NPTEL ONLINE CERTIFICATION COURSES

EARTHQUAKE SEISMOLOGY

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Module 12 :Numerical Problems in Seismology

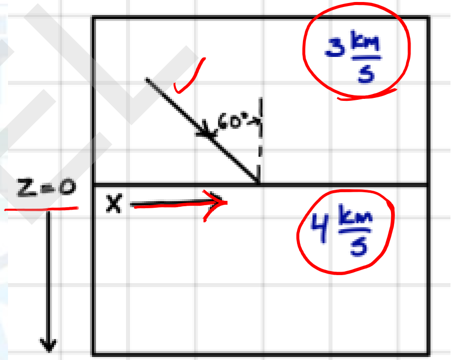
Lecture 05: Numerical Problems - Part II

CONCEPTS COVERED

- **Numerical Problems - Part-II**

Problem 1. Consider a SH wave coming in at an angle of 60 degrees, and some energy travels as a head wave along the interface. At what depth is the amplitude of the head wave 1/e of that at z=0?

a) for a 1 Hz wave?
b) for a .1 Hz wave?



The z-component of the wave is

$$-k_x\left(1-\frac{c_x^2}{\beta_2^2}\right)^{1/2}$$

$$p = \frac{\sin 60}{v_1} = 0.2887$$

$$\eta = \frac{\cos 60}{v_1} = 0.1667$$

1 Hz: $\omega = 2\pi\nu = 2\pi$

$$k_x = \omega p = 1.8138 \text{ km}^{-1}$$

$$k_z = \omega\eta = 1.0472 \text{ km}^{-1}$$

0.1 Hz

$$\omega = 2\pi\nu = 0.2\pi$$

$$k_x = \omega p = 0.1814 \text{ km}^{-1}$$

$$k_z = \omega\eta = 0.1047 \text{ km}^{-1}$$

$$c_x = \frac{1}{p} = 3.4641$$

Amplitude will be at $\frac{1}{e}$ when $-k_x\left(1 - \frac{c_x^2}{\beta_2^2}\right)^{1/2} z = -1$

$$z = 1.1 \text{ km for } 1 \text{ Hz}$$

$$= 11.0 \text{ km for } 0.1 \text{ Hz}$$

Problem 2. For a Rayleigh wave, calculate the depth at which $u_x = 0$ as a function of λ ($k_x = 2\pi/\lambda$).

$$u_x = A k_x \left[\exp(-0.85 k_x z) - 0.58 \exp(-0.39 k_x z)\right] = 0$$

for this to hold at all t : $e^{-0.85 k_x z} = 0.58\, e^{-0.39 k_x z}$

$$-0.85 k_x z = \ln(0.58) + (-0.39 k_x z)$$

$$z = \frac{\ln(0.58)}{-0.46 k_x} = \frac{\ln(0.58)\, \lambda_x}{2\pi(-0.46)}$$

$$\boxed{z = 0.1885 \lambda_x}$$

Problem 3. Show how the moment tensor for a vertical dipole can be decomposed into an isotropic source and a CLVD.

An isotropic source has the same eigen values, which means the same values along it's diagonal.

A CLVD source has one dipole that is -2 times the magnitude of the others.

$$M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_{22} \end{bmatrix} = \begin{bmatrix} M_{iso}/3 & 0 & 0 \\ 0 & M_{iso}/3 & 0 \\ 0 & 0 & M_{iso}/3 \end{bmatrix} + \begin{bmatrix} M_c & 0 & 0 \\ 0 & M_c & 0 \\ 0 & 0 & -2M_c \end{bmatrix}$$

Now we have a system of equations to solve

$$\frac{M_{iso}}{3} + M_c = 0 \Rightarrow M_c = -\frac{1}{3} M_{iso}$$

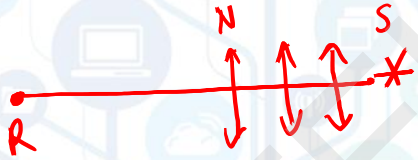
$$\frac{M_{iso}}{3} - 2M_c = M_{22}$$

$$\frac{M_{iso}}{3} - 2\left(-\frac{1}{3} M_{iso}\right) = M_{22}$$

$$M_{iso} = M_{22}$$

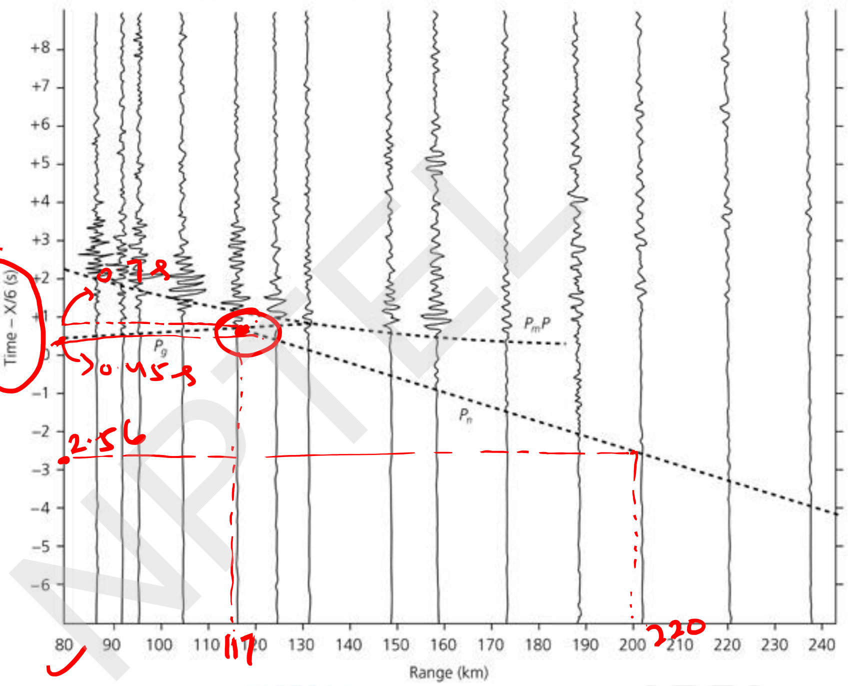
$$M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_{22} \end{bmatrix} = \begin{bmatrix} M_{22}/3 & 0 & 0 \\ 0 & \frac{M_{22}}{3} & 0 \\ 0 & 0 & \frac{M_{22}}{3} \end{bmatrix} + \begin{bmatrix} -M_{22}/3 & 0 & 0 \\ 0 & -\frac{M_{22}}{3} & 0 \\ 0 & 0 & \frac{2M_{22}}{3} \end{bmatrix}$$

Problem 4. A Love wave has a backazimuth of 270. On what channel of the seismometer would the energy be on (E,N,Z)?



Problem 5. Use the data from the refraction experiment in the figure to find the crust and mantle velocities and the crustal thickness. Remember that this is a reduced travel time plot.

Figure 3.2-5: Example of seismograms from a refraction profile.



1) For crustal velocity, use P_g.

At $x = 117$, $t_{red} = 0.78$

At $x = 80$, $t_{red} = 0.458$

$t_{red} = t - \frac{x}{6} \Rightarrow t = t_{red} + \frac{x}{6}$

$t_1 = 0.45 + \frac{80}{6} = 13.7833$ s ✓

$t_2 = 0.7 + \frac{117}{6} = 20.2000$ s ✓

Slope $= \frac{t_2 - t_1}{x_2 - x_1} = 0.2292$ s/Km

crustal vel. $= \frac{1}{\text{slope}} = 4.36$ Km/s

For mantle, use P_n

At $x = 117$ $\quad t_{red} = 0.7$

At $x = 220$ $\quad t_{red} = -2.55$

$$t_1 = 0.7 + \frac{117}{6} = 20.200\ \text{s}$$

$$t_2 = -2.55 + \frac{220}{6} = 34.1167\ \text{s}$$

slope = 0.1357 s/km $\Rightarrow$ $v_{P_n} = 7.4$ km/s

For crustal thickness.

$$x_d = 2h_0\left(\frac{v_1+v_0}{v_1-v_0}\right)^{1/2}$$

$x_d = 117$ km

$h_0 = 29.7$ km

Problem 6. Stein & Wysession, Page 118, Problem 33. Find the displacements for ${}_0T_3^2$ as functions of θ and ϕ in the manner done for ${}_0T_0^2$ in Eqns 2.9.12 and 2.9.13.

Note that modes are typically referred to in this format: ${}_nT_l^m$

so here $n=0$, $l=3$, $m=0$

$T(r, \theta, \phi)$ is

$$T_l^m = \left(0, \frac{1}{\sin\theta}\frac{\partial Y_l^m(\theta,\phi)}{\partial\phi}, -\frac{\partial Y_l^m(\theta,\phi)}{\partial\theta}\right)$$

$$Y_l^m(\theta,\phi) = (-1)^m\left(\left(\frac{2l+1}{4\pi}\right)\frac{(l-m)!}{(l+m)!}\right)^{1/2} P_l^m(\cos\theta)e^{imy}$$

so for $m=0$ & $l=3$

$$Y_3^0(\theta,\phi) = (-1)^0\left[\left(\frac{7}{4\pi}\right)\frac{3!}{3!}\right]^{1/2} P_3^0(\cos\theta)\, e^{i0\phi}$$

$$Y_3^0(\theta,\phi) = \sqrt{\frac{7}{4\pi}}\, P_3^0(\cos\theta)$$

P_3^0

$$u_\theta \propto \frac{1}{\sin\theta} P_3^0(\cos\theta)(im)\, e^{im}$$

$m=0 \qquad u_\theta = 0$

$$P_l^m(x) = \left[\frac{(1-x^2)^{m/2}}{2^l\, l!}\right]\left[\frac{d^{l+m}}{dx^{l+m}}(x^2-1)^l\right]$$

$$P_3^0(x) = \left[\frac{(1-x^2)^0}{2^3\, 3!}\right]\left[\frac{d^3}{dx^3}(x^2-1)^3\right]$$

$$= \frac{1}{2}(5x^3 - 2x)$$

substitute cosθ in for x

$$P_3^0(\cos\theta) = \frac{1}{2}\left[5(\cos\theta)^3 - 3\cos\theta\right]$$

$$u_\phi = -\frac{\partial}{\partial\theta} Y_0^3 \checkmark$$

$$Y_3^0 = \sqrt{\frac{7}{4\pi}}\, P_3^0(\cos\theta) = \sqrt{\frac{7}{4\pi}}\, \frac{1}{2}\left[5(\cos\theta)^3 - 3\cos\theta\right]$$

$$u_\phi = -\frac{\partial}{\partial\theta}\sqrt{\frac{7}{16\pi}}\left[5(\cos\theta)^3 - 3\cos\theta\right]$$

$$= -\sqrt{\frac{7}{16\pi}}\left[15\cos^2\theta\,\sin\theta - 3\cos\theta\right]$$

$$T_l^m = \left(0,\ 0,\ -\sqrt{\frac{7}{16\pi}}\left[15\cos^2\theta\sin\theta - 3\cos\theta\right]\right)$$

$\Rightarrow$

REFERENCES

- Stein, Seth, and Michael Wysession. An introduction to seismology, earthquakes, and earth structure. John Wiley & Sons, 2009.
- Lowrie, William, and Andreas Fichtner. Fundamentals of geophysics. Cambridge university press, 2020.
- Kearey, Philip, Michael Brooks, and Ian Hill. An introduction to geophysical exploration. Vol. 4. John Wiley & Sons, 2002.
- https://geologyscience.com/geology-branches/structural-geology/stress-and-strain/
- Seismology course, Professor Derek Schutt, Colorado State Univ., USA.

THANK YOU !