) \approx \frac{2}{\omega T_R} \quad \operatorname{sinc}\left(\frac{\omega T_D}{2}\right) \approx 1$$

$$\log\left[A(\omega)\right] \approx \log M_0 + \log\left(\frac{2}{\omega T_R}\right) + \log(1)$$

$$\log\left(\frac{2}{\omega T_R}\right) = \log\left(\frac{\omega T_R}{2}\right)^{-1}$$

$$= -1 \log\left(\frac{\omega T_R}{2}\right)$$

$$= -1\left[\log\left(\frac{T_R}{2}\right) + \log(\omega)\right]$$

$$\log\left[A(\omega)\right] \approx \log M_0 - \log\left(\frac{T_R}{2}\right) - \log(\omega)$$

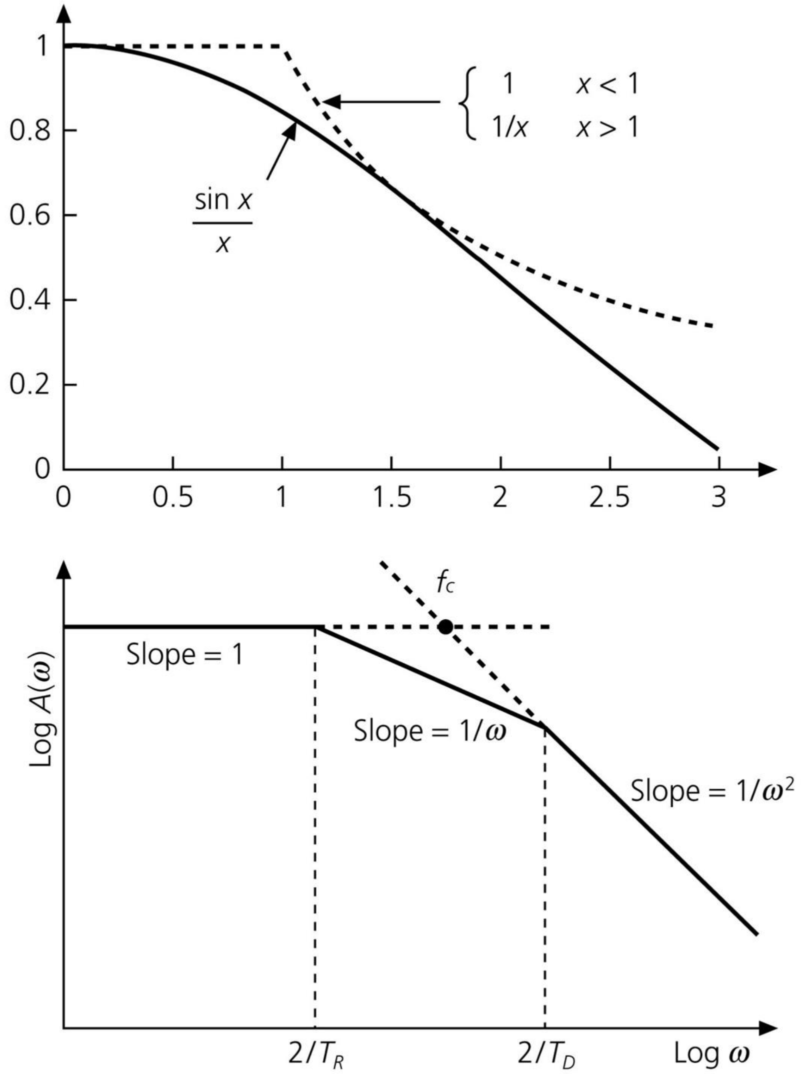
$\frac{2}{T_D} < \omega$ then $\operatorname{sinc}\left(\frac{\omega T_R}{2}\right) \approx \frac{2}{\omega T_R}$ $\operatorname{sinc}\left(\frac{\omega T_D}{2}\right) \approx \frac{2}{\omega T_D}$

$$\log\left[A(\omega)\right] = \log M_0 + \log\left[\left(\frac{2}{\omega T_R}\right)\left(\frac{2}{\omega T_D}\right)\right]$$

$$= \log M_0 - \log\left(\frac{T_R T_D}{4}\right) - \log(\omega^2)$$

$$= \log M_0 - \log\left(\frac{T_R T_D}{4}\right) - 2\log(\omega)$$

Figure 4.6-4: Approximation of the (sin x)/x function, and derivation of corner frequencies.



Assuming $T_R > T_D$ we have

$$\log|A(\omega)| =$$

$$\begin{array}{ll} \log M_0 & \omega < 2/T_R \\ \log M_0 - \log\left(T_R/2\right) - \log\omega & 2/T_R < \omega < 2/T_D \\ \log M_0 - \log\left(T_R T_D/2\right) - 2\log\omega & 2/T_D < \omega \end{array}$$

- ➔ **The spectrum is flat for frequencies less than the first corner, goes as ω^{-1} between the corners, and decays as ω^{-2} for the high frequencies.**
- ➔ **Thus the spectrum is parametrized by three factors: seismic moment, rise time, and rupture time.**

Key Points!!

- We can use other source spectral model to add third corner frequency to this model. Model representing the effects of fault width and yielding an ω^{-3} segment at high frequency.

- As a result, the interpretation of observed earthquake spectra depends somewhat on the source model.

- Seismic moment is the scale factor for the spectral amplitude at low frequencies $\omega \rightarrow 0$. This is the reason why it is also called the "static" moment.

- It is defined as the $M_0 = \mu \bar{D} S = \mu \bar{D} f L^2$

  Here, the fault area is written in terms of a shape factor f and the square of a dimension L

Key Points!!

- The rupture time needed for the rupture to propagate along the fault is approximately

$$T_R = L/v_R = L/(0.7\beta)$$

we assume that the rupture velocity is about 0.7 times the shear velocity.

- The rise time needed for the dislocation to reach its full value at any point on the fault has been predicted to be about

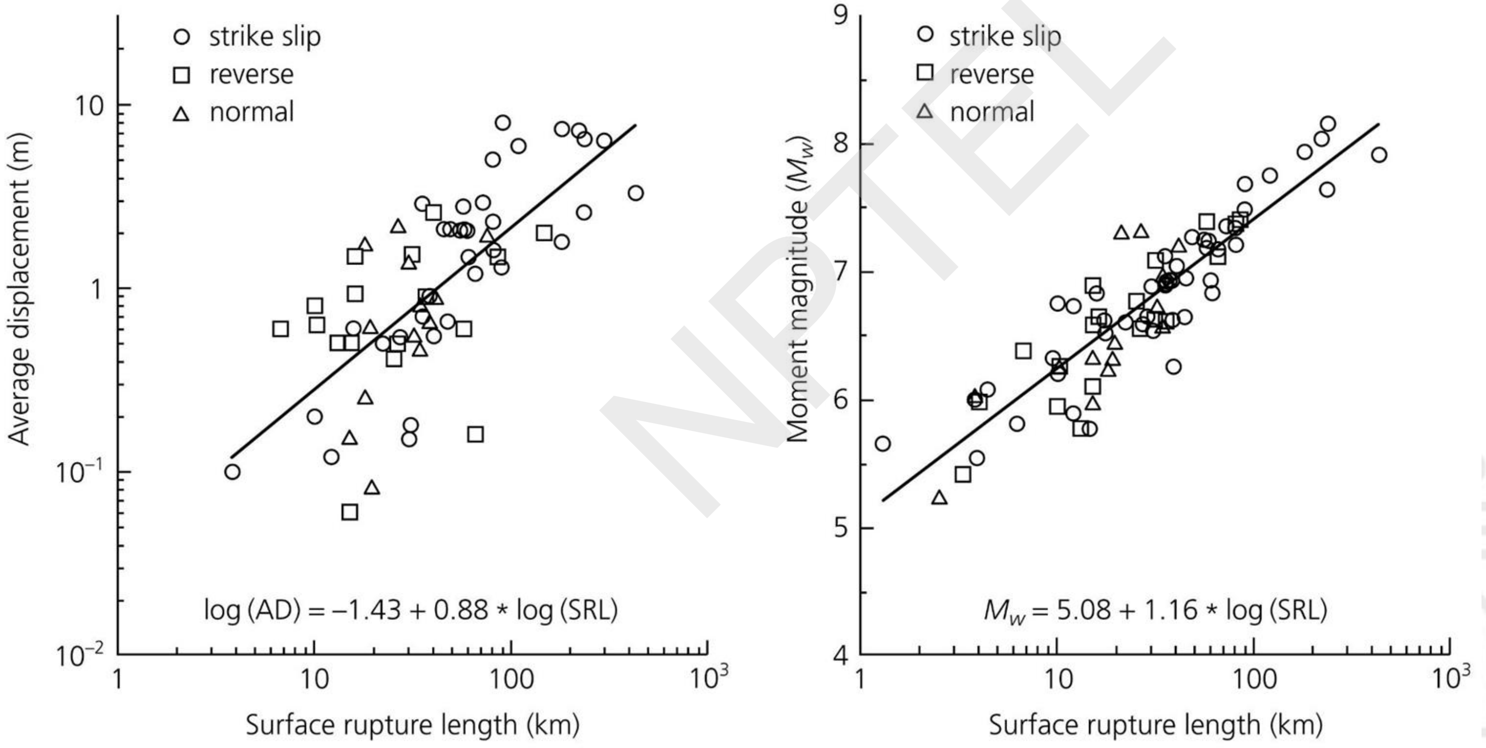
$$T_D = \mu\bar{D}/(\beta\Delta\sigma) = 16f^{(1/2)}/\left(7\beta\pi^{1.5}\right)$$

where Δσ is the stress drop in the earthquake.

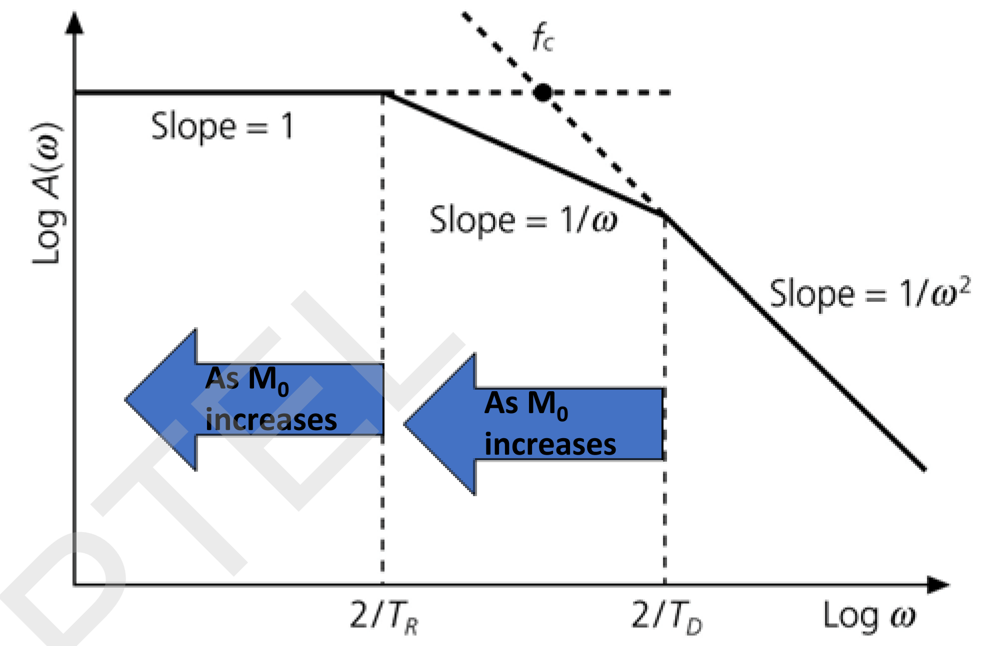
- As the fault length increases, the seismic moment, rupture time, and rise time increase.

- **The moment, M_0, determines the zero-frequency level, which rises as the earthquake becomes larger.**

Figure 4.6-7: Empirical relations between slip, fault length, and moment.



As T_R increases, $2/T_R$ will decrease. Likewise, the $2/T_D$ also increases. Hence, the point at which log(A(ω)) starts to roll off will move to lower and lower frequencies.



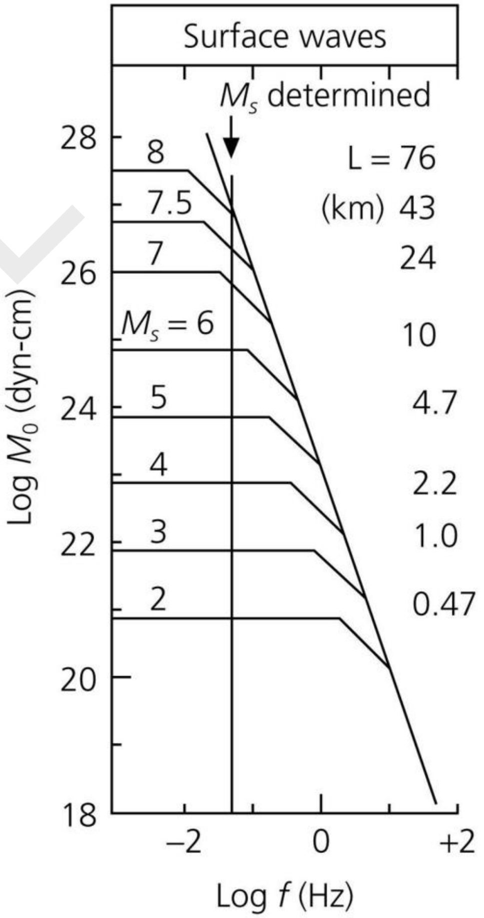
Saturation Of M_s

- The surface wave magnitude, M_s, is measured at a period of 20 s, and so depends on the spectral amplitude at this period.

- For earthquakes with moments less than about 10^{26} dyn-cm, a 20 s period corresponds to the flat part of the spectrum, so M_s increases with moment.

- However, for larger moments, 20 s is to the right of the first corner frequency, so M_s does not increase at the same rate as the moment.

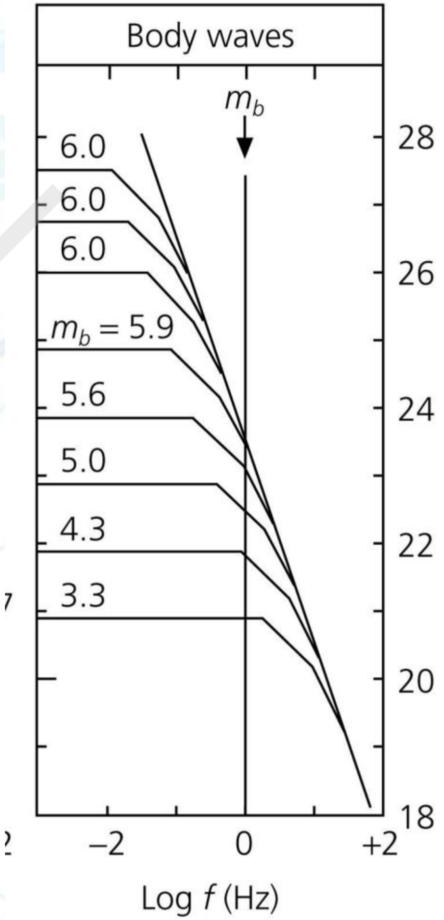
- Once the moment exceeds about 5×10^{27} dyn-cm, 20 s is to the right of the second corner, on the ω^{-2} portion of the spectrum. Thus M_s saturates at about 8.2, even if the moment increases.



Saturation Of M_b

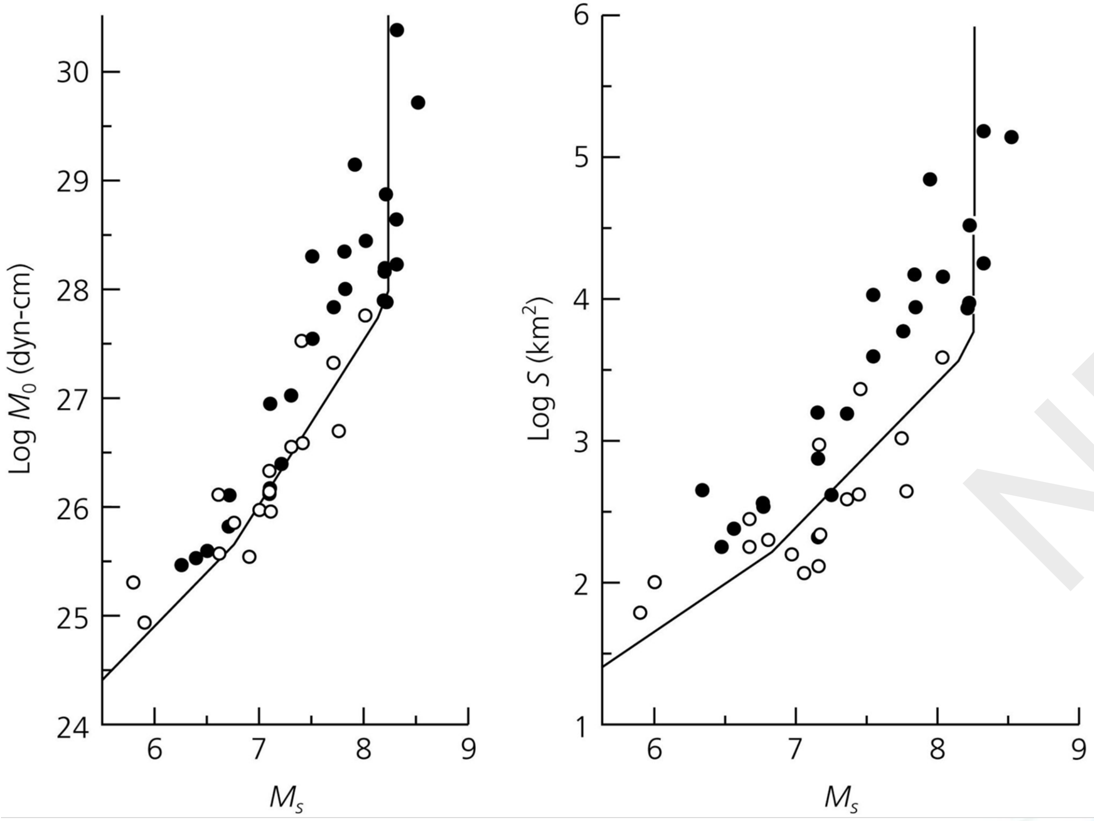
- A similar effect occurs for body wave magnitude, which depends on the amplitude at a period of 1 s. Because this period is shorter than the 20 s used for M_s, m_b saturates at a lower moment (about 10^{25} dyn-cm), and remains about 6 even for much larger earthquakes.

- Similar saturation effects occur for other magnitude scales which are measured at specific frequencies.



Saturation Of M_s

Figure 4.6-6: Demonstration of the saturation of body and surface wave magnitudes.



- **M_s saturates even as the moment and fault areas increase. Open and closed circles denote intraplate and interplate earthquakes, respectively.**

- **For earthquakes above about 10^{28} dyn-cm, M_s saturates even for progressively larger fault areas and thus seismic moments.**

- **As a result, M_s is not a useful measure of the size of very large earthquakes.**

Summary

- The relations between the moment and various magnitudes arise from the spectrum of the radiated seismic waves

- Seismic moment is the scale factor for the spectral amplitude at low frequencies ω → 0. This is the reason why it is also called the "static" moment and defined as the

$$M_0 = \mu \bar{D} S = \mu \bar{D} f L^2$$

Here, the fault area is written in terms of a shape factor f and the square of a dimension L

- The rupture time needed for the rupture to propagate along the fault is approximately

$$T_R = L / v_R$$

- The rise time needed for the dislocation to reach its full value at any point on the fault has been predicted to be about

$$T_D = \mu \bar{D} / (\beta \Delta\sigma) = 16 f^{(1/2)} / \left(7 \beta \pi^{1.5}\right)$$

where Δσ is the stress drop in the earthquake

Summary

- The moment, M_0, determines the zero-frequency level, which rises as the earthquake becomes larger.
- Once the moment exceeds about 5×10^{27} dyn-cm, 20 s is to the right of the second corner, on the ω^{-2} portion of the spectrum. Thus M_s saturates at about 8.2, even if the moment increases. M_s saturates even as the moment and fault areas increase.
- m_b saturates at a lower moment (about 10_{25} dyn-cm), and remains about 6 even for much larger earthquakes.

REFERENCES

- Stein, Seth, and Michael Wysession. An introduction to seismology, earthquakes, and earth structure. John Wiley & Sons, 2009.
- Lowrie, William, and Andreas Fichtner. Fundamentals of geophysics. Cambridge university press, 2020.
- Kearey, Philip, Michael Brooks, and Ian Hill. An introduction to geophysical exploration. Vol. 4. John Wiley & Sons, 2002.
- https://geologyscience.com/geology-branches/structural-geology/stress-and-strain/
- https://www.wikipedia.org/
- Seismology course, Professor Derek Schutt, Colorado State Univ., USA.

THANK YOU !