EARTHQUAKE SEISMOLOGY

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Module 11: Source parameters, Earthquake statistics.

Lecture 04: Frequency –magnitude relations

CONCEPTS COVERED

- Frequency –magnitude relations
  - Examples
  - Limitations
  - Magnitude invariance relation
- Summary

Recap

- The relationship between the slip in an earthquake, its fault dimensions, and its seismic moment is closely tied to the magnitude of the stress released by the earthquake, or stress drop.

- The best-constrained quantity is the seismic moment, so we estimate the average slip, $\bar{D}$, from the seismic moment as

$$\bar{D} \approx cM_0/(\mu L^2),$$

  "c" is a factor depending on the fault's shape.

- Stress drop is given as: $\Delta\sigma = cM_0/L^3 = cM_0/S^{3/2}$

- The stress drop on a circular fault with a radius R is $\Delta\sigma = \frac{7}{16}\frac{M_0}{R^3}$

- Stress drop on strike-slip on a rectangular fault with length L and width w yields

$$\Delta\sigma = \frac{2}{\pi}\frac{M_0}{w^2 L},$$

Recap

- Stress drop on dip-slip on a rectangular fault gives $\Delta\sigma = \dfrac{4(\lambda+\mu)}{\pi(\lambda+2\mu)} \dfrac{M_0}{w^2 L}$

- Small differences in time function duration correspond to larger differences in stress drop, even for an assumed rupture velocity and fault geometry.

- Stress drop both characterizes earthquake source spectra and gives insight into the physics of faulting.

- The ratio of the slip to fault length is constant indicates that strain release in earthquakes is roughly constant, at about $\epsilon_{xx} \approx \bar{D}/L \approx \Delta\sigma/\mu \approx 10^{-4}$

Frequency–magnitude relations

- The number of earthquakes that occur yearly around the world varies with magnitude, with successively smaller earthquakes being more common.

- This observation was quantified by Gutenberg and Richter in the 1940s via the logarithmic earthquake frequency–magnitude relation

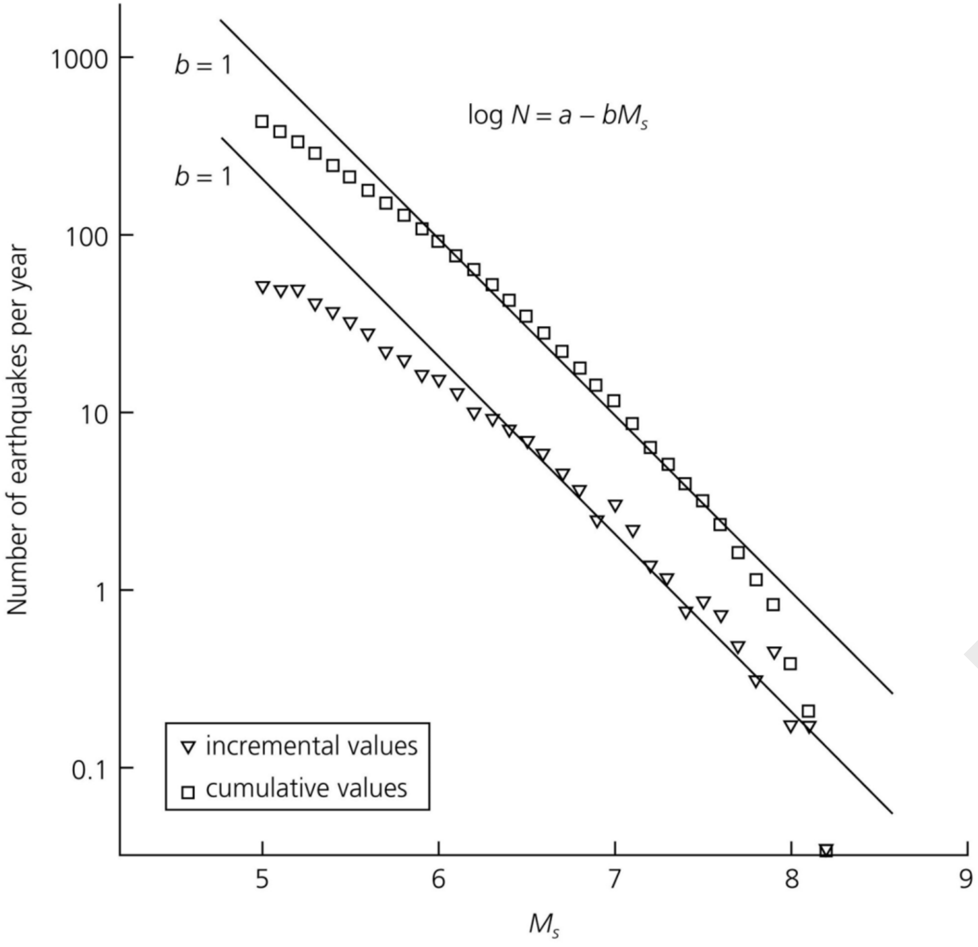
$$\log N = a_1 + bM$$

N is the number of earthquakes with magnitude greater than or equal to M occurring in a given time.

- The distribution is described by a linear relation, with constants a_1 and b.

- It turns out that although the intercept, a_1, depends on the number of earthquakes in the time and region sampled, the slope, b, is generally about 1.

Figure 4.7-1: Frequency-magnitude plot for earthquakes during 1968-1997.



Frequency–magnitude relations

- **There is an approximately tenfold increase in the number of earthquakes for successively smaller magnitudes.**

- **Annually around the world there are about one M_s = 8 earthquake, 10 M_s = 7 events, 100 M_s = 6 events, and so forth.**

- ➔ This relation also applies in individual seismic areas, with b generally about 1.
- ➔ The number of earthquakes depends on how seismically active an area is, the relative frequency (M > 6 earthquakes about 10 times more common than M > 7, etc.) still applies.
- ➔ For example, in the past 1300 years Japan is estimated to have had about 190 earthquakes with M > 7 and 20 with M > 8.
- ➔ Similarly, since 1816 southern California has had about 180 earthquakes with M > 6, 24 with M > 7, and 1 with M > 8; the New Madrid (central USA) seismic zone has had about 16 earthquakes with M > 5 and 2 with M > 6.

- **Such a pattern, called fractal scaling, self-similarity, or scale invariance, is common in nature.**

- **For instance, a coastline or river drainage pattern looks similar when viewed at scales of 1, 10, 100, or 1000 km.**

➔ The frequency–magnitude relation applies not only to the cumulative number N of earthquakes greater than a given magnitude, but to the incremental numbers n in a magnitude range M to M + dM.

$$N = 10^{a_1 - bM},$$

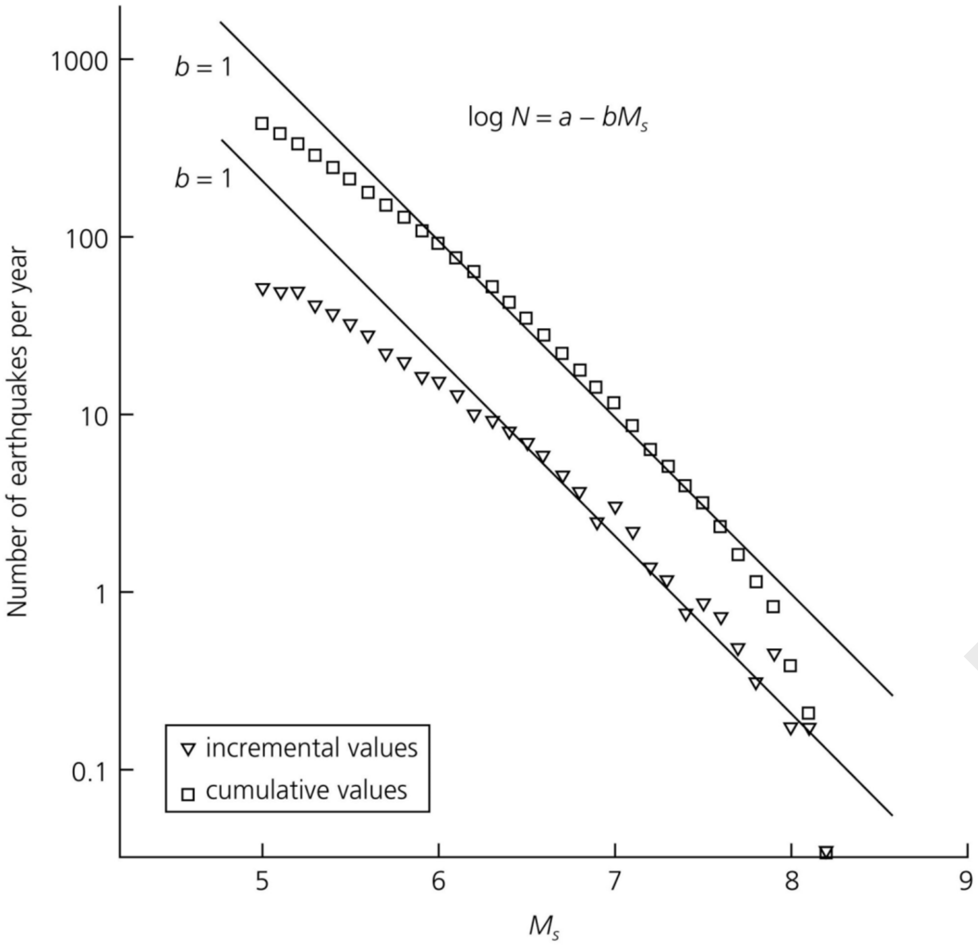
differentiate it with respect to M, and take the logarithm, so

$$\log\left(\frac{dN}{dM}\right) = \log n = a_2 - bM$$

where a_2 is a new constant.

Thus although the intercept ‘a’ changes, the slope b stays constant.

Figure 4.7-1: Frequency-magnitude plot for earthquakes during 1968-1997.



- ➔ **In the figure, show this effect, the fit is better for the cumulative number of earthquakes. Using more earthquakes by sampling longer intervals and/or larger areas produces better fits.**

- ➔ **Conversely, the shorter the time or the smaller the area, the more the fit is degraded by the statistics of small numbers, as discussed shortly.**

- Smaller earthquakes sometimes get unrecorded so the global earthquake catalog is incomplete. So, the fit deviates from the b=1 line for very small (M_s< 3) magnitudes.

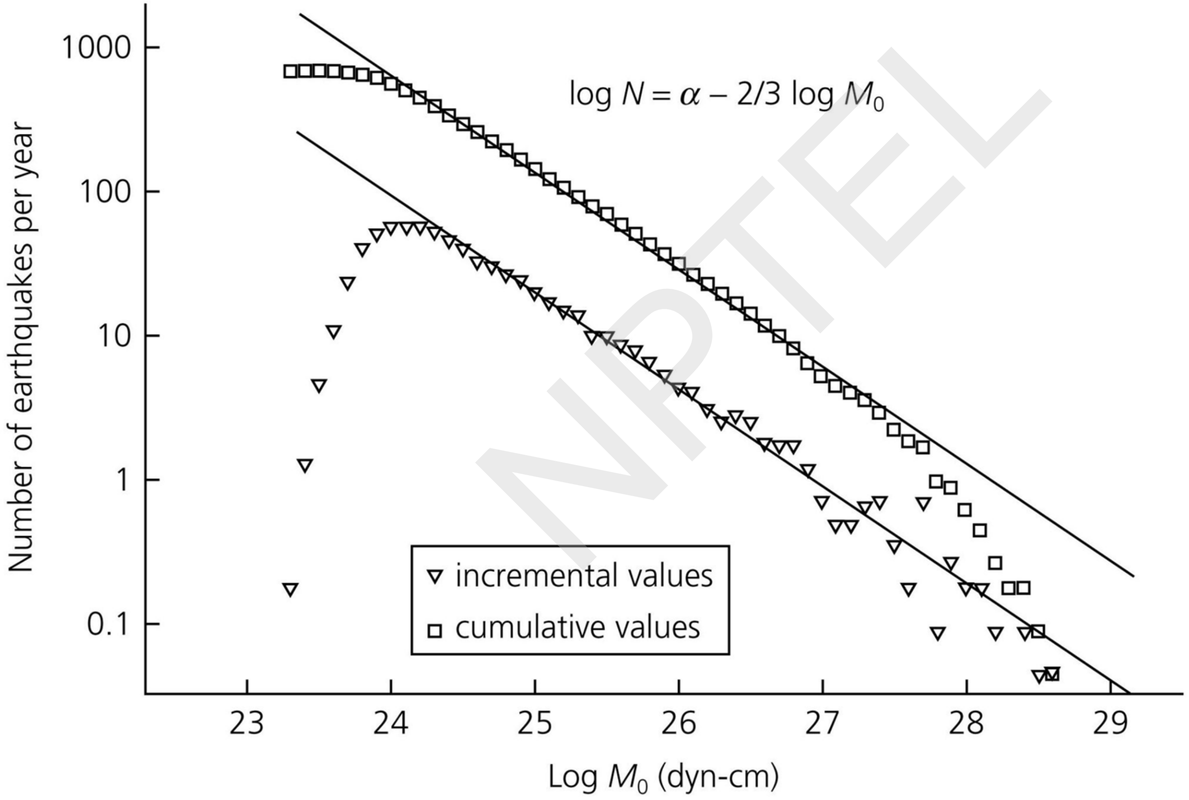
- The deviation for large (M_s>7.5) earthquakes is expected, because the surface wave magnitude saturates.

- ➔ **To address this issue we can use the seismic moment, which better indicates the size of large earthquakes.**

$$\log N = a_1 - b(\log M_0/1.5 - 10.73) = \alpha - \beta \log M_0$$

with slope β = b/1.5 ≈ 2/3

Figure 4.7-2: Frequency-moment plot for earthquakes during 1976-1998.



- **The deviation for small earthquakes is likely in part to be due to the incomplete earthquake catalog, but is also expected from energy considerations**

- **However, the deviation at large moments is more puzzling, since moments do not saturate.**

- **For moments above 10^{27} dyn-cm, the data are more consistent with $\beta \geq 1$ than $\beta = 2/3$.**

- **In other words, there are fewer earthquakes than expected for a given moment.**

Frequency –magnitude relations : Magnitude invariance

- A model for this phenomenon based on the concept of scale invariance assumes that the probability of an earthquake of a given size on a fault is inversely proportional to the area of faulting involved, so the number N of earthquakes with fault area greater than S should obey a frequency–area relation like those for magnitude or moment

$$\log N = c - \log S$$

We know that for constant stress drop the moment is proportional to $S^{3/2}$, or the fault dimension L cubed, so we expect

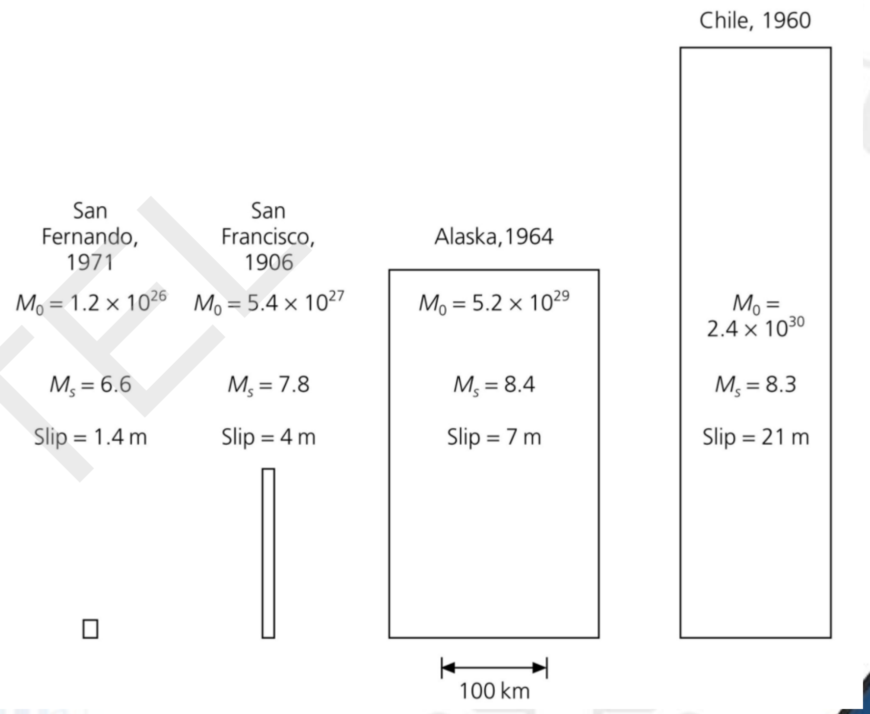
$$\log N = c - 2/3 \log M_0$$

Frequency –magnitude relations : Magnitude invariance

- **However, we have seen that for large transform fault earthquakes, which occur on vertical faults, the width (down-dip extent) stays narrow even as fault length increases**

- **As a result, the seismic moment for such earthquakes is no longer proportional to L^3, and is smaller than for other earthquakes of comparable fault length.**

Figure 4.6-3: Comparison of the magnitudes of four earthquakes.

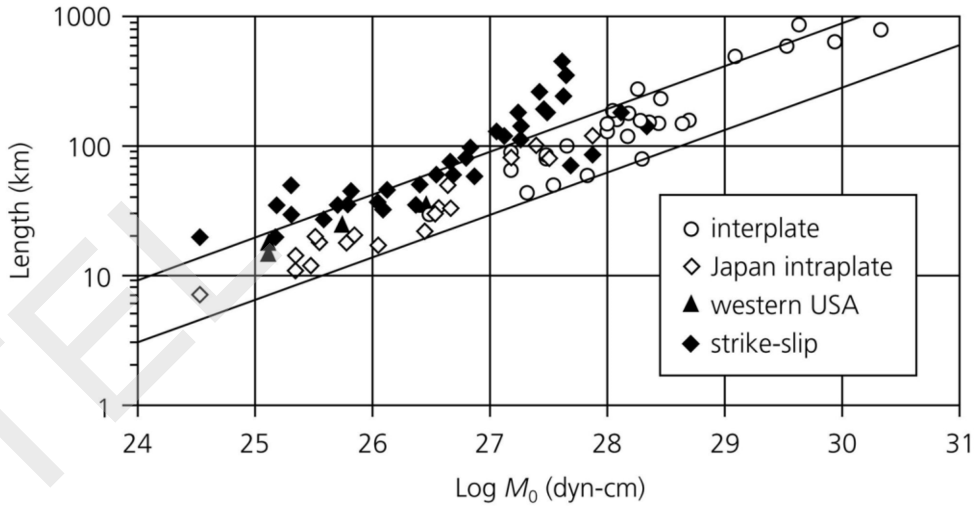


Frequency –magnitude relations : Magnitude invariance

- **If both the fault slip and the fault width no longer increase with length, then the fault area, moment, and number of earthquakes should be proportional to L.**

- **The frequency–moment data give insight into earthquake energy release because the radiated energy is proportional to the seismic moment**

Figure 4.7-3: Relationship between seismic moment and fault length for different earthquake types.



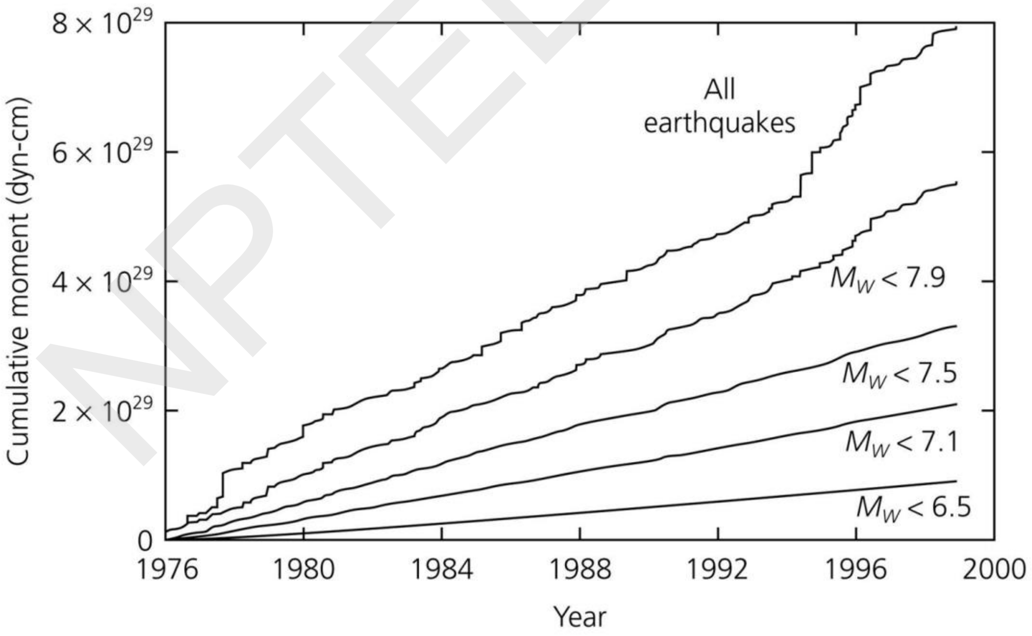
Frequency –magnitude relations : Energy

- The few largest earthquakes release much more energy than the many smaller earthquakes.
- In fact, the largest earthquake in a given year often releases more energy than the rest of the year's earthquakes.

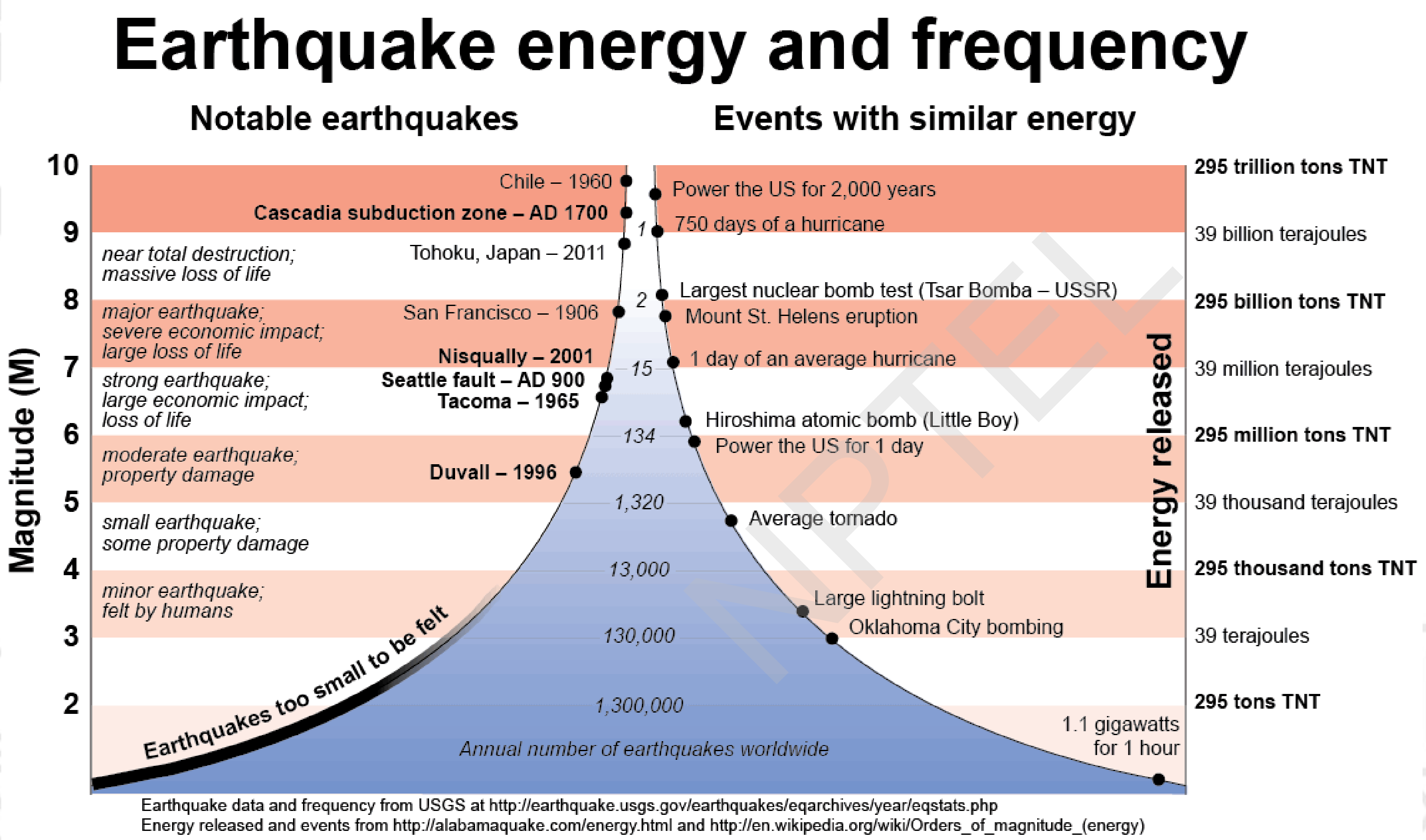
Earthquake magnitude (M_s)	Number per year	Energy released (10^{15} J/yr)
≥8.0	0–1	0–1,000
7–7.9	12	100
6–6.9	110	30
5–5.9	1,400	5
4–4.9	13,500	1
3–3.9	>100,000	0.2

Based upon data from the US Geological Survey National Earthquake Information Center. Energy estimates are based upon an empirical formula of Gutenberg and Richter (Gutenberg, 1959), and the magnitude scaling relations of Geller (1976), and are very approximate.

Figure 4.7-4: Cumulative seismic moment for earthquakes 1976-1998.



Frequency –magnitude relations : **Example**



Summary

- Frequency–magnitude relation $\log N = a_1 + bM$

  N is the number of earthquakes with magnitude greater than or equal to M occurring in a given time.

- The data deviate from the b = 1 line for very small (M_s < 3) magnitudes, because the global earthquake catalog is incomplete, with many small earthquakes not detected.

- Modified Frequency–magnitude relation $\log N = a_1 - b(\log M_0/1.5 - 10.73) = \alpha - \beta \log M_0$

  with slope β = b/1.5 ≈ 2/3

- For moments above 10^{27} dyn-cm, the data are more consistent with β ≥ 1 than β = 2/3.

- Number N of earthquakes with fault area greater than S should obey a frequency–area relation like those for magnitude or moment

$$\log N = c - \log S$$

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THANK YOU !