NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 11: Source parameters, Earthquake statistics.

Lecture 03: Magnitude scaling relations and Stress drop

CONCEPTS COVERED

- Scaling relation
- Stress Drop
  - For different fault geometry
  - Uncertainty Estimation
- Summary

Summary

- The relations between the moment and various magnitudes arise from the spectrum of the radiated seismic waves

- Seismic moment is the scale factor for the spectral amplitude at low frequencies ω → 0. This is the reason why it is also called the “static” moment and defined as the

$$M_0 = \mu \bar{D} S = \mu \bar{D} f L^2$$

Here, the fault area is written in terms of a shape factor f and the square of a dimension L

- The rupture time needed for the rupture to propagate along the fault is approximately

$$T_R = L/v_R$$

- The rise time needed for the dislocation to reach its full value at any point on the fault has been predicted to be about

$$T_D = \mu \bar{D}/(\beta \Delta\sigma) = 16 f^{(1/2)} / \left(7 \beta \pi^{1.5}\right)$$

where Δσ is the stress drop in the earthquake

Summary

- The moment, M_0, determines the zero-frequency level, which rises as the earthquake becomes larger.
- Once the moment exceeds about 5×10^{27} dyn-cm, 20 s is to the right of the second corner, on the ω^{-2} portion of the spectrum. Thus M_s saturates at about 8.2, even if the moment increases. M_s saturates even as the moment and fault areas increase.
- m_b saturates at a lower moment (about 10_{25} dyn-cm), and remains about 6 even for much larger earthquakes.

Scaling relations

Table 4.6-2 Earthquake scaling relations.

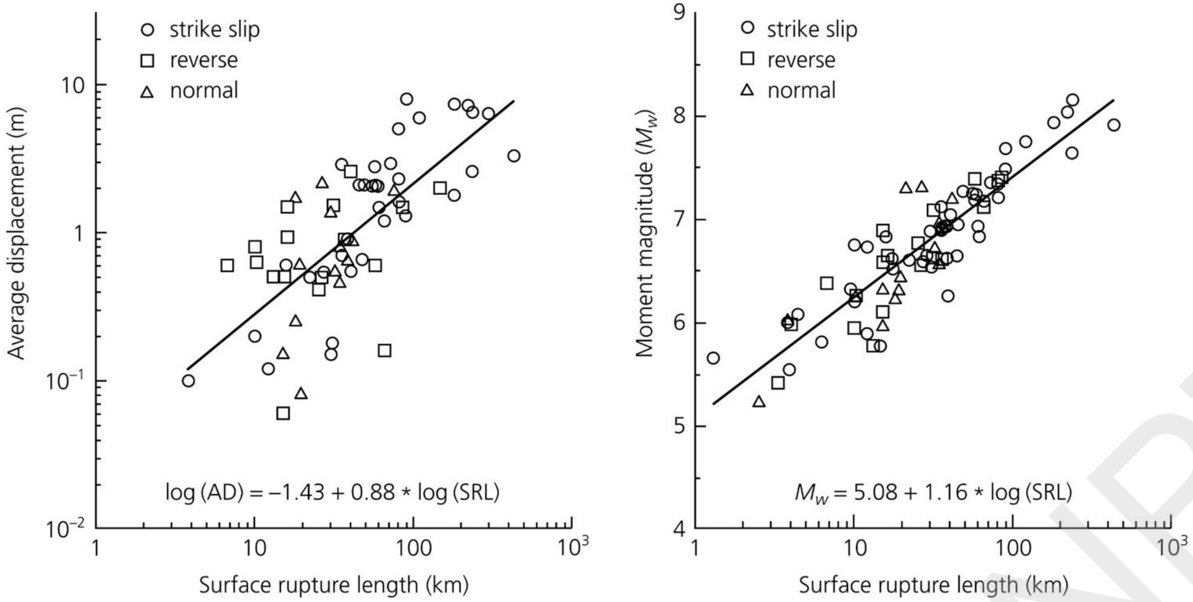
m_b and M_s are related by	
$m_b = M_s + 1.33$	$M_s < 2.86$
$m_b = 0.67M_s + 2.28$	$2.86 < M_s < 4.90$
$m_b = 0.33M_s + 3.91$	$4.90 < M_s < 6.27$
$m_b = 6.00$	$6.27 < M_s$.
Assuming $L = 2W$, M_s and fault area (in km^2) are related by	
$\log S = 0.67M_s - 2.28$	$M_s < 6.76$
$\log S = M_s - 4.53$	$6.76 < M_s < 8.12$
$\log S = 2M_s - 12.65$	$8.12 < M_s < 8.22$
$M_s = 8.22$	$S > 6080$ km^2.
Assuming a stress drop of 50 bars, $\log M_0$ (in dyn-cm) and M_s are related by	
$\log M_0 = M_s + 18.89$	$M_s < 6.76$
$\log M_0 = 1.5M_s + 15.51$	$6.76 < M_s < 8.12$
$\log M_0 = 3M_s + 3.33$	$8.12 < M_s < 8.22$
$M_s = 8.22$	$\log M_0 > 28$.

Source: Geller (1976).

- ➔ **Scaling relations provide insight into the relation between source parameters.**
- ➔ **These are used to estimate source parameters for earthquakes that have not yet occurred, or for which parameters of interest are unknown.**

Scaling relations

Figure 4.6-7: Empirical relations between slip, fault length, and moment.



- ➔ These relations do not provide reasoning behind magnitude saturation but they offer useful inferences about past and potential earthquakes.
- ➔ For example, 100 km long fault would have an average slip of 2m and M_w about 7.4.
- ➔ Whereas 10 km long fault will have 0.3 m slip and M_w about 6.2.

Stress drop

The relationship between the slip in an earthquake, its fault dimensions, and its seismic moment is closely tied to the magnitude of the stress released by the earthquake, or stress drop.

The earthquake releases the strain that has accumulated over time near the fault, so the radiated seismic waves are used to estimate the stress change.

To do this, let's assume earthquake's slip, D, occurs on a fault with characteristic dimension L, and so causes a strain change of approximately

$$\epsilon_{xx} = \frac{\partial u_x}{\partial x} \approx \frac{\bar{D}}{L},$$

so the stress drop averaged over the fault is approximately

$$\Delta\sigma \approx \mu\bar{D}/L.$$

Stress drop

The best-constrained quantity is the seismic moment, so we estimate the average slip, $\bar{D}$, from the seismic moment as

$$\bar{D} \approx cM_0/(\mu L^2),$$

"c" is a factor depending on the fault's shape.

Thus, stress drop is proportional to the moment and inversely proportional to the fault dimension cubed or the 3/2 power of the fault area:

$$\Delta\sigma = cM_0/L^3 = cM_0/S^{3/2}$$

The specific relation and values of c depend on the fault shape and the rupture direction.

Stress drop

For example, the stress drop on a circular fault with a radius R is

$$\Delta\sigma = \frac{7}{16}\frac{M_0}{R^3}$$

and strike-slip on a rectangular fault with length L and width w yields

$$\Delta\sigma = \frac{2}{\pi}\frac{M_0}{w^2 L},$$

and dip-slip on a rectangular fault gives

$$\Delta\sigma = \frac{4(\lambda+\mu)}{\pi(\lambda+2\mu)}\frac{M_0}{w^2 L} \approx \frac{8}{3\pi}\frac{M_0}{w^2 L}, \qquad \text{for } \lambda = \mu \text{ .}$$

These equations let us estimate the stress drop from an observed seismic moment and inferred fault dimensions.

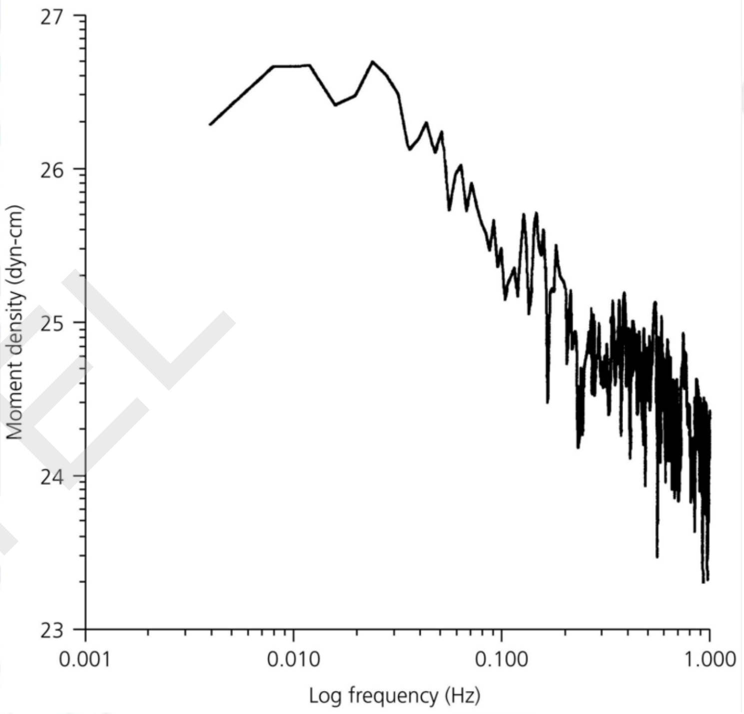
Stress drop

➔ To estimate the stress drop, one may use the spectrum to identify corner frequencies and estimate the rupture time and hence fault dimensions.

➔ M_w 7.1 earthquake occurring at 165 km depth in the subduction zone beneath Mexico.

➔ Analysis of the spectrum with a single corner frequency model and assuming a circular fault with rupture velocity of 3 km/s, yielded a rupture duration of 22 s and a stress drop of about 65 bars.

Figure 4.6-8: Sample amplitude spectrum, for the 1995 Chiapas, Mexico, earthquake.



Stress drop

- **In many cases the spectrum is not directly amenable to corner frequency analyses.**

- **The deep earthquake the spectrum of the direct P wave could be found without contamination from later-arriving surface reflections.**

Figure 4.6-9: Effect of free surface effects on theoretical source spectra.

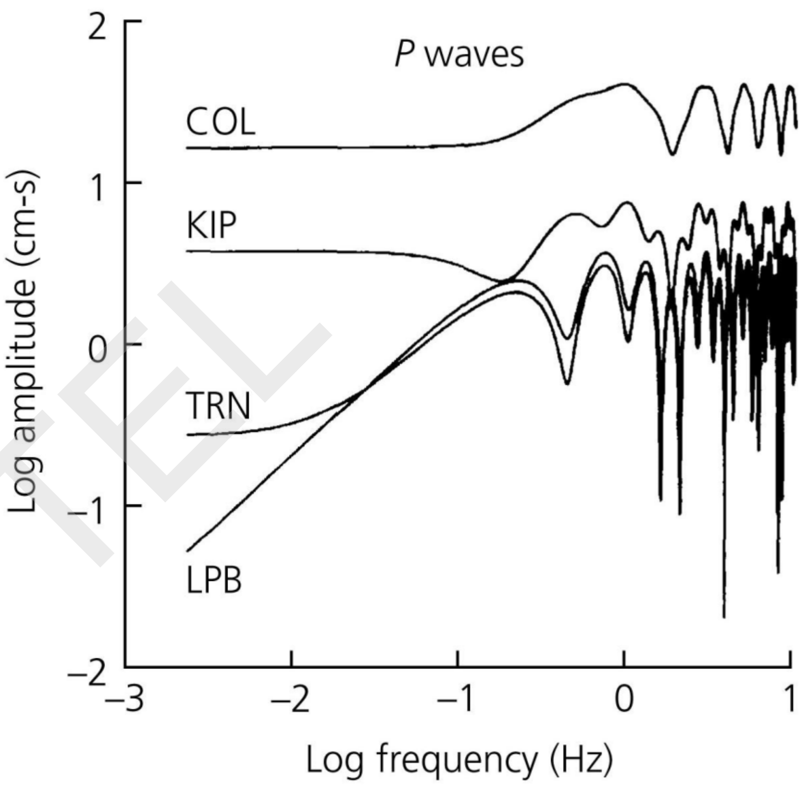
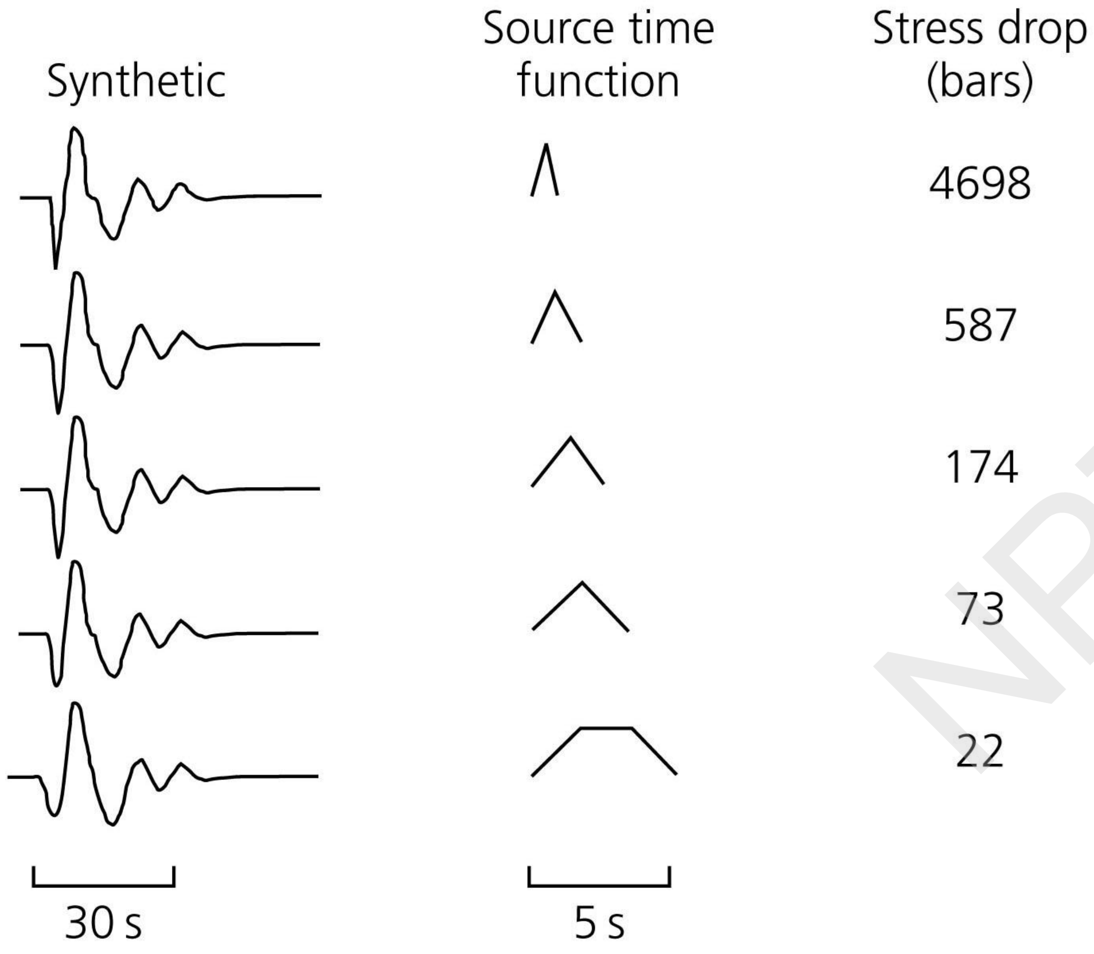


Figure 4.6-10: Effect of source time function and stress drop on synthetic seismograms.



- **As shown in the left figure, the seismogram depends only moderately on the source time function.**

- **However, small differences in time function duration correspond to larger differences in stress drop, even for an assumed rupture velocity and fault geometry.**

Stress drop Points to remember

- Compared to ridge earthquakes, transform earthquakes often have large M_s relative to m_b and large M_w relative to M_s, suggesting that seismic wave energy is relatively greater at longer periods.

- Earthquakes that preferentially radiate at longer periods are called "slow" earthquakes.

- Slow earthquakes underwater in the appropriate locations and focal geometry can cause very large tsunamis that are not predicted by tsunami warning systems based on real-time assessments of m_b or M_s.

Mandariga faulting model

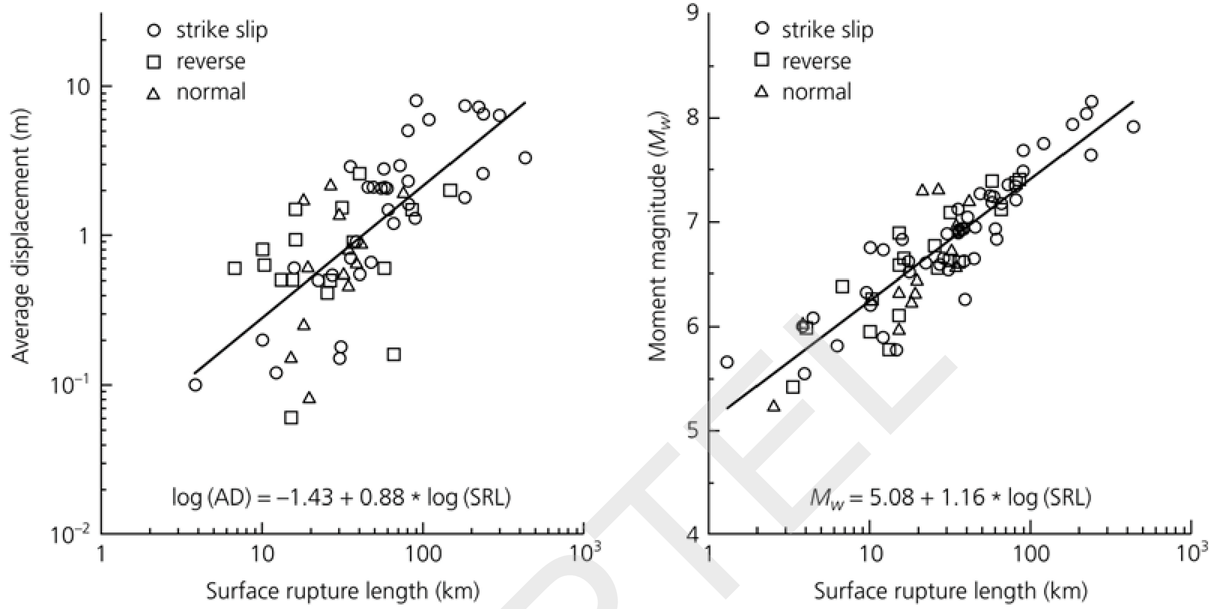
To estimate stress drop, we need a faulting model. Perhaps the most widely used in that of Mandariga (1976).

Assumptions: circular fault, and rupture velocity vr = 0.9b.

$$\Delta\sigma = \frac{7}{16}\left(\frac{f_c}{k\beta}\right)^3 M_0$$

where fc is the corner frequency, k = 0.32 for P-waves and 0.21 for S-waves, and b is the S-wave velocity rupture velocity vr = 0.9b.

Figure 4.6-7: Empirical relations between slip, fault length, and moment.



It is important to recognize that stress drop may only be a small portion of the total stress that the fault rock can withstand. Generally it's thought that an earthquake does not relieve all of the stress on a fault.

This means that when a rupture starts we don't know how big of an earthquake will occur.

Summary

- The relationship between the slip in an earthquake, its fault dimensions, and its seismic moment is closely tied to the magnitude of the stress released by the earthquake, or stress drop.

- The best-constrained quantity is the seismic moment, so we estimate the average slip, $\bar{D}$, from the seismic moment as

$$\bar{D} \approx cM_0/(\mu L^2),$$

"c" is a factor depending on the fault's shape.

- Stress drop is given as: $$\Delta\sigma = cM_0/L^3 = cM_0/S^{3/2}$$

- The stress drop on a circular fault with a radius R is $$\Delta\sigma = \frac{7}{16}\frac{M_0}{R^3}$$

- Stress drop on strike-slip on a rectangular fault with length L and width w yields

$$\Delta\sigma = \frac{2}{\pi}\frac{M_0}{w^2 L},$$

Summary

- Stress drop on dip-slip on a rectangular fault gives $$\Delta\sigma = \frac{4(\lambda+\mu)}{\pi(\lambda+2\mu)}\frac{M_0}{w^2 L}$$

- Small differences in time function duration correspond to larger differences in stress drop, even for an assumed rupture velocity and fault geometry.

- The standard deviation, or uncertainty, in the stress drop is thus approximately

$$\sigma_f^2 = \sigma_c^2\left(\frac{\partial f}{\partial c}\right)^2 + \sigma_{M_0}^2\left(\frac{\partial f}{\partial M_0}\right)^2 + \sigma_{v_R}^2\left(\frac{\partial f}{\partial v_R}\right)^2 + \sigma_{T_R}^2\left(\frac{\partial f}{\partial T_R}\right)^2$$

- Stress drop both characterizes earthquake source spectra and gives insight into the physics of faulting.

- The ratio of the slip to fault length is constant indicates that strain release in earthquakes is roughly constant, at about $\epsilon_{xx} \approx \bar{D}/L \approx \Delta\sigma/\mu \approx 10^{-4}$

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THANK YOU !