NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 12 :Numerical Problems in Seismology

Lecture 04: Numerical Problems - Part I

CONCEPTS COVERED

- **Numerical Problems - Part-I**

Problem 1. Calculate the stress and strain tensors for these two cases where monochromatic (i.e., a single frequency) plane waves are traveling in the x direction in a medium with homogeneous and isotropic properties. The two cases are:

(a) P-wave: $u(x) = A \sin(\omega t - kx)$ {thus displacements in the x-direction}
(b) S-wave: $u(y) = A \sin(\omega t - kx)$ {thus displacements in the y-direction}

(a) we have $u_x = A\sin(\omega t - kx)$

$u_y = 0$

$u_z = 0$

Strain Tensor

$$e_{ij} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) & \frac{\partial u_y}{\partial y} & \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right) & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

$u_y = u_z = 0$ & u_x is only a f^n of x so,

$$\frac{\partial u_x}{\partial y} = \frac{\partial u_x}{\partial z} = 0$$

we need to calculate $\frac{\partial u_x}{\partial x} = \frac{\partial}{\partial x}(A \sin(\omega t - kx))$

$$\frac{\partial u_x}{\partial x} = -kA \cos(\omega t - kx)$$

so, $e_{11} = -kA\cos(\omega t - kx)$ & all other element are '0'.

To get stress.

$$\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu e_{ij}$$

$$\theta = \frac{\partial u_x}{\partial x} + \underbrace{\frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}}_{0} = -kA\cos(\omega t - kx)$$

[redacted]

(b) S-wave $u(y) = A\sin(\omega t - kx)$

$$u_x = u_z = 0$$

So, $\dfrac{\partial u_y}{\partial x} = \dfrac{\partial}{\partial x}\left[A\sin(\omega t - kx)\right] = -kA\cos(\omega t - kx)$

All other derivatives are '0'.

$$e_{ij} = \begin{bmatrix} 0 & \frac{1}{2}(-kA\cos(\omega t - kx)) & 0 \\ \frac{1}{2}(-kA\cos(\omega t - kx)) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

To get stress, $\therefore \theta = 0$

$$\sigma = -\frac{1}{2} k A \cos(\omega t - kx) \begin{bmatrix} 0 & 2\mu & 0 \\ 2\mu & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Problem 2. Give me a specific example of a deformation function, u(x), where the total dilatation is 5 for all values of x.

The dilation is $\theta = e_{ij} = \underbrace{\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}} = \nabla \cdot \vec{u}$

So, we just need to come up with a function where the derivatives add upto five. There are many option.

Here's one.

$$u(x) = 5x_1$$

$$\theta = e_{ij} = \frac{\partial}{\partial x}(5x_1) + \frac{\partial}{\partial x_2}(5x_1) + \frac{\partial}{\partial x_3}(5x_1)$$

$$= 5 + 0 + 0 = 5$$

Problem 3. Two guitar strings are attached and pulled taught with a tension of 50N. String 1 has a density of 3.5×10^{-3} kg/m. String 2 has a density of 5×10^{-4} kg/m. A wave with initial amplitude 0.1 mm, and a wavelength of 5 cm propagates down string 1 and interacts with string 2.

a) What is the amplitude of the transmitted wave in string 2?
b) What is the amplitude of the reflected wave in string 1?
c) What is the sum of reflection and transmission coefficients?
d) What is the wavelength of the transmitted wave?
e) What is the energy flux in string 1, before any energy enters string 2?
f) What is the energy flux for the reflected and transmitted waves? Don’t worry about the direction of the energy travel (i.e. assume both the reflected and transmitted energy flux has the same sign).

(a) first calculate v_1 & v_2

$$v_1 = \left(\frac{T}{\rho}\right)^{1/2} = \left(\frac{50\times10^{3}}{3.5\times10^{-3}}\right)^{1/2} = 119.5 \text{ m/s}$$

$$v_2 = \left(\frac{T}{\rho}\right)^{1/2} = \left(\frac{50\times10^{3}}{5\times10^{-3}}\right)^{1/2} = 316.2 \text{ m/s}$$

$$I_1 = \rho_1 v_1 = 0.4183 \text{ kg/m}^3 \cdot \frac{\text{m}}{\text{s}} \qquad I_2 = \rho_2 v_2 = 0.1581 \frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{m}}{\text{s}}$$

$$T = \frac{2I_2}{I_1 + I_2} = 1.4514$$

$$\text{Amplitude} = (T)(1\times10^{-4}) = 1.4514\times10^{-4} \text{ m}$$

(b)

$$R = \frac{I_1 - I_2}{I_1 + I_2} = 0.4514 \qquad \text{Amplitude} = (1\times10^{-4})(R)$$

$$= 4.514\times10^{-5} \text{ m}$$

(c) $R + T = 1.9028$

(d) $$v_1 \frac{2\pi}{d_1} = v_2 \frac{2\pi}{d_2}$$

$$\Rightarrow d_2 = \frac{v_2\, 2\pi d_1}{v_1\, 2\pi} = \frac{v_2}{v_2} d_1 = 0.1323\, m$$

(e) $$\dot{E} = \frac{A^2 \omega^2 \rho_1 v_1}{2} = \frac{(0.1 \times 10^{-3})^2 (2\pi\nu)(\rho_1)(v_2)}{L}$$

$$= 0.4719 \quad s^{-2}\, \frac{kg}{m}\, \frac{m}{s} = \frac{J}{m \cdot s}$$

$$\dot{E}_R = \frac{(A\cdot R)^2 \omega^2 \rho_1 v_1}{2} = 0.0962 \ \frac{J}{m\cdot s}$$

$$\dot{E}_T = \frac{(A\cdot T)^2 \omega^2 \rho_2 v_2}{2} = 0.3757 \ \frac{J}{m\cdot s}$$

Problem 4. Assume a harmonic P-wave is traveling through a solid with velocity α = 10 km/sec. If the maximum strain is 10^{-8}, what is the maximum particle displacement for waves with periods of: (a) 1 s, (b) 10 s, (c) 100 s? Assume the P-wave is moving in the x-direction, so $\vec{u} = \left[Ae^{i(\omega t - kx)}, 0, 0\right]$

Lets assume the P-wave is moving in the x-direction.

Then, $u_x = Ae^{i(\omega t - kx)}$, $u_y = u_z = 0$. We are given wave periods of $T = 1, 10$ & 100 s.

$$e_{ij} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) & \frac{\partial u_y}{\partial y} & \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right) & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

The only term here that is non-zero is

$$\frac{\partial u_x}{\partial x} = \frac{\partial}{\partial x} A e^{i(\omega t - kx)} = -i\,kA e^{i(\omega t - kx)}$$

Note that the strain is maximized when $(\omega t - kx) = \frac{\pi}{2}$

then $-ikA e^{i\pi} = kA$

so; $kA = 10^{-8}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\alpha T} = \frac{2\pi}{[10, 100, 1000]}$$

$$A = \frac{10^{-8}}{k} = 1.59 \times [10^{-6}, 10^{-7}, 10^{-8}]\ \text{km}$$

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THANK YOU !